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# A Summation Formula of Half Argument Associated to Bailey Theorem

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## Abstract

*The main objective of the present paper is to derive a summation formula of half argument associated to Bailey theorem. The result presented here is presumably new.*

**Keywords:** *Gaussian Hypergeometric function, Contiguous function, Recurrence relation, Bailey summation theorem and Legendre duplication formula.*

## 1 Introduction

Generalized Gaussian hypergeometric function of one variable is defined as

$${}_A F_B \left[ \begin{matrix} a_1, a_2, \dots, a_A ; \\ b_1, b_2, \dots, b_B ; \end{matrix} z \right] = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \dots (a_A)_k z^k}{(b_1)_k (b_2)_k \dots (b_B)_k k!}$$

or

$${}_A F_B \left[ \begin{matrix} (a_A) ; \\ (b_B) ; \end{matrix} z \right] \equiv {}_A F_B \left[ \begin{matrix} (a_j)_{j=1}^A ; \\ (b_j)_{j=1}^B ; \end{matrix} z \right] = \sum_{k=0}^{\infty} \frac{((a_A))_k z^k}{((b_B))_k k!} \quad (1)$$

where the parameters  $b_1, b_2, \dots, b_B$  are neither zero nor negative integers and  $A, B$  are non-negative integers.

Contiguous Relation is defined as follows

[E. D. p.51(10), Andrews p.363(9.16), H.T. F. I p.103(32)]

$$(a-b) {}_2F_1 \left[ \begin{matrix} a, b \\ c \end{matrix} ; z \right] = a {}_2F_1 \left[ \begin{matrix} a+1, b \\ c \end{matrix} ; z \right] - b {}_2F_1 \left[ \begin{matrix} a, b+1 \\ c \end{matrix} ; z \right] \quad (2)$$

Recurrence relation of gamma function is defined as follows

$$\Gamma(z+1) = z \Gamma(z) \quad (3)$$

Legendre duplication formula is defined by

$$\sqrt{\pi} \Gamma(2z) = 2^{(2z-1)} \Gamma(z) \Gamma\left(z + \frac{1}{2}\right) \quad (4)$$

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi} = \frac{2^{(b-1)} \Gamma\left(\frac{b}{2}\right) \Gamma\left(\frac{b+1}{2}\right)}{\Gamma(b)} \quad (5)$$

$$= \frac{2^{(a-1)} \Gamma\left(\frac{a}{2}\right) \Gamma\left(\frac{a+1}{2}\right)}{\Gamma(a)} \quad (6)$$

Bailey summation theorem [Prud, p.491(7.3.7.8)] is as follows

$${}_2F_1 \left[ \begin{matrix} a, 1-a \\ c \end{matrix} ; \frac{1}{2} \right] = \frac{\Gamma\left(\frac{c}{2}\right) \Gamma\left(\frac{c+1}{2}\right)}{\Gamma\left(\frac{c+a}{2}\right) \Gamma\left(\frac{c+1-a}{2}\right)} = \frac{\sqrt{\pi} \Gamma(c)}{2^{c-1} \Gamma\left(\frac{c+a}{2}\right) \Gamma\left(\frac{c+1-a}{2}\right)} \quad (7)$$

## 2 Main Summation Formula

$$\begin{aligned} & {}_2F_1 \left[ \begin{matrix} a, -a-34 \\ c \end{matrix} ; \frac{1}{2} \right] = \frac{\sqrt{\pi} \Gamma(c)}{2^{c+34}} \times \left[ \frac{-179725396599156215808000a}{\Gamma\left(\frac{c-a+1}{2}\right) \Gamma\left(\frac{c+a+34}{2}\right)} + \right. \\ & + \frac{176278465478072598681600a^2 - 53998442915060528352000a^3 + 5212143058443128698752a^4}{\Gamma\left(\frac{c-a+1}{2}\right) \Gamma\left(\frac{c+a+34}{2}\right)} + \\ & + \frac{163094646060428607216a^5 - 34870989959766941120a^6 - 631093555519267400a^7}{\Gamma\left(\frac{c-a+1}{2}\right) \Gamma\left(\frac{c+a+34}{2}\right)} + \\ & + \frac{86522703948738904a^8 + 2692481789166367a^9 - 51357622524560a^{10} - 3216119281100a^{11}}{\Gamma\left(\frac{c-a+1}{2}\right) \Gamma\left(\frac{c+a+34}{2}\right)} + \\ & + \frac{-33810573664a^{12} + 474997418a^{13} + 12624080a^{14} + 81500a^{15} + 8a^{16} - a^{17}}{\Gamma\left(\frac{c-a+1}{2}\right) \Gamma\left(\frac{c+a+34}{2}\right)} + \\ & + \frac{179725396620079005696000c - 480079894993878915993600ac + 286607538218612078876160a^2c}{\Gamma\left(\frac{c-a+1}{2}\right) \Gamma\left(\frac{c+a+34}{2}\right)} + \end{aligned}$$

$$\begin{aligned}
 & + \frac{-58923681600767092598016a^3c + 2936554833613340252448a^4c + 289401323546004593280a^5c}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
 & + \frac{-17122889417978895600a^6c - 960678744526651920a^7c + 21786918677930802a^8c}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
 & + \frac{1678621152574080a^9c + 12060050654520a^{10}c - 730595054112a^{11}c - 15707443668a^{12}c}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
 & + \frac{-59928960a^{13}c + 1038120a^{14}c + 9648a^{15}c + 18a^{16}c + 303801429586540599705600c^2}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
 & + \frac{-453783256859086929100800ac^2 + 186831429964357434200064a^2c^2 - 25914595017961243846656a^3c^2}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
 & + \frac{322902132293612363520a^4c^2 + 132690881259963290880a^5c^2 - 1755729792096314496a^6c^2}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
 & + \frac{-319627807778725248a^7c^2 - 1938186903906000a^8c^2 + 279199198531440a^9c^2 + 5844089982192a^{10}c^2}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
 & + \frac{-16398985680a^{11}c^2 - 1467565680a^{12}c^2 - 12531120a^{13}c^2 - 10800a^{14}c^2 + 144a^{15}c^2}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
 & + \frac{221174161658527623413760c^3 - 226618912242782832033792ac^3 + 67036582057662381981696a^2c^3}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
 & + \frac{-6046549868935001753600a^3c^3 - 130812769244615790080a^4c^3 + 27665106534009564928a^5c^3}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
 & + \frac{375525399579949696a^6c^3 - 43434993087041600a^7c^3 - 1030384714673600a^8c^3 + 13899929883840a^9c^3}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
 & + \frac{623936233536a^{10}c^3 + 4417385280a^{11}c^3 - 33364800a^{12}c^3 - 450240a^{13}c^3 - 960a^{14}c^3}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
 & + \frac{93499020907827110608896c^4 - 70187939957957661523968ac^4 + 15063413339614862888960a^2c^4}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
 & + \frac{-801025517520925777920a^3c^4 - 49095298146212102400a^4c^4 + 2980691863620575104a^5c^4}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
 & + \frac{111731381007400000a^6c^4 - 2433965084610400a^7c^4 - 118394832346560a^8c^4 - 477481102560a^9c^4}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
 & + \frac{24869718720a^{10}c^4 + 295864800a^{11}c^4 + 477120a^{12}c^4 - 3360a^{13}c^4 + 25966303486948763762688c^5}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} +
 \end{aligned}$$

