An Overview of Linear and Nonlinear Rayleigh-Taylor Instability

M. H. Obied Allah

Mathematics Department, Faculty of Science
Assiut University, Assiut, Egypt
E-mail: mhobied@hotmail.com

Abstract

The subject of Rayleigh- Taylor instability is really fascinating. A survey is given to discuss what is known and what is not known about the phenomenon. Many promising extensions and applications are recovered.

1   Introduction

Rayleigh- Taylor instability occurs when a heavy fluid is supported by a lighter fluid in a gravitational field [1]. The Rayleigh- Taylor instability is of great concern in the inertial confinement fusion experiments [2] and astrophysical phenomena [3]. For simple demonstration of the phenomenon, it comes from the different densities of the paints (see Fig. 1).
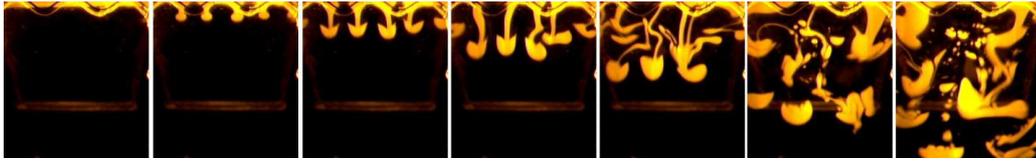
Figure 1: Shows the development of the Rayleigh-Taylor Instability with time

The same thing can be seen in the natural world. In the ocean and in the atmosphere, where, for reasons of temperature or salinity, denser gas and liquid is occasionally dumped over less-dense material and tunnels downwards, just like the paint. (see Fig. 2, Crab Nebula Image).

Figure 2: Understanding the Rayleigh- Taylor instability would give us how the Crab Nebula is

In nuclear fission, we get energy from splitting one atom into two atoms. But in nuclear fusion, we get energy when two atoms join together to form one. This would be a cleaner, safer and more abundant source of power. With current technology, deuterium-tritium can be achieved, but the tritium is radioactive, and it must be made from lithium. It is better to make deuterium- deuterium fusion, because it yields more energy and it is easer to extract deuterium from sea water But requires higher temperature that may be possible in the future. For the plasma to stay contained long enough, Inertial confinement fusion scientists must be aware of the time scales of the Rayleigh- Taylor instability in their capsule design and make sure that the fusion is faster than the

hydrodynamic instability. The Rayleigh-Taylor instability is the principal physical process inhibiting the achievement of sufficiently high compression of the fuel in inertial laser fusion. Tokamaks are some kind of experimental devices to confine a hot hydrogen gas, or plasma, long enough for nuclear reactions to produce useful amount of energy. A strong program is underway to increase the energy density and improve confinement devices such that their range of operation is limited by instabilities.

2 Plasma Physics and Magnetohydrodynamics

Plasma is an interesting object for investigation, experimentally and theoretically. The variations in density and temperature of plasma have opened incredible possibilities for research and applications. One of the most important factors in the flourishing of plasma physics has been the hope to realize controlled fusion. This world opens an energy source of such possibilities that it may change the face of the world. Moreover there is the realization of magneto hydrodynamic generators of much higher efficiency than the conventional ones. Magnetohydrodynamics [4] is a sub domain of plasma physics. It is not only restricted to plasma put also of use to molten metals or conducting fluids. Like in all physics, the equilibria, steady states and their stability play an important role in nuclear fusion. Stability has received an extra interest because it determines to a large extent whether or not has the plasma sufficiently long at ones disposition to realize fusion or to experiment with it.

