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Allee Effect in a New Population Model and Stability Analysis

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Abstract

In this paper, we investigate the local stability conditions of fixed point of a general nonlinear discrete-time population model with and without Allee effect which occur at low population density. We conclude that the Allee effect at different times increase the local stability of equilibrium point of the population dynamic model. The numerical simulations confirm the analytical results.

Keywords: *Local stability analysis, Fixed point, Allee effect.*

1 Introduction

Time evolution of population of species is generally modelled by either the continuous-time models or the discrete-time models. Although a discrete-time population model represents a richer dynamic picture (specifically, in terms of numerical simulation), the continuous-time population models are more appropriate to the nature except for the non-overlapping generations. The study of the stability in population models with and without Allee effect have been a very important topic in many areas of ecology and biology [1, 2, 3, 5]. This effect is firstly introduced by Allee in 1931 as "negative density dependence when the growth rate of the population decreases in low population density

[5]".The main causes of the Allee effect are the difficulty in finding mates, imbreeding depression, social dysfunction at small population sizes, predator avoidance and food exploitation.The most commonly observed mechanism is mate limitation, which causes Allee effects in both animals and plants. In late years, many authors have studied the stability of different population models with and without Allee effect [4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15].

The aim of this paper is to investigate and compare the stability analysis of equilibrium solutions with and without Allee effect by considering a more general state of the model studied in [[10]]. We consider a general nonlinear discrete-time population model with of the following form

$$N_{t+1} = \lambda N_t f(N_t, N_{t-1}, N_{t-2}), \qquad \lambda > 0 \tag{1}$$

where λ is per capita growth rate, which is always nonnegative, N_t represents the population density at time t and $f(N_t, N_{t-1}, N_{t-2})$ is the function describing interactions among mature individuals.

The general assumptions on the f are as follows:

1) $\frac{\partial f}{\partial N_t}(N,N,N) < 0$, $\frac{\partial f}{\partial N_{t-1}}(N,N,N) < 0$ and $\frac{\partial f}{\partial N_{t-2}}(N,N,N) < 0$ for $N \in [0,\infty)$, that is, f continuously decreases as density increases. From a biological point of view, this means that the reproductive output of an individual never increases as the population size increases. In other words, individual fitness never increases as the population size increases.

2) $f(0,0,0)$ is a positive finite number [10].

The remainder of this manuscript is organized as follows: In the following section, we present a characterization of the stability of the positive fixed points of Eq.(1) with and without Allee effect. In Section 3 we present some numerical simulations for supporting the analytical results obtained. The final section , we summarize obtained results.

2 Local Stability Analysis

In this section, we studied the local stability conditions of the nonnegative fixed point of Eq. (1) with and without Allee effect. We compared the local stability of these models. Assume that Eq. (1) has one nonnegative fixed point.

2.1 Local Stability Analysis of Eq. (1)

Theorem 2.1 *The fixed point N^* of Eq. (1) is locally stable if the inequalities*

$$N^* \frac{f_{N_{t-1}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)} - N^* \frac{f_{N_t}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)} - N^* \frac{f_{N_{t-2}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)} < 2 \tag{2}$$

$$N^*\frac{f_{N_{t-2}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}\left[N^*\frac{f_{N_{t-2}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}-N^*\frac{f_{N_t}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}-1\right] -N^*\frac{f_{N_{t-1}}(N^*,N^*,N^*)}{f(N^*,N^*)}<1 \tag{3}$$

$$N^*\frac{f_{N_{t-2}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}\left[N^*\frac{f_{N_t}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}+N^*\frac{f_{N_{t-2}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}+1\right] +N^*\frac{f_{N_{t-1}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}<1 \tag{4}$$

hold.