$$\begin{aligned}
& + \frac{-14657409272023175741440ac^5 + 2262698965968343705600a^2c^5 - 53980880947959178752a^3c^5}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
& + \frac{-7653568354604736256a^4c^5 + 147484321071669120a^5c^5 + 12085862944232000a^6c^5}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
& + \frac{9530353117824a^7c^5 - 6071200056768a^8c^5 - 70813733760a^9c^5 + 241348800a^{10}c^5 + 5943168a^{11}c^5}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
& + \frac{14784a^{12}c^5 + 5063113354325031649280c^6 - 2167153902409193062400ac^6}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
& + \frac{234714970109273206784a^2c^6 + 72965393203383296a^3c^6 - 690673666259709440a^4c^6}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
& + \frac{-1363969989536000a^5c^6 + 686661075881088a^6c^6 + 8198063803776a^7c^6 - 134697911040a^8c^6}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
& + \frac{-2501452800a^9c^6 - 6179712a^{10}c^6 + 29568a^{11}c^6 + 721356330294449274880c^7}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
& + \frac{-233641252827241971712ac^7 + 16956797879563452416a^2c^7 + 366636901242163200a^3c^7}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
& + \frac{-38601213445386240a^4c^7 - 621398831133696a^5c^7 + 20731335668736a^6c^7 + 425308477440a^7c^7}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
& + \frac{-241612800a^8c^7 - 33960960a^9c^7 - 101376a^{10}c^7 + 76967242257705992192c^8}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
& + \frac{-18676816839483883520ac^8 + 836470098009722880a^2c^8 + 36299867555466240a^3c^8}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
& + \frac{-1305758760176640a^4c^8 - 37509602473728a^5c^8 + 247552757760a^6c^8 + 9713932800a^7c^8}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
& + \frac{34974720a^8c^8 - 126720a^9c^8 + 6239553251386064896c^9 - 1114738046535393280ac^9}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
& + \frac{26007420905523200a^2c^9 + 1945362268129280a^3c^9 - 22331646937600a^4c^9 - 1141610229760a^5c^9}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
& + \frac{-2588185600a^6c^9 + 98109440a^7c^9 + 366080a^8c^9 + 386834950982205440c^{10} - 49581031165952000ac^{10}}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
& + \frac{350996352458752a^2c^{10} + 65277038002176a^3c^{10} + 30350960640a^4c^{10} - 18809190400a^5c^{10}}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} +
\end{aligned}$$

$$\begin{aligned}
 & + \frac{-100452352a^6c^{10} + 292864a^7c^{10} + 18329372050063360c^{11} - 1624283296563200ac^{11}}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
 & + \frac{-7763585449984a^2c^{11} + 1386477281280a^3c^{11} + 8971755520a^4c^{11} - 149839872a^5c^{11} - 745472a^6c^{11}}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
 & + \frac{658306213347328c^{12} - 38248757886976ac^{12} - 490285752320a^2c^{12} + 17632276480a^3c^{12}}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
 & + \frac{152821760a^4c^{12} - 372736a^5c^{12} + 17603540025344c^{13} - 619456839680ac^{13} - 10762035200a^2c^{13}}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
 & + \frac{115261440a^3c^{13} + 860160a^4c^{13} + 339403079680c^{14} - 6364364800ac^{14} - 117227520a^2c^{14} + 245760a^3c^{14}}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
 & + \frac{4456448000c^{15} - 35127296ac^{15} - 524288a^2c^{15} + 35651584c^{16} - 65536ac^{16} + 131072c^{17}}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
 & + \frac{830034394580628357120000 - 1377065451088614086553600a + 650917761399667233123840a^2}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+35}{2})} + \\
 & + \frac{-106441825212269888974848a^3 + 1797340338149203219104a^4 + 775661574779850021264a^5}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+35}{2})} + \\
 & + \frac{-14952856152307795120a^6 - 2905674724521079336a^7 - 15241731062500822a^8}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+35}{2})} + \\
 & + \frac{4510465479373073a^9 + 119693002445720a^{10} - 757636334884a^{11} - 72635836868a^{12}}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+35}{2})} + \\
 & + \frac{-1001843738a^{13} - 1104440a^{14} + 80068a^{15} + 586a^{16} + a^{17} + 1990782488296072809676800c}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+35}{2})} + \\
 & + \frac{-2318612026053359499816960ac + 799293299722875962131968a^2c - 86397059455847148780288a^3c}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+35}{2})} + \\
 & + \frac{-2060870095990900419552a^4c + 585055100899622895360a^5c + 8575377898850894544a^6c}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+35}{2})} + \\
 & + \frac{-1505688369793705680a^7c - 43691973648968238a^8c + 962487629468640a^9c + 55486260156888a^{10}c}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+35}{2})} + \\
 & + \frac{551069327424a^{11}c - 8772675828a^{12}c - 220815840a^{13}c - 1385400a^{14}c + 144a^{15}c + 18a^{16}c}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+35}{2})} + \\
 & + \frac{1881474425370799405793280c^2 - 1632458853872025818923008ac^2 + 414369892955453180203008a^2c^2}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+35}{2})} +
 \end{aligned}$$