3 Linear and Nonlinear Stability

The stability problems in fluid mechanics can be treated mathematically through: The linear theory which presupposes that the perturbations are infinitesimal. So, we neglect all products and powers through fluid media and the vibration of solid bodies are usually considered to be linear because the amplitudes involved are small. One needs to know the equilibrium state of a realistic configuration. Next, the problem of stability with respect to small perturbations about the equilibrium must be studied. If one could only show that static equilibria are possible, but they are unstable, fusion of magnetically confined system would be impossible. The nonlinear theories which attempt to allow for the finite amplitudes of the perturbations. It is often useful to derive special solutions of nonlinear evolution equations. The simplest types of such equations contain one time and one space variable, and the special solutions of interest include the traveling wave solutions. It is important to know if these are stable solutions of the evolution equation and analyze the stability of special solutions of the resulted ordinary or partial differential equations. In boundary layers, at high Reynolds number, the effects of viscosity are confined to a narrow region adjacent to a solid surface. In some cases these boundary layers, under the influence of pressure gradients, can significantly affect the entire flow field and the investigation of the linear and nonlinear stability of such flows becomes very important.

5   Normal Mode Analysis and Variational Principle

The normal mode analysis is due to Lord Rayleigh [5]. Its well known linearized version consists of linearizing and Fourier analyzing the system of relevant equations, for any arbitrary perturbation. The resulting eigenvalue problem is then solved. If there exist at least one mode which grows indefinitely in time, the system is linearly unstable. Otherwise it is linearly stable. By the variational analysis, an equilibrium state is stable if the change in potential energy is positive for all perturbations allowed the fluid equation; otherwise, the equilibrium is unstable. In other word, one has to calculate the minimal change in potential energy (or, more generally, to include dissipative effects; the minimal change in free energy) in order to get an answer whether the system is stable or not, without studying every possible detail of the whole configuration. The variational method is preferable to the normal mode analysis if one is only interesting in stability, yes or no. It needs fewer calculations than the normal mode analysis although it is still so difficult that only a few stability analyses were performed analytically. A nonlinear normal analysis was developed by Callebaut and Khater [6].When the growth rate is relatively small, normal mode method can not describe the theory of Rayleigh- Taylor instability. This is happened in elastic plastic media. Then, a new approach to Rayleigh- Taylor instability based on the Newton second law is applied [7].

6   Computer Simulation

The first attempted computer simulation of the nonlinear evolution of a large-scale instability was made by Roperts and Curtist in 1973. They have studied the nonlinear evolution of a resistive tearing made as a model for solar flares. Now a highly sophisticated computer cods are used to study the nonlinear stability problems. There is a complex phenomenology associated with the evolution of Rayleigh- Taylor instability. Very high resolution simulations are needed, to have an insight at the physics of turbulence and turbulent mixing in detail, hopefully contributing to a significant advance in understanding of these phenomena. A focusing on the modifications in the mixing layer structure and turbulence in response to the acceleration change, is given by Livescu et al. [8]. The results of 3D simulations of magnetohydrodynamic instabilities at the accretion disc- magnetospheric boundary [9], show that the instability is Rayleigh- Taylor. It manifests itself in the form of tall, thin tongues of plasma that penetrate the magnetosphere in the equatorial plane. Thus the magnetic field leads to suppression of the instability. Two- and three- dimensional simulations of Rayleigh- Taylor instability [10], with similar initial conditions, produce roughly comparable growth rates in the early stages of nonlinear growth. Magnetohydrodynamic simulations of Rayleigh- Taylor instability in young supernova remnants, has been studied by Byung et al. [11]. They show that Rayleigh- Taylor instability, can explain the observations. Their simulation shows also that Rayleigh- Taylor and Kelvin- Helmholtz instabilities amplify ambient magnetic fields locally and produce the clumpy radio shell. They have observed that strong magnetic field lines draped around the Rayleigh- Taylor fingers produce the radial magnetic field vector polarization. The emergence of a magnetic field into the atmosphere is a highly

