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Some Aspects of Pairwise Fuzzy Semi Preopen Sets in Fuzzy Bitopological Spaces

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Abstract

In this paper, considering a bitopological space, the concepts of pairwise fuzzy semi preopen sets, pairwise fuzzy semi pre-T_1 space, pairwise fuzzy semi pre-T_2 space and pairwise fuzzy semi pre-continuity are introduced. Using different conditions on two fuzzy topologies and their mixed fuzzy topology, some results are established on pairwise separation axioms and continuity.

Keywords: *Fuzzy semi pre-T_1 space, Fuzzy semi pre-T_2 space, Pairwise fuzzy semi preopen sets, Pairwise fuzzy semi pre-T_1 space, Pairwise fuzzy semi pre-T_2 space, Pairwise fuzzy semi pre-continuity, Fuzzy regular, Fuzzy bitopological space, Mixed fuzzy topology, Quasi-coincidence, Quasi-neighbourhood.*

1 Introduction

The concept of fuzzy set was introduced by the American mathematician L. A. Zadeh in 1965, in his celebrated paper [16]. Since its inception, fuzzy set theory has entered into a wide variety of disciplines of science, technology and humanities. General Topology is one of the important branches of mathematics in which fuzzy set theory has been applied systematically. The synthesis of ideas, notions and methods of fuzzy set theory with general topology has resulted in fuzzy topology as a new branch of mathematics. Chang [2] introduced the concept of fuzzy topological space and considered fuzzy continuity, fuzzy compactness etc. After that, several authors have successfully attempted to relate numerous concepts of general topology to the fuzzy topology. The study of mixed topology and some of its related topics is known since the middle of this century. The study of mixed topology originated from the work of Polish mathematicians Alexiewicz and Semadeni. N. R. Das, P. C. Baishya and P. Das have studied various aspects of mixed fuzzy topological spaces [4, 5]. In paper [4], N. R. Das and P. C. Baishya have constructed a fuzzy topology on a set X called mixed fuzzy topology from two given fuzzy topologies on X with the help of closure of neighbourhoods of one topology with respect to the other topology. Analogous to the concept of Bitopological spaces studied by J. C. Kelly [8] and others [7, 14, 15], the concept of "fuzzy bitopological space" was introduced by Wu Congxin and Wu Jianrong [3] in 1992. The study of fuzzy bitopological spaces was continued further by N. R. Das and P. C. Baishya [4] who proposed different pairwise separation axioms as generalizations of natural separation axioms in the sense that such notions reduce to the natural separation axioms of a fuzzy topological space provided the two fuzzy topologies coincide. Also, the relations between the pairwise separation axioms and natural fuzzy separation axioms of the mixed fuzzy topological space are investigated. In paper [1] D. Andijevic has given the definitions of semi pre open sets and semi pre continuity and also some theorems on semi preopen sets and semi pre cntinuity. J. C. Kelly [8] first generalized a few separation axioms of topological spaces to bitopological spaces and called them pairwise separation axioms. E. P. Lane [9] and C. W. Patty [14] and others studied further in this direction. So far, no attempt has been made to relate the above concepts to bitopological space. We have worked in this direction and following the definition of fuzzy semi preopen set and fuzzy semi pre continuity [13], we have defined pairwise fuzzy semi preopen set, pairwise fuzzy semi pre-T_1 space and pairwise fuzzy semi pre-T_2 space. We have also defined pairwise fuzzy semi pre-continuity using the definition of fuzzy semi pre-continuity [13].

2 Preliminaries

We recall some definitions and results used in this sequel. The remaining definitions and notations which are not explained can be referred to [4, 12, 16].