$$\begin{aligned}
& + \frac{-26959982583521703297024a^3c^2 - 1940300862028789774080a^4c^2 + 154977103409202885888a^5c^2}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+35}{2})} + \\
& + \frac{7009302342933356928a^6c^2 - 224562920489457792a^7c^2 - 13797969990591600a^8c^2}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+35}{2})} + \\
& + \frac{-61945879319280a^9c^2 + 6774945182544a^{10}c^2 + 130356127440a^{11}c^2 + 359874480a^{12}c^2}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+35}{2})} + \\
& + \frac{-10088400a^{13}c^2 - 84240a^{14}c^2 - 144a^{15}c^2 + 986226901373753993723904c^3}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+35}{2})} + \\
& + \frac{-653423370678042386006016ac^3 + 121223070338904522633216a^2c^3 - 3667640265557653038080a^3c^3}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+35}{2})} + \\
& + \frac{-618211944332780994560a^4c^3 + 16410099785215273984a^5c^3 + 1646990973593760256a^6c^3}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+35}{2})} + \\
& + \frac{-676053491748800a^7c^3 - 1669935333999680a^8c^3 - 28248215244480a^9c^3 + 203190542016a^{10}c^3}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+35}{2})} + \\
& + \frac{8832626880a^{11}c^3 + 64653120a^{12}c^3 - 6720a^{13}c^3 - 960a^{14}c^3 + 330786223711946433626112c^4}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+35}{2})} + \\
& + \frac{-168860896838707866058752ac^4 + 22383105664203945687040a^2c^4 - 16726986497183746560a^3c^4}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+35}{2})} + \\
& + \frac{-104239686251248880640a^4c^4 - 130245440433036544a^5c^4 + 187765957371977600a^6c^4}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+35}{2})} + \\
& + \frac{3001342598673760a^7c^4 - 85207747806720a^8c^4 - 2541885524640a^9c^4 - 11611797120a^{10}c^4}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+35}{2})} + \\
& + \frac{201764640a^{11}c^4 + 1962240a^{12}c^4 + 3360a^{13}c^4 + 76696311130024412971008c^5}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+35}{2})} + \\
& + \frac{-30155794084574210375680ac^5 + 2737703814670849841152a^2c^5 + 71925252918545478144a^3c^5}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+35}{2})} + \\
& + \frac{-10436737037868418816a^4c^5 - 215952412981683840a^5c^5 + 11177621240050496a^6c^5}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+35}{2})} + \\
& + \frac{335521585779072a^7c^5 - 610214582208a^8c^5 - 97158821760a^9c^5 - 853435968a^{10}c^5 + 88704a^{11}c^5}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+35}{2})} + \\
& + \frac{14784a^{12}c^5 + 12882155956746673192960c^6 - 3873167547042866888704ac^6}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+35}{2})} +
\end{aligned}$$

$$\begin{aligned}
 & + \frac{223078849412160974848a^2c^6 + 12307957883649455104a^3c^6 - 626568475975805440a^4c^6}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+35}{2})} + \\
 & + \frac{-24714927694057216a^5c^6 + 279373545365376a^6c^6 + 17078857724544a^7c^6 + 117524816640a^8c^6}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+35}{2})} + \\
 & + \frac{-1479582720a^9c^6 - 17238144a^{10}c^6 - 29568a^{11}c^6 + 1614739852415051235328c^7}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+35}{2})} + \\
 & + \frac{-366289727284047609856ac^7 + 11428642405941026816a^2c^7 + 1124217772101980160a^3c^7}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+35}{2})} + \\
 & + \frac{-19620139023513600a^4c^7 - 1448430437465088a^5c^7 - 4638476688384a^6c^7 + 444154275840a^7c^7}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+35}{2})} + \\
 & + \frac{4876861440a^8c^7 - 506880a^9c^7 - 101376a^{10}c^7 + 153904706400973225984c^8}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+35}{2})} + \\
 & + \frac{-25814484746842316800ac^8 + 266052344275998720a^2c^8 + 65419232628034560a^3c^8}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+35}{2})} + \\
 & + \frac{-2322924172800a^4c^8 - 49487966744832a^5c^8 - 513930869760a^6c^8 + 5072770560a^7c^8 + 73751040a^8c^8}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+35}{2})} + \\
 & + \frac{126720a^9c^8 + 11274134322544377856c^9 - 1358960792457748480ac^9 - 8269540861204480a^2c^9}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+35}{2})} + \\
 & + \frac{2522723887523840a^3c^9 + 26144095577600a^4c^9 - 962333532160a^5c^9 - 14088954880a^6c^9 + 1464320a^7c^9}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+35}{2})} + \\
 & + \frac{366080a^8c^9 + 637294279304151040c^{10} - 53034831151464448ac^{10} - 996988880183296a^2c^{10}}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+35}{2})} + \\
 & + \frac{63623584088064a^3c^{10} + 1083194112000a^4c^{10} - 8792655872a^5c^{10} - 170153984a^6c^{10} - 292864a^7c^{10}}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+35}{2})} + \\
 & + \frac{27731905852997632c^{11} - 1503483152138240ac^{11} - 43006124621824a^2c^{11} + 979829760000a^3c^{11}}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+35}{2})} + \\
 & + \frac{21518049280a^4c^{11} - 2236416a^5c^{11} - 745472a^6c^{11} + 920131294724096c^{12} - 29722879606784ac^{12}}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+35}{2})} + \\
 & + \frac{-1082306068480a^2c^{12} + 7460311040a^3c^{12} + 216186880a^4c^{12} + 372736a^5c^{12} + 22843387019264c^{13}}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+35}{2})} + \\
 & + \frac{-376857313280ac^{13} - 16552632320a^2c^{13} + 1720320a^3c^{13} + 860160a^4c^{13} + 410617118720c^{14}}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+35}{2})} +
 \end{aligned}$$

$$\begin{aligned}
& + \frac{-2459402240ac^{14} - 142295040a^2c^{14} - 245760a^3c^{14} + 5044699136c^{15} - 524288ac^{15} - 524288a^2c^{15}}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+35}{2})} + \\
& + \left. \frac{37879808c^{16} + 65536ac^{16} + 131072c^{17}}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+35}{2})} \right] \quad (8)
\end{aligned}$$

### 3 Derivation of Result (8)

putting  $b = -a - 34$ ,  $z = \frac{1}{2}$  in known result (2), we get

$$\begin{aligned}
& (2a + 34) {}_2F_1 \left[ \begin{matrix} a, & -a - 34 & ; & 1 \\ c & & & 2 \end{matrix} \right] \\
& = a {}_2F_1 \left[ \begin{matrix} a + 1, & -a - 34 & ; & 1 \\ c & & & 2 \end{matrix} \right] + (a + 34) {}_2F_1 \left[ \begin{matrix} a, & -a - 33 & ; & 1 \\ c & & & 2 \end{matrix} \right]
\end{aligned}$$