nonlinear process. So, many numerical simulations have been carried out to investigate the dynamics of emerging flux. The two- dimensional simulations, explained the interaction of the emerging flux and a preexisting magnetic field, shoed that fast magnetic field reconnection between the coronal and a newly emerging magnetic field produced an X- ray brightening [12]. Shibata [13] showed that a horizontal flux sheet below the photosphere is unstable to the undular mode of magnetic field buoyancy instability. This instability which is also known as Parker instability forms an upward expanding magnetic loop rising in a self similar fashion. The model of Yokoyama et al. [14] predicts velocities of the rising magnetic field loop and dawnflows along the magnetic field consistent with observations. The horizontal magnetic field has been observed in quiet parts of the photosphere. The behavior of magnetohydrodynamic waves in the solar atmosphere permeated by a uniform straight magnetic field, has been studied by Danilko [15].He has shown that fast waves in a medium permeated by a horizontal magnetic field obey the magnetoacoustic Klein- Gordan equation.This implies that the propagation of these waves creates a wake, oscillating at a cutoff period of the medium. A numerical study by Byung et al. [16], of Rayleigh- Taylor instability in magnetic fluids, has showed that a normal magnetic field tends to increase the growth rate of fingers in the nonlinear regime, depending on the strength of the field. This is why the field lines tend to collimate the flow. On the other hand, they have shoed that a tangential magnetic field stabilizes the flow in general and suppresses structures on the scales smaller than the critical wavelength.

7   Factors Influencing the Development of Rayleigh-Taylor Stability

There are several physical mechanisms influencing the Rayleigh- Taylor instability, such as density gradient effects [17], surface tension, viscosity, resistivity, compressibility and the ablative stabilization. The magnetic field was found to have a significant effect that decreases the growth rate of Rayleigh Taylor instability. Several studies were carried out to investigate the effects of magnetic field on the Rayleigh Taylor instability which indicated that the magnetic field can reduce the linear growth rate of the Rayleigh- Taylor instability and if the magnetic field is strong enough, the Rayleigh- Taylor instability can even be damped. For instance, Yang et al. [18] indicated that the magnetic field effects strongly reduced the linear growth rate of the Rayleigh Taylor instability, especially when the perturbation wavelength is short. Stone and Gardiner [19] considered magnetic fields that are initially parallel to the interface, but have a variety of configurations, including uniform everywhere, uniform in the light fluid only, and fields that change direction at the interface. They found that strong magnetic fields do not suppress instability. However, by inhibiting secondary shear instabilities they reduce mixing between the heavy and light fluid. Moreover, in case of fields parallel to the interface produce long, isolated fingers separated by the critical wavelength k_c, which may be relevant to the morphology of the optical filaments in the Crab nebula. Wei et al. [20] investigated the effect of magnetic field on the Rayleigh-Taylor instability. They reported that the magnetic field can change the growth rate and, in particular, a very strong magnetic field suppresses the growth rate. Hoshoudy and El-

Ansary [21] studied the effect of a homogeneous horizontal magnetic field pervading both fluids. They found that the horizontal magnetic field helps to stabilize the instability. The horizontal magnetic field has critical strength in order to suppress the instability completely. Elhefnawy [22-23] studied the nonlinear evolution of interfacial waves separating two magnetic fluids subjected to tangential, normal and oblique magnetic fields in two dimensions, with the use of the method of multiple scales. He studied analytically and numerically the stability for both the tangential and normal magnetic fields. His results based on stability diagram indicated the effect of magnetic fields.