Proof: Let N^* be a positive fixed point of Eq.(1). By the fixed point definition, one has

$$1=\lambda f(N^*_, N^*,N^*). \tag{5}$$

If linearization of Eq. (1) is made around N^*,we obtain Eq.(6)

$$\begin{aligned} N_{t+1} &= \left(1+N^*\frac{f_{N_t}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}\right)N_t-\left(N^*\frac{f_{N_{t-1}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}\right)N_{t-1} \\ &\quad +\left(N^*\frac{f_{N_{t-2}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}\right)N_{t-2} \end{aligned} \tag{6}$$

so that its characteristic polynomial is

$$\begin{aligned} p(\mu) &= \mu^3-\left(1+N^*\frac{f_{N_t}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}\right)\mu^2-\left(N^*\frac{f_{N_{t-1}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}\right)\mu \\ &\quad -\left(N^*\frac{f_{N_{t-2}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}\right) \end{aligned} \tag{7}$$

Then, from Eq. (7), we write

$$\begin{aligned} p &= 1+N^*\frac{f_{N_t}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}, q=N^*\frac{f_{N_{t-1}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}, \\ r &= N^*\frac{f_{N_{t-2}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)} \end{aligned}$$

We conclude that N^* is locally stable if

$$|p+r| < 1-q, \tag{8}$$

$$|pr+q| < 1-r^2 \tag{9}$$

with the help of Schur-Cohn Criteria [1, 2]. From where, we have

$$\left|1+N^*\frac{f_{N_t}(N^*,N^*,N^*)}{f(N^*,N^*N^*)}+N^*\frac{f_{N_{t-2}}(N^*,N^*,N^*)}{f(N^*,N^*N^*)}\right| <1-N^*\frac{f_{N_{t-1}}(N^*,N^*N^*)}{f(N^*,N^*N^*)} \tag{10}$$

$$\left|\left(1+N^*\frac{f_{N_t}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}\right)N^*\frac{f_{N_{t-2}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}+N^*\frac{f_{N_{t-1}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}\right| < 1-N^{*2}\frac{f_{N_{t-2}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)} \tag{11}$$

From (10) inequality, we get

$$N^*\frac{f_{N_{t-1}}(N^*,N^*N^*)}{f(N^*,N^*N^*)}-N^*\frac{f_{N_t}(N^*,N^*,N^*)}{f(N^*,N^*N^*)}-N^*\frac{f_{N_{t-2}}(N^*,N^*N^*)}{f(N^*,N^*N^*)}<2 \tag{12}$$

and

$$N^*\frac{f_{N_t}(N^*,N^*,N^*)}{f(N^*,N^*N^*)}+N^*\frac{f_{N_{t-2}}(N^*,N^*,N^*)}{f(N^*,N^*N^*)}+N^*\frac{f_{N_{t-1}}(N^*,N^*,N^*)}{f(N^*,N^*N^*)}<0. \tag{13}$$

(13) inequality is always provided for $f_{N_t}<0, f_{N_{t-1}}<0, f_{N_{t-2}}<0$. From (11) inequality, we obtain as follows:

$$N^*\frac{f_{N_{t-2}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}\left[N^*\frac{f_{N_{t-2}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}-N^*\frac{f_{N_t}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}-1\right] -N^*\frac{f_{N_{t-1}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}<1 \tag{14}$$

$$N^*\frac{f_{N_{t-2}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}\left[N^*\frac{f_{N_t}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}+N^*\frac{f_{N_{t-2}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}+1\right] +N^*\frac{f_{N_{t-1}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}<1 \tag{15}$$

So the proof is completed.

2.2 Local Stability Analysis on the Discrete Delay Model (1) with Allee Effect

In this subsection we study the local stability analysis of the nonnegative fixed point of Eq. (1) with Allee effect at different times as t and $t-1, t-2$.

2.3 Allee Effect at Time t

We consider the following nonlinear delay difference equation with Allee effect to discrete delay model (1)

$$N_{t+1}=\lambda^* N_t \alpha(N_t) f(N_t, N_{t-1,}N_{t-2}), \qquad \lambda^*>0 \tag{16}$$

where the function f satisfies the conditions (1) and (2). Biologically, it is assumed that α satisfies the following assumptions:

(3) "If $N = 0$, then $\alpha(N) = 0$; that is, there is no reproduction without partners.