Now with the help of Bailey theorem, we get

$$\begin{aligned}
L.H.S & = a \frac{\sqrt{\pi} \Gamma(c)}{2^{c+33}} \times \left[ \frac{711374856192000 - 11649855761356867891200a + 10583929300529509332480a^2}{\Gamma(\frac{c+a+34}{2}) \Gamma(\frac{c-a-1}{2})} + \right. \\
& + \frac{-2770431466954494954240a^3 + 166141223384654926368a^4 + 16881939126090505600a^5}{\Gamma(\frac{c+a+34}{2}) \Gamma(\frac{c-a-1}{2})} + \\
& + \frac{-990097205418458480a^6 - 70284980830747760a^7 + 819352191011266a^8 + 122125343956000a^9}{\Gamma(\frac{c+a+34}{2}) \Gamma(\frac{c-a-1}{2})} + \\
& + \frac{2029138765720a^{10} - 29755418560a^{11} - 1266103636a^{12} - 12426400a^{13} - 1720a^{14} + 560a^{15} + 2a^{16}}{\Gamma(\frac{c+a+34}{2}) \Gamma(\frac{c-a-1}{2})} + \\
& + \frac{11649858166322469273600c - 29489382908047876423680ac + 15803721228081422794752a^2c}{\Gamma(\frac{c+a+34}{2}) \Gamma(\frac{c-a-1}{2})} + \\
& + \frac{-2603061665564232499200a^3c + 34509627471413821440a^4c + 15923755104187392000a^5c}{\Gamma(\frac{c+a+34}{2}) \Gamma(\frac{c-a-1}{2})} + \\
& + \frac{-108339111861657600a^6c - 42555650026982400a^7c - 667295789276160a^8c + 30528173491200a^9c}{\Gamma(\frac{c+a+34}{2}) \Gamma(\frac{c-a-1}{2})} + \\
& + \frac{1046643569664a^{10}c + 6401203200a^{11}c - 140743680a^{12}c - 2257920a^{13}c - 9216a^{14}c}{\Gamma(\frac{c+a+34}{2}) \Gamma(\frac{c-a-1}{2})} + \\
& + \frac{18905457109254691553280c^2 - 26095266336651004846080ac^2 + 9300673308101047898112a^2c^2}{\Gamma(\frac{c+a+34}{2}) \Gamma(\frac{c-a-1}{2})} + \\
& + \left. \frac{-938625371327885606400a^3c^2 - 25907684009995307520a^4c^2 + 4843152866792782080a^5c^2}{\Gamma(\frac{c+a+34}{2}) \Gamma(\frac{c-a-1}{2})} \right]
\end{aligned}$$



$$\begin{aligned}
 & + \frac{111992885096522496a^6c^2 - 7353458915695200a^7c^2 - 270702336595680a^8c^2 + 259890708000a^9c^2}{\Gamma(\frac{c+a+34}{2}) \Gamma(\frac{c-a-1}{2})} + \\
 & + \frac{121466489760a^{10}c^2 + 1645106400a^{11}c^2 + 2483040a^{12}c^2 - 70560a^{13}c^2 - 288a^{14}c^2}{\Gamma(\frac{c+a+34}{2}) \Gamma(\frac{c-a-1}{2})} + \\
 & + \frac{13061979536167538786304c^3 - 12045951529532325888000ac^3 + 2954799499882633625600a^2c^3}{\Gamma(\frac{c+a+34}{2}) \Gamma(\frac{c-a-1}{2})} + \\
 & + \frac{-162960936100508958720a^3c^3 - 12447254083520413696a^4c^3 + 626455756286976000a^5c^3}{\Gamma(\frac{c+a+34}{2}) \Gamma(\frac{c-a-1}{2})} + \\
 & + \frac{33782469386547200a^6c^3 - 307005679411200a^7c^3 - 32922624430080a^8c^3 - 382491648000a^9c^3}{\Gamma(\frac{c+a+34}{2}) \Gamma(\frac{c-a-1}{2})} + \\
 & + \frac{3609661440a^{10}c^3 + 90316800a^{11}c^3 + 430080a^{12}c^3 + 5182200308055169499136c^4}{\Gamma(\frac{c+a+34}{2}) \Gamma(\frac{c-a-1}{2})} + \\
 & + \frac{-3403176565234662809600ac^4 + 574218537036537200640a^2c^4 - 11675761474038914560a^3c^4}{\Gamma(\frac{c+a+34}{2}) \Gamma(\frac{c-a-1}{2})} + \\
 & + \frac{-2354548838427070208a^4c^4 + 244651845900000000a^5c^4 + 3998692068536000a^6c^4}{\Gamma(\frac{c+a+34}{2}) \Gamma(\frac{c-a-1}{2})} + \\
 & + \frac{37835455996800a^7c^4 - 1600777442880a^8c^4 - 32469360000a^9c^4 - 94987200a^{10}c^4 + 1411200a^{11}c^4}{\Gamma(\frac{c+a+34}{2}) \Gamma(\frac{c-a-1}{2})} + \\
 & + \frac{6720a^{12}c^4 + 1335611511375690465280c^5 - 639175442138966261760ac^5 + 72305759593706225664a^2c^5}{\Gamma(\frac{c+a+34}{2}) \Gamma(\frac{c-a-1}{2})} + \\
 & + \frac{567380299203379200a^3c^5 - 242911920542269440a^4c^5 - 2654843532042240a^5c^5}{\Gamma(\frac{c+a+34}{2}) \Gamma(\frac{c-a-1}{2})} + \\
 & + \frac{232561509261312a^6c^5 + 4854166732800a^7c^5 - 17485332480a^8c^5 - 993484800a^9c^5 - 5677056a^{10}c^5}{\Gamma(\frac{c+a+34}{2}) \Gamma(\frac{c-a-1}{2})} + \\
 & + \frac{238892050047962644480c^6 - 83682981373071646720ac^6 + 5998212953711863808a^2c^6}{\Gamma(\frac{c+a+34}{2}) \Gamma(\frac{c-a-1}{2})} + \\
 & + \frac{199402745248537600a^3c^6 - 14518616774520320a^4c^6 - 388060173768960a^5c^6 + 5913567061248a^6c^6}{\Gamma(\frac{c+a+34}{2}) \Gamma(\frac{c-a-1}{2})} + \\
 & + \frac{220118976000a^7c^6 + 1028966400a^8c^6 - 10348800a^9c^6 - 59136a^{10}c^6 + 30835747557468012544c^7}{\Gamma(\frac{c+a+34}{2}) \Gamma(\frac{c-a-1}{2})} + \\
 & + \frac{-7848001787265024000ac^7 + 317727678745804800a^2c^7 + 19817793174896640a^3c^7}{\Gamma(\frac{c+a+34}{2}) \Gamma(\frac{c-a-1}{2})} +
 \end{aligned}$$