Electrohydrodynamic linear stability of finitely conducting flows through porous fluids with mass and heat transfer has been studied by Moatimid and Obied Allah [24]. Obied Allah and Yahia [25] studied nonlinear Rayleigh- Taylor instability in the presence of magnetic field, mass and heat transfer. Thermal effects on linear and nonlinear Rayleigh-Taylor instability in the presence of mass, heat transfer and magnetic field have been studied by Obied Allah and Ibraheem [26]. Kelvin- Helmholtz instability in the presence of mass, heat transfer, porous media, surface tension and magnetic field has been studied by Obied Allah [27]. Hsieh [28] has found that the effect of mass and heat transfer tends to enhance the stability of the system when the vapor is hotter than the liquid. Ho [29] studied the linear Rayleigh- Taylor instability of two viscous fluid of equal kinematical viscosities in the presence of mass and heat transfer. Khodaparast and Kawaji [30] studied the Rayleigh- Taylor instability and Kelvin- Helmholtz instability stabilities of liquid- vapor interface. They concluded that coupled-viscosity –phase change has stabilizing effect on Rayleigh- Taylor instability whereas it has destabilizing effect on Kelvin- Helmholtz instability. Viscous potential flow analysis of interfacial stability with mass and heat transfer through porous media has been studied by Obied Allah [31]. Asthana and Agrawal [32] studied viscous potential flow analysis of Kelvin-Helmholtz instability with mass and heat transfer. They found that heat and mass transfer have destabilizing effect on relative velocity when lower fluid velocity is low while having a stabilizing effect when lower fluid viscosity is high. They also studied viscous potential flow analysis of Kelvin- Helmholtz instability with mass and heat transfer in the presence of a horizontal electric field. They found that the stability criterion is given by a critical value of relative velocity of two fluids as well as critical value of the applied electric field [33]. The Rayleigh Taylor instability and internal waves in quantum plasmas have been studied by Vitaly Bychkov et. al. [34]. They have showed that quantum pressure always stabilizes the Rayleigh Taylor instability. They have also declared that in the case of stable stratification, quantum pressure modifies the dispersion relation of the inertial waves. Moreover, because of the quantum effects, the inertia waves may propagate in the transverse direction, which was impossible in the classical case. Quantum effects on Rayleigh Taylor instability has been studied by Jintao et al. [35]. It has been shown that Rayleigh Taylor instability is remarkably affected by quantum effects. They have shown that the magnetic field has a stabilizing effect similar to the behavior in classical plasmas. Hoshoudy [36] studied quantum effects on Rayleigh-Taylor instability in a vertical inhomogeneous rotating plasma. He has revealed that the Rayleigh-Taylor instability is affected by quantum and magnetic field effects. The role of compressibility effects on the Rayleigh- Taylor instability has been studied by many authors [37-39]. Sharp [40] has found a stabilizing effect of

compressibility. On the other hand, Bernstien and Book [41] showed that compressibility has a destabilizing effect. Khater and Obied Allah [42], has showed the stabilizing effect of rotation on Rayleigh- Taylor instability of an accelerating, compressible, perfectly conducting plane layer.

8   Conclusions and Open Problems

We have presented how the problem of Rayleigh –Taylor instability is important in astrophysics and nuclear fusion. Linear and nonlinear theories are discussed. Then we have also presented some factors affecting the problem. Of course this article can not recover all sides of the problem, but we would like to refer to some points and future problems.

a- During the motion of a fluid interface undergoing Rayleigh-Taylor instability, we have noticed that, vorticity is generated on the interface baronclinically. This vorticity is then subject to Kelvin-Helmholtz instability. When the fluids are immiscible, a sharp interface exists between them which deforms into a pattern containing rising bubbles of lighter fluid and falling spikes of heavier fluid. A strong shearing flow develops on the sides of the spike as the lighter and the heavier fluid pass by each other. This part of the interface is then susceptible to the Kelvin-Helmholtz instability. This mechanism is explained by sharp and also in the interface.

b- It is believed that the stability fluids will continue to be an exciting research area for many years because of its frequent occurrence in many science and technological applications. Towards controlled thermonuclear fusion, the plasma confinement time with magnetic field is still, very small (a small fraction of the second).This means that the goal will take a long time to use the energy released from nuclear fusion.

c- Many stability problems in micro polar and nano fluids, have not yet studied. For example, the Rayleigh-Taylor instability and related problems.

d- The boundaries in almost of the stability problems are taken rigid. In the view of the elastic boundaries, a lot of stability problems will be opened.

e- Rayleigh Taylor instability has not yet been demonstrated in quantum fluids.

f- Simulation of Rayleigh Taylor instability in three dimensions, still need further study.

g- Last and not least, the problems involving the instability of interfaces separating fluid bodies, the lows which govern the motion of the fluid where the interface meets a solid boundary, are only partially understood. Some of the difficulties at the foundation are more easily understood from the study of the equation which governs the energy of the two fluids. Any way, there are other methods which can be applied to linear and nonlinear stability problems. For examples, the role of negative energy waves gives an insight into the instability mechanism. A new mechanism for linear and nonlinear

significance and which appears to have potential implications for many stability problems.