(4) $\alpha'(N) > 0$ for $N \in (0, \infty)$; that is; Allee effect decreases as density increases.

(5) $\lim_{N\to\infty}\alpha(N) = 1$; that is, Allee effect vanishes at high densities[10]."

Theorem 2.2 *The fixed point N^* of Eq. (16) is locally stable if the inequalities*

$$N^*\left[\frac{f_{N_{t-1}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)} - \frac{f_{N_t}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)} - \frac{\alpha'(N^*)}{\alpha(N^*)} - \frac{f_{N_{t-2}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}\right] < 2 \tag{17}$$

$$N^*\left[\frac{f_{N_t}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)} + \frac{\alpha'(N^*)}{\alpha(N^*)} + \frac{f_{N_{t-2}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)} + \frac{f_{N_{t-1}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}\right] < 0 \tag{18}$$

$$\begin{aligned} &N^{*^2}\frac{f^2_{N_{t-2}}(N^*,N^*,N^*)}{f^2(N^*,N^*,N^*)} - N^*\frac{f_{N_{t-2}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)} \\ &-N^{*2}\frac{f_{N_t}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}\frac{f_{N_{t-2}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)} - N^{*2}\frac{\alpha'(N^*)}{\alpha(N^*)}\frac{f_{N_{t-2}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)} < 1 \end{aligned} \tag{19}$$

$$\begin{aligned} &N^{*^2}\frac{f^2_{N_{t-2}}(N^*,N^*,N^*)}{f^2(N^*,N^*,N^*)} + N^*\frac{f_{N_{t-2}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)} \\ &+N^{*2}\frac{f_{N_t}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}\frac{f_{N_{t-2}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)} + N^{*2}\frac{\alpha'(N^*)}{\alpha(N^*)}\frac{f_{N_{t-2}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)} < 1 \end{aligned} \tag{20}$$

hold.

Proof: Likewise, if p and q values are calculated for Eq. (16), we obtain

$$p = 1 + N^*\frac{f_{N_t}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}N^* + \frac{\alpha'(N^*)}{\alpha(N^*)}N^* \tag{21}$$

$$q = N^*\frac{f_{N_{t-1}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}, r = N^*\frac{f_{N_{t-2}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)} \tag{22}$$

If the values of p, q and r are written in the inequalities (8)-(9), we can get the result claimed.

2.4 Allee Effect at Time t-1

We now incorporate an Allee effect into the discrete model (1) as follow:

$$N_{t+1} = \lambda^* N_t \alpha(N_{t-1}) f(N_t, N_{t-1}, N_{t-2}),\ \lambda^* > 0. \tag{23}$$

Then, we can state the following theorem.

Theorem 2.3 *The fixed point N^* of Eq. (23) is locally stable if the inequalities*

$$N^*\frac{f_{N_{t-1}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}+N^*\frac{\alpha'(N^*)}{\alpha(N^*)}-N^*\frac{f_{N_t}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}-N^*\frac{f_{N_{t-2}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}<2, \tag{24}$$

$$N^*\frac{f_{N_t}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}+N^*\frac{f_{N_{t-1}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}-N^*\frac{f_{N_{t-2}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}+N^*\frac{\alpha'(N^*)}{\alpha(N^*)}<0, \tag{25}$$

$$\begin{aligned}&N^{*2}\left[\frac{f^2_{N_{t-2}}(N^*,N^*,N^*)}{f^2(N^*,N^*,N^*)}-\frac{f_{N_t}(N^*,N^*,N^*)f_{N_{t-2}}(N^*,N^*,N^*)}{f^2(N^*,N^*,N^*)}\right]\\&-N^*\left[\frac{f_{N_{t-2}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}+\frac{f_{N_{t-1}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}+\frac{\alpha'(N^*)}{\alpha(N^*)}\right]<1,\end{aligned} \tag{26}$$

$$\begin{aligned}&N^{*2}\left[\frac{f_{N_t}(N^*,N^*,N^*)f_{N_{t-2}}(N^*,N^*,N^*)}{f^2(N^*,N^*,N^*)}+\frac{f_{N_{t-2}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}\right]\\&+N^*\left[\frac{f_{N_{t-2}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}+\frac{f_{N_{t-1}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}+\frac{\alpha'(N^*)}{\alpha(N^*)}\right]<1\end{aligned} \tag{27}$$

hold.