$$\begin{aligned}
& + \frac{-464717794050048a^4c^7 - 21745395302400a^5c^7 - 21648506880a^6c^7 + 4541644800a^7c^7 + 32440320a^8c^7}{\Gamma(\frac{c+a+34}{2}) \Gamma(\frac{c-a-1}{2})} + \\
& + \frac{2940251351637360640c^8 - 534275237232844800ac^8 + 9119929360312320a^2c^8 + 1113438138193920a^3c^8}{\Gamma(\frac{c+a+34}{2}) \Gamma(\frac{c-a-1}{2})} + \\
& + \frac{-3712478482944a^4c^8 - 642216960000a^5c^8 - 4667520000a^6c^8 + 35481600a^7c^8 + 253440a^8c^8}{\Gamma(\frac{c+a+34}{2}) \Gamma(\frac{c-a-1}{2})} + \\
& + \frac{209708493108674560c^9 - 26445855365529600ac^9 - 6623131074560a^2c^9 + 38780348006400a^3c^9}{\Gamma(\frac{c+a+34}{2}) \Gamma(\frac{c-a-1}{2})} + \\
& + \frac{266998251520a^4c^9 - 9840230400a^5c^9 - 93716480a^6c^9 + 11228126063165440c^{10}}{\Gamma(\frac{c+a+34}{2}) \Gamma(\frac{c-a-1}{2})} + \\
& + \frac{-941756872376320ac^{10} - 11939938762752a^2c^{10} + 830166937600a^3c^{10} + 10065735680a^4c^{10}}{\Gamma(\frac{c+a+34}{2}) \Gamma(\frac{c-a-1}{2})} + \\
& + \frac{-61501440a^5c^{10} - 585728a^6c^{10} + 448974049771520c^{11} - 23494891929600ac^{11} - 495947612160a^2c^{11}}{\Gamma(\frac{c+a+34}{2}) \Gamma(\frac{c-a-1}{2})} + \\
& + \frac{10019143680a^3c^{11} + 143130624a^4c^{11} + 13201575575552c^{12} - 389546393600ac^{12} - 10216693760a^2c^{12}}{\Gamma(\frac{c+a+34}{2}) \Gamma(\frac{c-a-1}{2})} + \\
& + \frac{52183040a^3c^{12} + 745472a^4c^{12} + 276939407360c^{13} - 3853516800ac^{13} - 110100480a^2c^{13}}{\Gamma(\frac{c+a+34}{2}) \Gamma(\frac{c-a-1}{2})} + \\
& + \frac{3920691200c^{14} - 17203200ac^{14} - 491520a^2c^{14} + 33554432c^{15} + 131072c^{16}}{\Gamma(\frac{c+a+34}{2}) \Gamma(\frac{c-a-1}{2})} + \\
& + \frac{-26901550859285613772800 + 15783871134711276011520a + 24593676535902527926272a^2}{\Gamma(\frac{c+a+35}{2}) \Gamma(\frac{c-a}{2})} + \\
& + \frac{-15828364941706790000640a^3 + 2325705152937951598848a^4 + 44775895638604318720a^5}{\Gamma(\frac{c+a+35}{2}) \Gamma(\frac{c-a}{2})} + \\
& + \frac{-17978002545608932736a^6 - 383673734211743360a^7 + 47591517431400976a^8 + 1994601823244800a^9}{\Gamma(\frac{c+a+35}{2}) \Gamma(\frac{c-a}{2})} + \\
& + \frac{-10968601185344a^{10} - 1990519740160a^{11} - 37960379296a^{12} - 49047040a^{13} + 6463808a^{14} + 76160a^{15}}{\Gamma(\frac{c+a+35}{2}) \Gamma(\frac{c-a}{2})} + \\
& + \frac{272a^{16} - 35802594542933941616640c - 37574264549117193246720ac + 65814625686102352190208a^2c}{\Gamma(\frac{c+a+35}{2}) \Gamma(\frac{c-a}{2})} + \\
& + \frac{-20937965215995894568320a^3c + 1476447693352054299024a^4c + 129876084401552003200a^5c}{\Gamma(\frac{c+a+35}{2}) \Gamma(\frac{c-a}{2})} +
\end{aligned}$$

$$\begin{aligned}
 & + \frac{-8717352478665599960a^6c - 576440215573662440a^7c + 7725120933920929a^8c}{\Gamma(\frac{c+a+35}{2}) \Gamma(\frac{c-a}{2})} + \\
 & + \frac{+1037200401344800a^9c + 16903148413756a^{10}c - 257576594800a^{11}c - 10768889530a^{12}c}{\Gamma(\frac{c+a+35}{2}) \Gamma(\frac{c-a}{2})} + \\
 & + \frac{-105424480a^{13}c - 13804a^{14}c + 4760a^{15}c + 17a^{16}c + 5955399896870066061312c^2}{\Gamma(\frac{c+a+35}{2}) \Gamma(\frac{c-a}{2})} + \\
 & + \frac{-80136894750576674734080ac^2 + 55310995564176690266112a^2c^2 - 10365659387586746019840a^3c^2}{\Gamma(\frac{c+a+35}{2}) \Gamma(\frac{c-a}{2})} + \\
 & + \frac{197178364868858892288a^4c^2 + 64988146828735764480a^5c^2 - 601709564957850624a^6c^2}{\Gamma(\frac{c+a+35}{2}) \Gamma(\frac{c-a}{2})} + \\
 & + \frac{-179508800777644800a^7c^2 - 2696260771480320a^8c^2 + 131360474784000a^9c^2 + 4432837812480a^{10}c^2}{\Gamma(\frac{c+a+35}{2}) \Gamma(\frac{c-a}{2})} + \\
 & + \frac{26821267200a^{11}c^2 - 599988480a^{12}c^2 - 9596160a^{13}c^2 - 9168a^{14}c^2 + 28605280292405452210176c^3}{\Gamma(\frac{c+a+35}{2}) \Gamma(\frac{c-a}{2})} + \\
 & + \frac{-56250074639233383014400ac^3 + 23215886433559382133760a^2c^3 - 2589450583196524391680a^3c^3}{\Gamma(\frac{c+a+35}{2}) \Gamma(\frac{c-a}{2})} + \\
 & + \frac{-60350168215612705024a^4c^3 + 13576009027474087040a^5c^3 + 294771751117984448a^6c^3}{\Gamma(\frac{c+a+35}{2}) \Gamma(\frac{c-a}{2})} + \\
 & + \frac{-21044839640717200a^7c^3 - 757898503358480a^8c^3 + 920144794800a^9c^3 + 344692142256a^{10}c^3}{\Gamma(\frac{c+a+35}{2}) \Gamma(\frac{c-a}{2})} + \\
 & + \frac{4653138000a^{11}c^3 + 6997200a^{12}c^3 - 199920a^{13}c^3 - 816a^{14}c^3 + 19312444030256840441856c^4}{\Gamma(\frac{c+a+35}{2}) \Gamma(\frac{c-a}{2})} + \\
 & + \frac{-21335646430447281766400ac^4 + 5777811216404936540160a^2c^4 - 349633279306779381760a^3c^4}{\Gamma(\frac{c+a+35}{2}) \Gamma(\frac{c-a}{2})} + \\
 & + \frac{-24745988332879751168a^4c^4 + 1349271470267212800a^5c^4 + 70139710066511360a^6c^4}{\Gamma(\frac{c+a+35}{2}) \Gamma(\frac{c-a}{2})} + \\
 & + \frac{-686613116851200a^7c^4 - 69835596526080a^8c^4 - 805757568000a^9c^4 + 7710743040a^{10}c^4}{\Gamma(\frac{c+a+35}{2}) \Gamma(\frac{c-a}{2})} + \\
 & + \frac{191923200a^{11}c^4 + 913920a^{12}c^4 + 6936616375368644034560c^5 - 5103271371331692912640ac^5}{\Gamma(\frac{c+a+35}{2}) \Gamma(\frac{c-a}{2})} + \\
 & + \frac{925699382554314167296a^2c^5 - 21482322035391965440a^3c^5 - 3879536248379121792a^4c^5}{\Gamma(\frac{c+a+35}{2}) \Gamma(\frac{c-a}{2})} +
 \end{aligned}$$