References

[1]   G.I. Taylor, The instability of liquid surfaces when accelerated in a direction perpendicular to their planes, *I. Proc. Roy. Soc. (Lond.) Ser. A.*, 201(1950), 92-96.

[2]   R.G. Evans, The influence of self-generated magnetic fields on the Rayleigh-Taylor instability, *Plasma Physics and Controlled Fusion*, 28(7) (1986), 1021-1026.

[3]   A.J. Scannapieco, Atmospheric modes in laser fusion targets, *Phys. Fluids*, 24(9) (1981), 1699-1764.

[4]   J.P. Goedbloed, Lecture notes on ideal magnetohyrdrodynamics, *Rijnhuizen Report*, Brazil, (1979).

[5]   L. Rayleigh, *The Theory of Sound (2nd ed.)*, Dover ed., New York, (1877).

[6]   D.K. Callebaut and A.H. Khater, Variational principle for MHD stability, *Bulletin of the American Physical Society*, 24(8) (1979), 948-959.

[7]   A.R. Piriz, J.J.L. Cela, M.C.S. Moreno, O.D. Cortazar, N. Tahir and D.H.H. Hoffmann, A new approach to Rayleigh-Taylor instability: Application to accelerated elastic solids, *Nuclear Instruments and Methods in Physics Research A*, 577(2007), 250-256.

[8]   D. Levescu and T. Wei, Direct numerical simulations of Rayleigh-Taylor instability with gravity reversal, *Seventh International Computationl Fluid Dynamics (ICCFD7)*, Big Island, on Conference Hawaii, July 9-13 (2012).

[9]   A.K. Kulkarni and M.M. Romavova, Accretion of magnetized stars through Rayleigh-Taylor instability: Global 3D simulations, *Mon. Not. R. Astron. Soc.*, 386(2008), 673-687.

[10] W. Cabot, Comparizon of two- and three- dimensional simulations of miscible Rayleigh-Taylor instability, *Physics of Fluids*, UCRL-JRNL- 21993, February 27 (2006).

[11] B. IL Jun and M.L. Norman, MHD simulations of Rayleigh- Taylor instability in young supernova remnants, *Astrophysics and Space Science*, 233(1995), 267-272.

[12] H. Isope, T. Miyagoshi, K. Shibata and T. Yokoyama, Three–dimensional simulation of solar emerging flux using the earth simulate or I-magnetic Rayleigh-Taylor instability at the top of the emerging flux as the origin of filamentary structure, *Astron. Soc. Japan*, 85(2006), 423-440.

[13] K. Shibata, Numerical simulation of solar and astrophysical MHD flows, *Solar and Astrophysical Magnetohydrodynamic Flows*, 481(1996), 217-299.

[14] T. Yokoyama and K. Shibata, Magnetohydrodynamic simulation of a solar flare with chromospheric evaporation based on the magnetic reconnection model, *The Astrophysical Journal*, 549(2001), 1160-1174.

[15] D. Danilko, Numerical simulations of magnetohydrodynamic waves in the solar atmosphere, *Ph.D Thesis*, Maria Curie- Sklodowska University, Lublin, (2010).

[16] B. IL Jun, M.L. Norman and J.H. Stone, A numerical study of Rayleigh- Taylor instability in magnetic fluids, *The Astrophysical Journal*, 453(1995), 332-349.

[17] S. Chandrasekar, *Hydrodynamic and Hydromagnetic Stability*, Oxford University Press, (1961).

[18] L.F. Wang, B.L. Yang, W.H. Ye and X.T. He, Stabilization of the Rayleigh-Taylor instability in quantum magnetized plasmas, *Phys. Plasmas*, 19(2012), 072704-10.

[19] J.M. Stone and T. Gardiner, The magnetic Rayleigh- Taylor instability in three dimensions, *Astrophysical Journal*, 671(2007), 1726.

[20] W.Z. Wei, Z.W. Lu, L. Ding and Y.W. Hong, Effect of magnetic field and equilibrium flow on Rayleigh-Taylor instability, *Chin. Phys. Lett.*, 21(2004), 2001-2004.