Proof: According to Eq. (23), p, q and r expressions are as follows:

$$\begin{aligned}p &= 1+N^*\frac{f_{N_t}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)},\\ q &= N^*\left[\frac{\alpha'(N^*)}{\alpha(N^*)}+\frac{f_{N_{t-1}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}\right],\end{aligned}$$

$$r=N^*\frac{f_{N_{t-2}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}$$

For N^* fixed point to be stable, if the inequalities in (8-9) are evaluated according to p, q and r values, then we can reach the result claimed.

2.5 Allee Effect at Time t-2

We now incorporate an Allee effect into the discrete delay model (1) as follow:

$$N_{t+1}=\lambda^* N_t\alpha(N_{t-2})f(N_t,N_{t-1},N_{t-2}),\ \lambda^*>0. \tag{28}$$

Then, we can state the following theorem.

Theorem 2.4 *The fixed point N^* of Eq. (28) is locally stable if the inequalities*

$$N^*\frac{f_{N_{t-1}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}+N^*\frac{\alpha'(N^*)}{\alpha(N^*)}-N^*\frac{f_{N_t}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}-N^*\frac{f_{N_{t-2}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}<2, \tag{29}$$

$$N^*\frac{f_{N_t}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}+N^*\frac{f_{N_{t-1}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}-N^*\frac{f_{N_{t-2}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}+N^*\frac{\alpha'(N^*)}{\alpha(N^*)}<0, \quad (30)$$

$$\begin{aligned} &N^{*2}\left[\frac{f^2_{N_{t-2}}(N^*,N^*,N^*)}{f^2(N^*,N^*,N^*)}-\frac{f_{N_t}(N^*,N^*,N^*)f_{N_{t-2}}(N^*,N^*,N^*)}{f^2(N^*,N^*,N^*)}\right] \\ &-N^*\left[\frac{f_{N_{t-2}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}+\frac{f_{N_{t-1}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}+\frac{\alpha'(N^*)}{\alpha(N^*)}\right]<1, \end{aligned} \quad (31)$$

$$\begin{aligned} &N^{*2}\left[\frac{f_{N_t}(N^*,N^*,N^*)f_{N_{t-2}}(N^*,N^*,N^*)}{f^2(N^*,N^*,N^*)}+\frac{f_{N_{t-2}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}\right] \\ &+N^*\left[\frac{f_{N_{t-2}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}+\frac{f_{N_{t-1}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}+\frac{\alpha'(N^*)}{\alpha(N^*)}\right]<1 \end{aligned} \quad (32)$$

hold.

Proof: According to Eq. (23), p, q and r expressions are as follows:

$$p=1+N^*\frac{f_{N_t}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}, q=N^*\frac{f_{N_{t-1}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)},$$

$$r=N^*\frac{f_{N_{t-2}}(N^*,N^*,N^*)}{f(N^*,N^*,N^*)}+\frac{\alpha'(N^*)}{\alpha(N^*)}$$

For N^* fixed point to be stable, if the inequalities in (8-9) are evaluated according to p, q and r values, we get the result claimed.