$$\begin{aligned}
& + \frac{44853693242783040a^5c^5 + 6747252302225248a^6c^5 + 62451234310080a^7c^5 - 2730053792928a^8c^5}{\Gamma(\frac{c+a+35}{2}) \Gamma(\frac{c-a}{2})} + \\
& + \frac{-55109947200a^9c^5 - 160975584a^{10}c^5 + 2399040a^{11}c^5 + 11424a^{12}c^5 + 1603367675572063830016c^6}{\Gamma(\frac{c+a+35}{2}) \Gamma(\frac{c-a}{2})} + \\
& + \frac{-829852824563180830720ac^6 + 99282285705149431808a^2c^6 + 610483031046676480a^3c^6}{\Gamma(\frac{c+a+35}{2}) \Gamma(\frac{c-a}{2})} + \\
& + \frac{-339494666644238336a^4c^6 - 3546206191503360a^5c^6 + 329669012563968a^6c^6 + 6831698227200a^7c^6}{\Gamma(\frac{c+a+35}{2}) \Gamma(\frac{c-a}{2})} + \\
& + \frac{-25092587520a^8c^6 - 1407436800a^9c^6 - 8042496a^{10}c^6 + 258074757345826963456c^7}{\Gamma(\frac{c+a+35}{2}) \Gamma(\frac{c-a}{2})} + \\
& + \frac{-95671353853665300480ac^7 + 7178781155395075072a^2c^7 + 229611642814005760a^3c^7}{\Gamma(\frac{c+a+35}{2}) \Gamma(\frac{c-a}{2})} + \\
& + \frac{-17586477596517632a^4c^7 - 463944785082240a^5c^7 + 7233588861312a^6c^7 + 266885203200a^7c^7}{\Gamma(\frac{c+a+35}{2}) \Gamma(\frac{c-a}{2})} + \\
& + \frac{1246586880a^8c^7 - 12566400a^9c^7 - 71808a^{10}c^7 + 30161918441449062400c^8 - 8005151223147724800ac^8}{\Gamma(\frac{c+a+35}{2}) \Gamma(\frac{c-a}{2})} + \\
& + \frac{337589250492088320a^2c^8 + 20563595193630720a^3c^8 - 497143464136704a^4c^8 - 22979019571200a^5c^8}{\Gamma(\frac{c+a+35}{2}) \Gamma(\frac{c-a}{2})} + \\
& + \frac{-21806653440a^6c^8 + 4825497600a^7c^8 + 34467840a^8c^8 + 2620702677478014976c^9}{\Gamma(\frac{c+a+35}{2}) \Gamma(\frac{c-a}{2})} + \\
& + \frac{-491336637929984000ac^9 + 8780188866380800a^2c^9 + 1039831150167040a^3c^9 - 3648664963328a^4c^9}{\Gamma(\frac{c+a+35}{2}) \Gamma(\frac{c-a}{2})} + \\
& + \frac{-605666969600a^5c^9 - 4399915520a^6c^9 + 33510400a^7c^9 + 239360a^8c^9 + 171334040773722112c^{10}}{\Gamma(\frac{c+a+35}{2}) \Gamma(\frac{c-a}{2})} + \\
& + \frac{-22113941249720320ac^{10} + 2067629211648a^2c^{10} + 32807164149760a^3c^{10} + 224718061568a^4c^{10}}{\Gamma(\frac{c+a+35}{2}) \Gamma(\frac{c-a}{2})} + \\
& + \frac{-8364195840a^5c^{10} - 79659008a^6c^{10} + 8453266487377920c^{11} - 721105140654080ac^{11}}{\Gamma(\frac{c+a+35}{2}) \Gamma(\frac{c-a}{2})} + \\
& + \frac{-9052809945088a^2c^{11} + 640605521920a^3c^{11} + 7765395456a^4c^{11} - 47523840a^5c^{11} - 452608a^6c^{11}}{\Gamma(\frac{c+a+35}{2}) \Gamma(\frac{c-a}{2})} + \\
& + \frac{312938124541952c^{12} - 16571246182400ac^{12} - 349268541440a^2c^{12} + 7096893440a^3c^{12}}{\Gamma(\frac{c+a+35}{2}) \Gamma(\frac{c-a}{2})} +
\end{aligned}$$