Definition 2.1 *[10] A fuzzy set in X is called a fuzzy point if and only if it takes the value* 0 *for all $y \in X$ except one, say $x \in X$. If its value at x is*

λ $(0 < \lambda \leq 1)$ *we denote this fuzzy point by* x_λ*, where the point* x *is called its support.*

Definition 2.2 *[10] A fuzzy point* x_λ *is said to be quasi-coincident (q-coincident) with a fuzzy set* μ*, written as* $x_\lambda q\mu$*, if* $\lambda > \mu'(x)$ *i.e* $\lambda + \mu(x) > 1$*. A fuzzy set* μ *is said to be q-coincident with a fuzzy set* μ_1 *written as* $\mu q\mu_1$*, if there exists* $x \in X$ *such that* $\mu(x) > \mu_1'(x)$ *i.e* $\mu(x) + \mu_1(x) > 1$*.*

Definition 2.3 *[10] A fuzzy set* μ *in a fuzzy topological space* (X, τ) *is said to be a fuzzy q-nbd (fuzzy nbd) of a fuzzy point* x_λ *if there exists an* $U \in \tau$ *such that* $x_\lambda qU \subseteq \mu$ $(x_\lambda \in U \subseteq \mu)$*. Obviosuly,* 1_X *is a q-nbd of every fuzzy point in* X *but* 0_X *is not a q-nbd of any fuzzy point in* X*.*

Definition 2.4 *[6] A fuzzy topological space* (X, τ) *is called fuzzy regular if and only if* $\alpha \in (0, 1)$*,* $U \in \tau^c$*,* $x \in X$ *and* $\alpha < 1 - U(x)$ *imply that there exists* V *and* W *in* T *with* $\alpha < V(x)$*,* $U \subseteq W$ *and* $V \subseteq 1 - W$*.* τ^c *is the collection of all* τ*-closed fuzzy subsets of* X*.*

Definition 2.5 *[11] A topological space* X *is said to be semi pre-*T_1 *if for any two distinct points* x *and* y *of* X*, there exists semi preopen sets* U *and* V *such that* $x \in U$ *and* $y \in U$ *and also* $x \notin V$ *and* $y \in V$*.*

Definition 2.6 *[11] A topological space* X *is said to be semi pre-*T_2 *if for any pair of distinct points* x*,* y *of* X*, there exists disjoint semi preopen sets* U *and* V *such that* $x \in U$ *and* $y \in U$*.*

Definition 2.7 *[13] A fuzzy set* μ *in a fuzzy topological space* (X, τ) *is said to be fuzzy semi preopen if* $\mu \subseteq cl(int(cl\mu))$*.*

Definition 2.8 *[4] A triplet* (X, τ_1, τ_2) *of a non-empty set* X *together with two fuzzy topologies* τ_1 *and* τ_2 *on* X *is called a fuzzy bitopological space.*

Definition 2.9 *[4] Let* (X, τ_1) *and* (X, τ_2) *be two fuzzy topological spaces and let* $\tau_1(\tau_2) = \{A \in I^X|$ *for every* $x_\alpha qA$*, there exists a* τ_2*-q-nbd* A_α *of* x_α *such that* τ_1*-closure,* $cl(A_\alpha) \subseteq A\}$*. Then* $\tau_1(\tau_2)$ *is a fuzzy topology on* X*.*

Lemma 2.10 *[4] Let* τ_1 *and* τ_2 *be two fuzzy topologies on a set* X*. If every* τ_1 *quasi-neighbourhood of* x_α *is* τ_2 *quasi-neighbourhood of* x_α *for all fuzzy point* x_α *then* τ_1 *is coarser then* τ_2*.*

Theorem 2.11 *[4] Let* τ_1 *and* τ_2 *be two fuzzy topologies on a set* X*. Then the mixed topoloy* $\tau_1(\tau_2)$ *is coarser then* τ_2*. In symbol,* $\tau_1(\tau_2) \subseteq \tau_2$*.*

Proposition 2.12 *[4] If* τ_1 *is fuzzy regular and* $\tau_1 \subseteq \tau_2$ *then* $\tau_1 \subseteq \tau_1(\tau_2)$*.*