Theorem 2.5 *Allee effect at time t, $t-1$ and $t-2$ increases the stability of the Eq. (1).*

Proof: Let's $x=\frac{df((N^*,N^*,N^*)}{dN_t}<0$, $y=\frac{df((N^*,N^*,N^*)}{dN_{t-1}}<0$, $z=\frac{df((N^*,N^*,N^*)}{dN_{t-2}}<0$ and $t=N^*\frac{\alpha'(N^*)}{\alpha(N^*)}$. The equation (1) is stable if and only if the following

$$\begin{aligned} y-x-z &< 2 \\ z^2-xz-y-z &< 1 \\ z^2+xz+y+z &< 1 \end{aligned} \quad (33)$$

hold. Similarly, the Eq. (1) with Allee effect at time t, $t-1$ and $t-2$ is stable if and only if, respectively, as follows:

$$\begin{aligned} -x+y-z-t &< 2 \\ x+y+z+t &< 0 \\ z^2-z-xz-tz &< 1 \\ z^2+z+xz+tz &< 1, \end{aligned} \quad (34)$$

$$\begin{aligned} -x+y-z+t &< 2 \\ x+y+z+t &< 0 \\ z^2-z-xz-y-t &< 1 \\ z^2+z+xz+y+t &< 1 \end{aligned} \tag{35}$$

and

$$\begin{aligned} -x+y-z-t &< 2 \\ x+y+z+t &< 0 \\ z^2+2zt+t^2-xz-xt-z-t &< 1 \\ z^2+2zt+t^2+xz-xt-z-t &< 1. \end{aligned} \tag{36}$$

It is easily seen that the inequalities (34), (35) and (36) is provided for at least one value t under the inequalities (33).

3 Numerical Simulations

In this section, we report on numerical simulations of the presented model with and without Allee effect to support the analytical results obtained in previous sections. We used the Maple software for computations and Sigmaplot for graphic arts.

Let's define the functions f and α as follows,

$f(N_t, N_{t-1}, N_{t-2}) = 1 - \frac{N_t}{K} - \frac{N_{t-1}}{K} - \frac{N_{t-2}}{K}$, $\alpha(N_t) = \frac{N_t}{\alpha+N_t}$ where $K > 0$ represents the carrying capacity and α is a positive constant. Thus, Eqs.(1),(16),(23)and (28) reduce to following forms, respectively.

$$N_{t+1} = \lambda N_t\left(1 - \frac{N_t}{K} - \frac{N_{t-1}}{K} - \frac{N_{t-2}}{K}\right) \tag{37}$$

$$N_{t+1} = \lambda^* N_t\left(\frac{N_i}{\alpha+Ni}\right)\left(1 - \frac{N_t}{K} - \frac{N_{t-1}}{K} - \frac{N_{t-2}}{K}\right),\ i = t-2, t-1, t. \tag{38}$$

where $\lambda > 0$ and $\lambda^* = \frac{\lambda}{\alpha(N^*)}$. If initial conditions are taken as $N_{-2} = 0.1, N_{-1} = 0.2, N_0 = 0.3$ and we choose $K = 1, \lambda = 1.9, \alpha = 0.03$, in Eqs.(37) and (38), the positive fixed points and normalized growth rate are obtained as $N^* = 0.1578947368$ and $\lambda^* = 2.261$.

In Figures 1, 2 and 3, we graph the 2D trajectories of the population dynamics models (1) with and without Allee effect at time t, $t-1, t-2$, respectively.

$N_{t+1} = \lambda N_t(1 - N_t/K - N_{t-1}/K - N_{t-2}/K)$ and $N_{t+1} = \lambda^* N_t(\frac{N_t}{\alpha+Nt})$
$(1 - \frac{N_t}{K} - \frac{N_{t-1}}{K} - \frac{N_{t-2}}{K})$ with $\lambda = 1.9, K = 1, \alpha = 0.03, \lambda = \lambda^*\alpha(N^*)$
and the initial conditions $N_{-2} = 0.1, N_{-1} = 0.2, N_0 = 0.3$.