$$\begin{aligned}
 & + \frac{101384192a^4c^{12} + 8554111827968c^{13} - 254362214400ac^{13} - 6670397440a^2c^{13} + 34119680a^3c^{13}}{\Gamma(\frac{c+a+35}{2}) \Gamma(\frac{c-a}{2})} + \\
 & + \frac{487424a^4c^{13} + 167428751360c^{14} - 2339635200ac^{14} - 66846720a^2c^{14} + 2218754048c^{15} - 9748480ac^{15}}{\Gamma(\frac{c+a+35}{2}) \Gamma(\frac{c-a}{2})} + \\
 & + \left. \frac{-278528a^2c^{15} + 17825792c^{16} + 65536c^{17}}{\Gamma(\frac{c+a+35}{2}) \Gamma(\frac{c-a}{2})} \right] + \\
 & + (a + 34) \frac{\sqrt{\pi} \Gamma(c)}{2^{c+33}} \times \left[ \frac{25152557411534192640000 - 41703720849113941094400a}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+33}{2})} + \right. \\
 & + \frac{19709894815170352104960a^2 - 3248870872778027152128a^3 + 69382446182624045088a^4}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+33}{2})} + \\
 & + \frac{21315517728591461760a^5 - 484980289567046000a^6 - 72695620476572016a^7 - 184165753338974a^8}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+33}{2})} + \\
 & + \frac{100467072425280a^9 + 2276436092920a^{10} - 15530052384a^{11} - 1104968116a^{12} - 12344640a^{13}}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+33}{2})} + \\
 & + \frac{-9880a^{14} + 528a^{15} + 2a^{16} + 59564543360137533849600c - 68964072347421643898880ac}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+33}{2})} + \\
 & + \frac{23659975283974806306816a^2c - 2581145481501661003776a^3c - 45295126616923201536a^4c}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+33}{2})} + \\
 & + \frac{15721075550607888384a^5c + 168518501247731712a^6c - 36241646672360448a^7c}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+33}{2})} + \\
 & + \frac{-896073378444288a^8c + 20443171617792a^9c + 967577791488a^{10}c + 7917364224a^{11}c}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+33}{2})} + \\
 & + \frac{-112229376a^{12}c - 2128896a^{13}c - 9216a^{14}c + 55209390363959450664960c^2}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+33}{2})} + \\
 & + \frac{-47385363831224691621888ac^2 + 11914498521624672866304a^2c^2 - 789045168527390066688a^3c^2}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+33}{2})} + \\
 & + \frac{-48205140948895949568a^4c^2 + 4031935144489549440a^5c^2 + 155890351600475712a^6c^2}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+33}{2})} + \\
 & + \frac{-5192519337761184a^7c^2 - 267845484433824a^8c^2 - 864889480224a^9c^2 + 103554091872a^{10}c^2}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+33}{2})} + \\
 & + \frac{1609911072a^{11}c^2 + 3374112a^{12}c^2 - 66528a^{13}c^2 - 288a^{14}c^2 + 28212651848781652819968c^3}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+33}{2})} +
 \end{aligned}$$

$$\begin{aligned}
& + \frac{-18391716626436216324096ac^3 + 3363246502373945114624a^2c^3 - 107591945670749945856a^3c^3}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+33}{2})} + \\
& + \frac{-15064306286702903296a^4c^3 + 419108425471549440a^5c^3 + 35042521656750080a^6c^3}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+33}{2})} + \\
& + \frac{-57798078750720a^7c^3 - 29332454215680a^8c^3 - 413715456000a^9c^3 + 2644561920a^{10}c^3}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+33}{2})} + \\
& + \frac{85155840a^{11}c^3 + 430080a^{12}c^3 + 9168896117065599418368c^4 - 4577124117242454294528ac^4}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+33}{2})} + \\
& + \frac{594933018761420593152a^2c^4 - 2091476947101070848a^3c^4 - 2418326605000241408a^4c^4}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+33}{2})} + \\
& + \frac{1353153735584640a^5c^4 + 3691728941320640a^6c^4 + 49484637417600a^7c^4 - 1313057148480a^8c^4}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+33}{2})} + \\
& + \frac{-31443350400a^9c^4 - 110066880a^{10}c^4 + 1330560a^{11}c^4 + 6720a^{12}c^4 + 2046285303422994350080c^5}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+33}{2})} + \\
& + \frac{-781127808223150080000ac^5 + 69176081639221100544a^2c^5 + 1507999108109598720a^3c^5}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+33}{2})} + \\
& + \frac{-226320276065157120a^4c^5 - 3947419656069120a^5c^5 + 198175013363712a^6c^5 + 4958965186560a^7c^5}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+33}{2})} + \\
& + \frac{-8799436800a^8c^5 - 936714240a^9c^5 - 5677056a^{10}c^5 + 328359716767384207360c^6}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+33}{2})} + \\
& + \frac{-95025098827414339584ac^6 + 5316857729245196288a^2c^6 + 253485984947543040a^3c^6}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+33}{2})} + \\
& + \frac{-12497243244738560a^4c^6 - 418977988805376a^5c^6 + 4402402169088a^6c^6 + 211521784320a^7c^6}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+33}{2})} + \\
& + \frac{1119444480a^8c^6 - 9757440a^9c^6 - 59136a^{10}c^6 + 38981216231747485696c^7 - 8422253459609223168ac^7}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+33}{2})} + \\
& + \frac{255703027216023552a^2c^7 + 21459800509120512a^3c^7 - 356472231886848a^4c^7 - 21521946378240a^5c^7}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+33}{2})} + \\
& + \frac{-52531691520a^6c^7 + 4282122240a^7c^7 + 32440320a^8c^7 + 3482530005128052736c^8}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+33}{2})} + \\
& + \frac{-549163114458292224ac^8 + 5763691493615616a^2c^8 + 1121960460589056a^3c^8 - 572630598144a^4c^8}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+33}{2})} +
\end{aligned}$$

$$\begin{aligned}
 & + \frac{-613480919040a^5c^8 - 4908794880a^6c^8 + 33454080a^7c^8 + 253440a^8c^8 + 236109221739888640c^9}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+33}{2})} + \\
 & + \frac{-26317384691220480ac^9 - 121265189027840a^2c^9 + 37615827025920a^3c^9 + 314793656320a^4c^9}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+33}{2})} + \\
 & + \frac{-9277931520a^5c^9 - 93716480a^6c^9 + 12157122956492800c^{10} - 915427060973568ac^{10}}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+33}{2})} + \\
 & + \frac{-14369438932992a^2c^{10} + 789300695040a^3c^{10} + 10364456960a^4c^{10} - 57987072a^5c^{10} - 585728a^6c^{10}}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+33}{2})} + \\
 & + \frac{471963118075904c^{11} - 22473511796736ac^{11} - 525146259456a^2c^{11} + 9446621184a^3c^{11}}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+33}{2})} + \\
 & + \frac{143130624a^4c^{11} + 13580853837824c^{12} - 368959438848ac^{12} - 10368770048a^2c^{12} + 49201152a^3c^{12}}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+33}{2})} + \\
 & + \frac{745472a^4c^{12} + 280682823680c^{13} - 3633315840ac^{13} - 110100480a^2c^{13} + 3937402880c^{14}}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+33}{2})} + \\
 & + \frac{-16220160ac^{14} - 491520a^2c^{14} + 33554432c^{15} + 131072c^{16}}{\Gamma(\frac{c-a}{2}) \Gamma(\frac{c+a+33}{2})} + \\
 & + \frac{-89862698289116712960000a + 85324890802711872614400a^2 - 24339791709149468682240a^3}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
 & + \frac{1848662638364989276416a^4 + 142174831803544286208a^5 - 14128689229522969472a^6}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
 & + \frac{-691910888639279232a^7 + 29456241385999120a^8 + 2003112901726464a^9 + 8442002117824a^{10}}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
 & + \frac{-1541070913536a^{11} - 36768719008a^{12} - 131695872a^{13} + 5354048a^{14} + 71808a^{15} + 272a^{16}}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
 & + \frac{89862698310039502848000c - 237225605525247890073600ac + 136069971345811471921920a^2c}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
 & + \frac{-25391171053837880843904a^3c + 716896092330394563600a^4c + 169768665024302558016a^5c}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
 & + \frac{-4549433525270112824a^6c - 603006998552640264a^7c - 811735852298063a^8c + 856296032951568a^9c}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
 & + \frac{19055867672908a^{10}c - 136561582080a^{11}c - 9401762314a^{12}c - 104740944a^{13}c - 83164a^{14}c}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} +
 \end{aligned}$$