3 Pairwise Separation Axioms

Definition 3.1 *A fuzzy topological space (X, τ) is said to be fuzzy semi pre-T_1 if for any two distinct fuzzy points x_{λ_1} and x_{λ_2} of X, there exists fuzzy semi preopen sets ν_1 and ν_2 such that $x_{\lambda_1} \in \nu_1$ and $x_{\lambda_2} \notin \nu_1$, and also $x_{\lambda_1} \notin \nu_2$ and $x_{\lambda_2} \in \nu_2$.*

Definition 3.2 *A fuzzy topological space (X, τ_1) is said to be fuzzy semi pre-T_2 if for any pair of distinct fuzzy points x_{λ_1} and x_{λ_2} of X, there exists disjoint fuzzy semi preopen sets ν_1 and ν_2 such that $x_{\lambda_1} \in \nu_1$ and $x_{\lambda_2} \in \nu_2$.*

Definition 3.3 *Let (X, τ_1, τ_2) be a fuzzy bitopological space. A subset μ of X is said to be pairwise τ_1-τ_2 fuzzy semi preopen if $\mu \subseteq Cl_{\tau_2}(Int_{\tau_1}(Cl_{\tau_2}\mu))$.*

Definition 3.4 *A fuzzy bitopological space $(X,\ \tau_1,\ \tau_2)$ is said to be pairwise τ_1-τ_2 fuzzy semi pre-T_1 if for any two distinct fuzzy points x_{λ_1} and x_{λ_2} of X, there exists pairwise τ_1-τ_2 fuzzy semi preopen sets ν_1 and ν_2 such that $x_{\lambda_1} \in \nu_1$ and $x_{\lambda_2} \notin \nu_1$ and also $x_{\lambda_1} \notin \nu_2$ and $x_{\lambda_2} \in \nu_2$.*

Definition 3.5 *A fuzzy bitopological space $(X,\ \tau_1,\ \tau_2)$ is said to be pairwise τ_1-τ_2 fuzzy semi pre-T_2 if for any pair of distinct fuzzy points x_{λ_1} and x_{λ_2} of X, there exists disjoint pairwise τ_1-τ_2 fuzzy semi preopen sets ν_1 and ν_2 such that $x_{\lambda_1} \in \nu_1$ and $x_{\lambda_2} \in \nu_2$.*

Theorem 3.6 *Let τ_1 and τ_2 be two fuzzy topologies on a set X and let $\tau_1(\tau_2)$ be the mixed fuzzy topology. If μ is a fuzzy set which is τ_2 fuzzy semi preopen then μ is pairwise τ_2-$\tau_1(\tau_2)$ fuzzy semi preopen in the fuzzy bitopological space (X, τ_1, τ_2).*

Proof: We have $\tau_1(\tau_2) \subseteq \tau_2$, by Theorem 2.11, So,

$$Cl_{\tau_2}\mu \subseteq Cl_{\tau_1(\tau_2)}\mu \tag{1}$$

Since μ is τ_2 fuzzy semi preopen, we have $\mu \subseteq Cl_{\tau_2}(Int_{\tau_2}(Cl_{\tau_2}\mu))$ and from (1)

$$\mu \subseteq Cl_{\tau_2}(Int_{\tau_2}(Cl_{\tau_1(\tau_2)}\mu))$$

Therefore, μ is pairwise τ_2-$\tau_1(\tau_2)$ fuzzy semi preopen. Hence the result follows.

Theorem 3.7 *Let τ_1 and τ_2 be two fuzzy topologies on a set X such that τ_1 is fuzzy regular and $\tau_1 \subseteq \tau_2$ and $\tau_1(\tau_2)$ be the mixed fuzzy topology. If μ is a fuzzy set which is $\tau_1(\tau_2)$ fuzzy semi preopen then μ is pairwise $\tau_1(\tau_2)$-τ_1 fuzzy semi preopen in the fuzzy bitopological space $(X, \tau_1(\tau_2), \tau_1)$.*