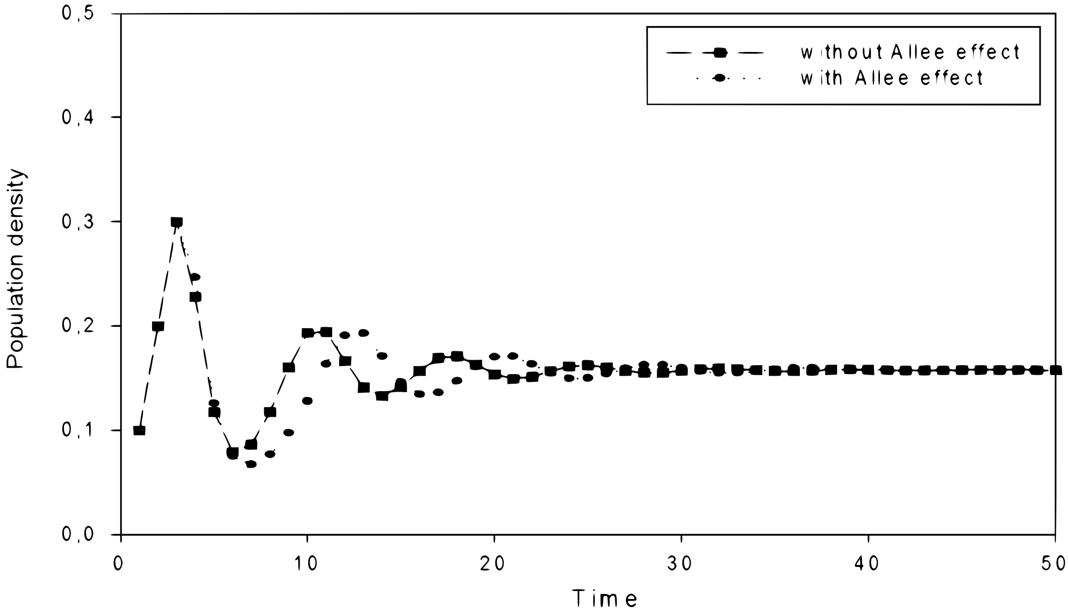

Figure 1: Density-time graphs model

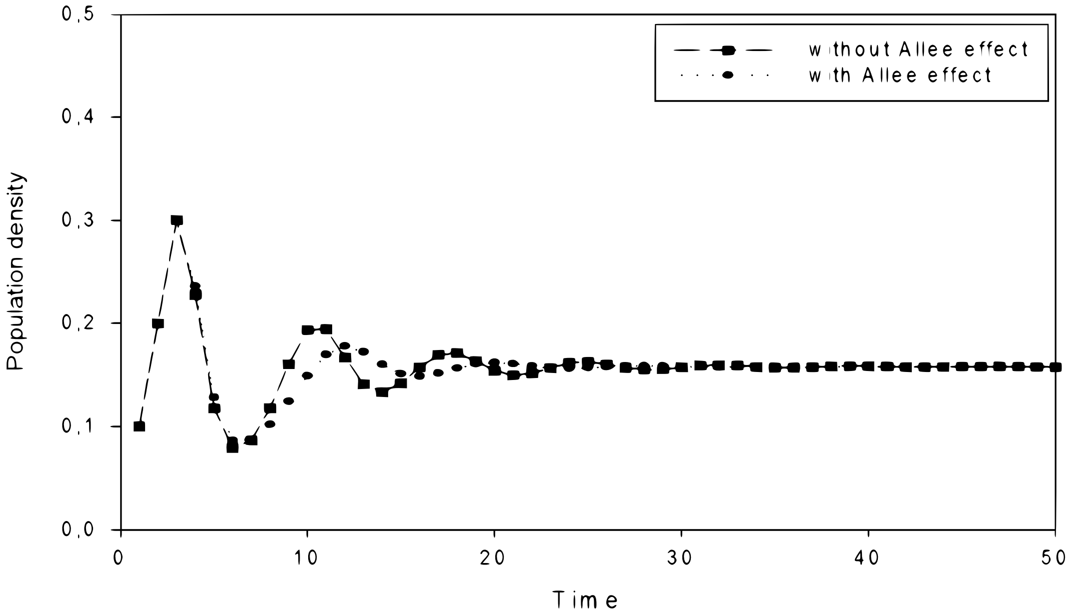

Figure 2: Density-time graphs model

$N_{t+1} = \lambda N_t(1 - N_t/K - N_{t-1}/K - N_{t-2}/K)$ and $N_{t+1} = \lambda^* N_t(\frac{N_{t-1}}{\alpha+Nt-1})$ $(1 - \frac{N_t}{K} - \frac{N_{t-1}}{K} - \frac{N_{t-2}}{K})$ with $\lambda = 1.9, K = 1, \alpha = 0.03, \lambda = \lambda^* \alpha\,(N^*)$ and the initial conditions$N_{-2} = 0.1, N_{-1} = 0.2, N_0 = 0.3$.

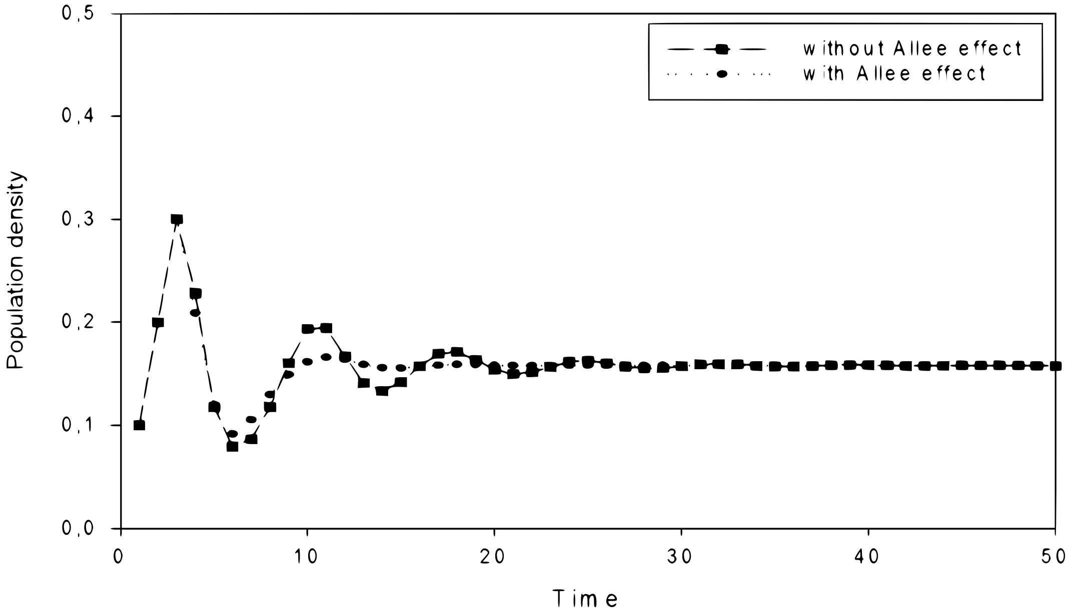

Figure 3: Density-time graphs model

$N_{t+1} = \lambda N_t(1 - N_t/K - N_{t-1}/K - N_{t-2}/K)$ and $N_{t+1} = \lambda^* N_t(\frac{N_{t-2}}{\alpha+Nt-2})$ $(1 - \frac{N_t}{K} - \frac{N_{t-1}}{K} - \frac{N_{t-2}}{K})$with $\lambda = 1.9, K = 1, \alpha = 0.03, \lambda = \lambda^* \alpha(N^*)$ and the initial conditions $N_{-2} = 0.1, N_{-1} = 0.2, N_0 = 0.3$.

4 Conclusion and Discussion

This study give the local stability analysis of the fixed point of a general population model with and without Allee effect. Also, we compare the local stability conditions of both models. In conclusion, Allee effect at different times increase the local stability of fixed point of (1).

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