Magnetohydrodynamic Boundary Layer Flow of Power-Law Fluids near a Suddenly Accelerated Flat Plate

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Abstract

The magnetohydrodynamic (MHD) flow near a flat plate suddenly set in motion has been studied for a particular class of non-Newtonian power-law fluids. Two different groups of transformations, namely linear group of transformation and a spiral group of transformation are used to derive similarity solutions of the present flow equations. The similarity solution is found in most general form. It is observed that controlled equations are in agreement to those standard equations found in literature.

Keywords: Linear Group of Transformation, Magnetohydrodynamic (MHD), Non-Newtonian Power-Law Fluid, Similarity Solution, Spiral Group of Transformation

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1 Introduction

Differential models either linear or non-linear often describe physical problems in engineering and science. There is also an abundance of transformations of various types that appear in the literature are generally aimed at obtaining some sort of simplification of a differential model. Academic curiosity and practical applications have demands to find the solution of differential equations governing the motion of non-Newtonian fluids. The property of these fluids is that the stress strain relationship is non-linear. In literature it is assumed that the non-Newtonian behavior can be described by Power-Law model. This assumption probably made because the power-law of fluid model is mathematically simpler to describe non-Newtonian behavior, and hence it poses the minimum mathematical difficulties in carrying out necessary theoretical and experimental studies.

Magnetohydrodynamic is the study of the motion of an electrically conducting fluid in the presence of a magnetic field. Due to the motion of an electrically conducting fluid in a magnetic field the electrical currents are induced in the fluid, which produces their own magnetic field, called induced magnetic field, and these modify the original magnetic field. In addition to this, the induced currents interact with the magnetic field to produce electromagnetic forces perturbing the original motion. Thus basic effects of magnetohydrodynamics are, the motion of fluid affects the magnetic field and the vice-versa. Teipel [8] studied the problem of impulsive motion of flat plate in a viscoelastic field. Following Teipel [8], Choubey [4] analyzed the hydromagnetic flow of an electrically conducting Rivlin-Ericksen liquid near an infinite horizontal flat plate started impulsively from rest in its own plane with constant velocity and is subjected to an applied uniform transverse magnetic field. In the present paper a theoretical study is made of the laminar incompressible magnetohydrodynamic boundary layer flow of non-Newtonian power-law fluids near a suddenly accelerated flat plate. We use group-theoretic method to derive the similarity solutions for the flow situation under consideration.

2 Problem Formulations

The MHD boundary layer flow of non-Newtonian Power-law fluids near a suddenly accelerated flat plate set in motion prescribed that a velocity component ‘u’ along the surface of the flat plate is a function of time and of the y-coordinate perpendicular to the plate, where x-coordinate is parallel to the plate. The constant impulsive velocity ‘U’ is given to the plate in its own plane and a uniform transverse magnetic field $B_x(y,t)$ be applied transversely to the plate. The fluid is assumed to be of low conductivity such that, the induced magnetic field is
negligible and the Lorenz force is \(-\sigma B_x^2(y,t)u\). Hence the flow field is governed by the equation,

\[
\frac{\partial u}{\partial t} = \nu \frac{\partial}{\partial y} \left( \frac{\partial u}{\partial y} + \frac{\partial u}{\partial y} \right) - S(y,t) u \tag{1}
\]

where \(S(y,t) = \frac{g \sigma B^2_x(y,t)}{\rho}\) is a magnetic strength,

with the boundary conditions,
\(y = 0: u = U(y, t)\)
\(y = \infty: u = 0\)

where \(y\) is a co-ordinate perpendicular to the plate.

### 3 Similarity Solution

The theory of group transformation was first introduced and developed extensively by Sophus Lie. In recent years, there has been a revival of interest in applying the principles of continuous group of transformation to the differential models, either linear or nonlinear. Birkhoff [3] proposed a method based upon simple group of transformations for obtaining invariant solutions for some problems in the general area of hydrodynamics. The method essentially involves algebraic manipulations, an aspect which makes the method attractive. Group-theoretic methods are powerful tool because they are not based on linear operators, superposition or any other aspects of linear solution techniques. Therefore, these methods are applicable to nonlinear differential models. Group-theoretic methods are powerful, versatile and fundamental to the development of systematic procedures that lead to invariant solution of boundary value problems. In dealing with differential boundary value problems in engineering and applied science however, physical aspects associated with the problems are of importance and need to be properly addressed. Consideration of boundary and initial conditions as an integral part of the mathematical description becomes an essential part of any group-theoretic analysis.

The invocation of invariance of a partial differential system under a group of transformations would lead to reduction in the number of independent variables. The concept of invariance plays a key role in the mathematical formulation of the group-theoretic procedures. The main advantage of using group-theoretic procedures is that they are systematic. Invariance can be invoked either by using an “assumed” group or by starting out with general groups and then applying
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deductive procedures. In the next section we introduce two types of one parameter group transformations namely linear group of transformation and spiral group of transformation to obtain similarity solution of governing flow situation.

Case-I:

A one-parameter group of transformation is chosen in the form,

\[ t = A_{1}^{\alpha_{1}} \bar{t}, \quad y = A_{2}^{\alpha_{2}} \bar{y}, \quad u = A_{3}^{\alpha_{3}} \bar{u}, \quad U(y,t) = A_{4}^{\alpha_{4}} \bar{U}(y,t), \quad S(y,t) = A_{5}^{\alpha_{5}} \bar{S}(y,t) \]  

(3)

where \( \alpha_{1}, \alpha_{2}, \alpha_{3}, \alpha_{4}, \alpha_{5} \) and \( A \) are constants.