$$\begin{aligned}
& + \frac{4488a^{15}c + 17a^{16}c + 151900714793270299852800c^2 - 222317260362933570109440ac^2}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
& + \frac{86935826462889670705152a^2c^2 - 10498578498005560934400a^3c^2 - 130709572641046333440a^4c^2}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
& + \frac{64995157204393743360a^5c^2 + 569240425477565952a^6c^2 - 153732368174609664a^7c^2}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
& + \frac{-3683737613723904a^8c^2 + 88632480980736a^9c^2 + 4100909928192a^{10}c^2 + 33286885632a^{11}c^2}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
& + \frac{-478802688a^{12}c^2 - 9047808a^{13}c^2 - 39168a^{14}c^2 + 110587080829263811706880c^3}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
& + \frac{-110142828414252585566208ac^3 + 30491219362452157532160a^2c^3 - 2218879345457652218880a^3c^3}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
& + \frac{-123125165678847436160a^4c^3 + 11407910424275800512a^5c^3 + 420857420752347040a^6c^3}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
& + \frac{-14988359804811888a^7c^3 - 751432709419568a^8c^3 - 2272534730928a^9c^3 + 294025799760a^{10}c^3}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
& + \frac{4553874864a^{11}c^3 + 9521904a^{12}c^3 - 188496a^{13}c^3 - 816a^{14}c^3 + 46749510453913555304448c^4}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
& + \frac{-33834863483688908095488ac^4 + 6665806998098833293312a^2c^4 - 238579594717061419008a^3c^4}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
& + \frac{-30421003984057837568a^4c^4 + 917821747203609600a^5c^4 + 73059820249433600a^6c^4}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
& + \frac{-157798295424000a^7c^4 - 62268009914880a^8c^4 - 872510284800a^9c^4 + 5659906560a^{10}c^4}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
& + \frac{180956160a^{11}c^4 + 913920a^{12}c^4 + 12983151743474381881344c^5 - 7003414714095863095296ac^5}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
& + \frac{966520417081477172224a^2c^5 - 5648251088804062464a^3c^5 - 4004965917763569792a^4c^5}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
& + \frac{5827636174155840a^5c^5 + 6238246488744160a^6c^5 + 82327806259776a^7c^5 - 2241698356128a^8c^5}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
& + \frac{-53370757440a^9c^5 - 186611040a^{10}c^5 + 2261952a^{11}c^5 + 11424a^{12}c^5 + 2531556677162515824640c^6}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} +
\end{aligned}$$



$$\begin{aligned}
 & + \frac{-1025247629248753827840ac^6 + 95454131591298056192a^2c^6 + 1926646652820578304a^3c^6}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
 & + \frac{-317059290769227776a^4c^6 - 5379524729542656a^5c^6 + 281261068290048a^6c^6 + 6982736302080a^7c^6}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
 & + \frac{-12787568640a^8c^6 - 1327011840a^9c^6 - 8042496a^{10}c^6 + 360678165147224637440c^7}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
 & + \frac{-109272096592974594048ac^7 + 6389169743822018560a^2c^7 + 295182703698313728a^3c^7}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
 & + \frac{-15167501990930432a^4c^7 - 501813103118976a^5c^7 + 5401337369472a^6c^7 + 256468734720a^7c^7}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
 & + \frac{1356453120a^8c^7 - 11848320a^9c^7 - 71808a^{10}c^7 + 38483621128852996096c^8}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
 & + \frac{-8616765095449657344ac^8 + 273144966851936256a^2c^8 + 22322981949751296a^3c^8}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
 & + \frac{-382741945749504a^4c^8 - 22748774400000a^5c^8 - 54620037120a^6c^8 + 4549754880a^7c^8}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
 & + \frac{34467840a^8c^8 + 3119776625693032448c^9 - 505765929255088128ac^9 + 5644793400046592a^2c^9}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
 & + \frac{1048458298094592a^3c^9 - 687484956928a^4c^9 - 578577162240a^5c^9 - 4627786240a^6c^9 + 31595520a^7c^9}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
 & + \frac{239360a^8c^9 + 193417475491102720c^{10} - 22020595230965760ac^{10} - 94923107794944a^2c^{10}}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
 & + \frac{31826243125248a^3c^{10} + 265344155648a^4c^{10} - 7886241792a^5c^{10} - 79659008a^6c^{10}}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
 & + \frac{9164686025031680c^{11} - 701109000683520ac^{11} - 10927565688832a^2c^{11} + 609077753856a^3c^{11}}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
 & + \frac{7996225536a^4c^{11} - 44808192a^5c^{11} - 452608a^6c^{11} + 329153106673664c^{12} - 15851823955968ac^{12}}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
 & + \frac{-369950916608a^2c^{12} + 6691356672a^3c^{12} + 101384192a^4c^{12} + 8801770012672c^{13}}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} + \\
 & + \frac{-240921010176ac^{13} - 6769831936a^2c^{13} + 32169984a^3c^{13} + 487424a^4c^{13} + 169701539840c^{14}}{\Gamma(\frac{c-a+1}{2}) \Gamma(\frac{c+a+34}{2})} +
 \end{aligned}$$

$$+ \frac{-2205941760ac^{14} - 66846720a^2c^{14} + 2228224000c^{15} - 9191424ac^{15} - 278528a^2c^{15}}{\Gamma\left(\frac{c-a+1}{2}\right) \Gamma\left(\frac{c+a+34}{2}\right)} + \left. \frac{17825792c^{16} + 65536c^{17}}{\Gamma\left(\frac{c-a+1}{2}\right) \Gamma\left(\frac{c+a+34}{2}\right)} \right]$$

On simplification , we prove the result (8).

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