[21] G.A. Hoshoudy and N.F. El-Ansary, Study of the effect of viscosity and homogeneous horizontal magnetic field on Rayleigh-Taylor instability, *Z. Angew. Math. Mech.*, 83(2003), 61-68.

[22] A.F. Elhefnawy and Z. Angew, Stability properties of a cylindrical flows in magnetic fluids, *Math Phys*, 44(3) (1993), 495-509.

[23] A.F. Elhefnawy and Z. Angew, Nonlinear instability of superposed magnetic fluids in the presence of an oblique magnetic field, *Math Phys*, 46(1995), 39-46.

[24] G.M. Moatimid and M.H.O. Allah, Electrohydrodynamic linear stability of finitely conducting flows through porous fluids with mass and heat transfer, *Applied Mathematical Modeling*, 34(2010), 3118-3129.

[25] M.H.O. Allah and A.A. Yahia, Nonlinear Rayleigh-Taylor instability in the presence of magnetic field, mass and heat transfer, *Astrophysics and Space Science*, 181(1991), 183-194.

[26] M.H.O. Allah and M.A. Ibraheem, *Indian Journal of Pure and Applied Mathematics*, 31(2000), 1545-1558.

[27] M.H.O. Allah, Kelvin-Helmholtz instability in the presence of mass, heat transfer, porous media, surface tension and magnetic field, *Mechanics and Mechanical Engineering*, 9(2005), 179-187.

[28] D.Y. Hsieh, Effects of heat and mass transfer on Rayleigh- Taylor instability, *Trans. ASME*, 94(D) (1972), 156-162.

[29] D.Y. Hsieh, Interfacial stability with mass and heat transfer, *Phys. Fluids*, 21(1978), 745-758.

[30] S.P. Ho, Linear Rayleigh-Taylor stability of viscous fluids with mass and heat transfer, *J. Fluid Mech.*, 101(1980), 111-128.

[31] M.H.O. Allah, Viscous potential flow analysis of interfacial stability with mass transfer through porous media, *Applied Mathematics and Computation*, 217(2011), 7920-7931.

[32] R. Asthana and G.S. Agrawal, Viscous potential flow analysis of Kelvin-Helmholtz instability with mass transfer and vaporization, *Physica*, A 382(2007), 389-404.

[33] R. Asthana and G.S. Agrawal, Viscous potential flow analysis of electrohydrodynamic Kelvin-Helmholtz instability with mass and heat transfer, *International Journal of Engineering Science*, 48(2010), 1925-1936.

[34] V. Bychkov, M. Marklund and M. Modestov, The Rayleigh-Taylor instability and inertial waves in quantum plasmas, *Physics Letters*, A (2008), 1-6.

[35] J. Cao, H. Ren, Z. Wu and P.K. Chu, Quantum effects on Rayleigh-Taylor instability in magnetized plasma, *Physics of Plasmas*, 15(2008), 012110-4.

[36] G.A. Hoshoudy, Quantum effects on Rayleigh-Taylor instability in a vertical inhomogeneous rotating plasma, *Physics of Plasmas*, 16(2009), 024501-4.

[37] H. Yong, H.X. Wei and J.Z. He, Compressibility effects on the Rayleigh-Taylor instability growth rates, *Chin. Phys. Lett.*, 25(2008), 1015-1518.

[38] B.K. Shivamoggi, Rayleigh-Taylor instability of superposed barotropic fluids, *ZAAMP*, 3(2012), 521-583.

[39] L. Baker, Compressible Rayleigh-Taylor instability, *Phys Fluids*, 26(1983), 950-953.

[40] D.H. Sharp, An overview of the Rayleigh-Taylor instability, *Physica*, 12(D) (1984), 3-18.

[41] I.B. Bernstein and D.L. Book, Effects of compressibility on Rayleigh-Taylor instability, *Phys. Fluids*, 26(1983), 453-458.

[42] A.H. Khater and M.H.O. Allah, Effects of rotation on Rayleigh-Taylor instabilities of an accelerating, compressible perfectly conducting plane layer, *Astrophysics and Space Science*, 106(2) (1984), 245-255.