Applying the above group transformation to “Eq. (1)”, we obtain

\[
A_{1}^{\alpha_{1}} \frac{\partial \bar{u}}{\partial \bar{t}} = A_{2}^{\alpha_{2}} \alpha_{3} \nu \frac{\partial \bar{u}}{\partial \bar{y}} \left( \frac{\partial \bar{u}}{\partial \bar{y}} \right)^{\alpha_{1}-1} \frac{\partial \bar{u}}{\partial \bar{y}} - \frac{\partial \bar{S}}{\partial \bar{y}} \bar{u} \]

(4)

For “Eq. (4)” to be invariant under the group transformation (3), we must have

\[ \alpha_{3} - \alpha_{1} = n \quad \alpha_{3} - (n + 1) \alpha_{2} = \alpha_{5} + \alpha_{3} \]

Divide throughout by \( \alpha_{s} \) and let \( m = \frac{a_{3}}{\alpha_{1}}, \quad b = \frac{a_{5}}{\alpha_{1}} \), and \( \beta = \frac{a_{3}}{\alpha_{1}} = -1 \) then,

\[ b = \frac{1 + (n-1) m}{(n+1)} \]

(5)

The “absolute invariants” under the above group of transformation are

\[ \eta = \frac{y}{t^{\beta}}; \quad F(\eta) = \frac{u}{t^{\eta m}}; \quad U = C t^{m}; \quad S = \frac{S_{0}}{t} \]

(6)

Substituting these invariants in the basic equation (1), the reduced ordinary differential equation is,

\[
\nu \frac{d}{d\eta} \left( \frac{F'(\eta)^{n-1}}{F'(\eta)} \right) + \frac{[1 + (n-1) m]}{(n+1)} \eta F'(\eta) - m F(\eta) - S_{0} F(\eta) = 0 \]

(7)

with the transformed boundary conditions,

\[ A \quad \eta = 0 : F(0) = C \]

\[ \eta = \infty : F(\infty) = 0 \quad \text{which are independent of } t. \]
Deductions:

- For \( n = 1 \) and \( m = 0 \), “Eq. (7)” reduces to Stoke’s solution & Blasius first case [7, 2].
- For \( n = 1 \) and \( m = 1 \), “Eq. (7)” reduces to Blasius second case [2].
- For \( n = 0 \) and \( m = 1 \), “Eq. (7)” reduces to Watson’s first case [9].
- For \( m = 1 \) and \( n \)-arbitrary, “Eq. (7)” reduces to solution of Bird and Wells [1, 10].
- For the non-magnetic case \( S_0 = 0 \) and \( n = 1 \) “Eq. (7)” reduces to an equation which is well agreed with the equation found in the literature [6].

Case-II:

A one-parameter spiral group of transformation is chosen in the form

\[
t = t + \beta_1 b, \quad y = e^{\beta_2 b} y, \quad u = e^{\beta_3 b} u, \quad U(y,t) = e^{\beta_4 b} U, \quad S(y,t) = e^{\beta_5 b} S
\]

where \( \beta_1, \beta_2, \beta_3, \beta_4, \beta_5 \) and \( b \) are constants.

Substituting these quantities in the basic “Eq. (1)” we obtain,

\[
\left( e^{\beta b} \right) \frac{\partial u}{\partial t} = \nu e^{\beta y} \left( \frac{1}{\beta_3 b} \frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y} \right) - e^{(\beta_3 + \beta_4) b} S \frac{\partial u}{\partial y} \tag{10}
\]

For “Eq. (10)” to be invariant under a spiral group of transformation (9), we must have,

\[
\beta_3 = \frac{(n+1)}{(n-1)} \beta_2 \quad \text{(for } n \neq 1) \quad \beta_4 = \beta_3 \quad \text{and} \quad \beta_5 = 0
\]

We therefore obtain the transformed independent and dependent variables as,

\[
\eta = \frac{y}{e^{pt}}, \quad F(\eta) = \frac{u}{e^{\left(\frac{(n+1)}{(n-1)}\right)pt}}, \quad U(t) = C e^{\left(\frac{(n+1)}{(n-1)}\right)pt} \quad \text{where} \quad p = \frac{\beta_2}{\beta_1} \quad \text{and}
\]

\[
S=S_0 \tag{12}
\]

Substituting these quantities in the basic equation (1) we obtain,

\[
\nu \frac{d}{d\eta} \left\{ F(\eta) \frac{dF(\eta)}{d\eta} \right\} + p \eta F(\eta) - \left[\frac{(n+1)}{(n-1)}\right] p F(\eta) - S_0 F(\eta) = 0 \tag{13}
\]

with the transformed boundary conditions,
At $\eta = 0$; $F(0) = C$

$\eta = \infty$; $F(\infty) = 0$ which are independent of $t$. \hfill (14)

**Deduction:**

For the non-magnetic case $S_0 = 0$ and $n = 1$, equation (1) becomes a ‘Diffusion Equation’ and then following the above spiral group of transformation it reduces to Watson’s second case [10] namely,$\nu F''(\eta) - p F(\eta) = 0$ this is a linear differential equation with constant coefficients. The analytical solution of this equation is given by; $u(y, t) = C e^{\left[\nu t \cdot \frac{\sqrt{\nu}}{\nu} y\right]}$ this is well agreed with the equations found in literature [6].

4 Conclusion

Similarity solutions for the MHD boundary layer flow of power-law fluid near a suddenly accelerated flat plate are obtained in its most general form by two different group transformations, namely one parameter linear group of transformation and a spiral group of transformation. The restriction to be imposed on transverse magnetic field parameter $S(y, t)$ for the existence of similarity solutions is systematically derived. It is shown that present similarity equations are reduced to some standard equations found in literature. The analytical solution for the diffusion equation is also obtained.

**References**