Proof: Since τ_1 is fuzzy regular and $\tau_1 \subseteq \tau_2$, we have $\tau_1 \subseteq \tau_1(\tau_2)$, by Proposition 2.12. So,

$$Cl_{\tau_1(\tau_2)}\mu \subseteq Cl_{\tau_1}\mu.$$

Again, since μ is $\tau_1(\tau_2)$ fuzzy semi preopen, we have

$$\mu \subseteq Cl_{\tau_1(\tau_2)}(Int_{\tau_1(\tau_2)}(Cl_{\tau_1(\tau_2)}\mu))$$

and combining, we have

$$\mu \subseteq Cl_{\tau_1(\tau_2)}(Int_{\tau_1(\tau_2)}(Cl_{\tau_1}\mu)) \subseteq Cl_{\tau_1}(Int_{\tau_1(\tau_2)}(Cl_{\tau_1}\mu))$$

Therefore, μ is pairwise $\tau_1(\tau_2)$-τ_1 fuzzy semi preopen. This proves the theorem.

Theorem 3.8 *Let τ_1 and τ_2 be two fuzzy topologies on a set X such that $\tau_1 \subseteq \tau_2$. If μ is a fuzzy set which is τ_2-fuzzy semi preopen then μ is pairwise τ_2-τ_1 fuzzy semi preopen in the fuzzy bitopological space (X, τ_2, τ_1).*

Proof: We have $\tau_1 \subseteq \tau_2$. It follows that $Cl_{\tau_2}\mu \subseteq Cl_{\tau_1}\mu$. Since μ is τ_2-fuzzy semi preopen, we have $\mu \subseteq Cl_{\tau_2}(Int_{\tau_2}(Cl_{\tau_2}\mu))$ and combining it follows that $\mu \subseteq Cl_{\tau_1}(Int_{\tau_2}(Cl_{\tau_1}\mu))$. Therefore, μ is pairwise τ_2-τ_1 semi preopen. This completes the proof.

Theorem 3.9 *Let τ_1 and τ_2 be two fuzzy topologies on a set X such that $\tau_2 \subseteq \tau_1$. If μ is a fuzzy set which is τ_1-fuzzy semi preopen then μ is pairwise τ_1-τ_2 fuzzy semi preopen in the fuzzy bitopological space (X, τ_2, τ_1).*

Proof: Since $\tau_2 \subseteq \tau_1$, we have $Cl_{\tau_1}\mu \subseteq Cl_{\tau_2}\mu$. Also, μ is τ_1-fuzzy semi preopen. So, we have $\mu \subseteq Cl_{\tau_1}(Int_{\tau_1}(Cl_{\tau_1}\mu))$ and combining, we get $\mu \subseteq Cl_{\tau_2}(Int_{\tau_1}(Cl_{\tau_2}\mu))$. Therefore, μ is pairwise τ_1-τ_2 fuzzy semi preopen. This completes the proof.

Theorem 3.10 *Let τ_1 and τ_2 be two fuzzy topologies and let $\tau_1(\tau_2)$ be the mixed fuzzy topology. Let $(X, \tau_2, \tau_1(\tau_2))$ be a fuzzy bitopological space. If (X, τ_2) is a fuzzy topological space which is fuzzy semi pre-T_1 then $(X, \tau_2, \tau_1(\tau_2))$ is pairwise τ_2-$\tau_1(\tau_2)$ fuzzy semi pre-T_1.*

Proof: Suppose (X, τ_2) is fuzzy topological space which is fuzzy semi pre-T_1. Thus, for any two distinct fuzzy points x_{λ_1} and x_{λ_2} of X, there exists τ_2 fuzzy semi preopen sets ν_1 and ν_2 such that $x_{\lambda_1} \in \nu_1$ and $x_{\lambda_2} \notin \nu_1$ and also $x_{\lambda_1} \notin \nu_2$ and $x_{\lambda_2} \in \nu_2$. Now, any fuzzy sets which is τ_2 fuzzy semi preopen is pairwise τ_2-$\tau_1(\tau_2)$ fuzzy semi preopen, by Theorem 3.6 Thus, $(X, \tau_2, \tau_1(\tau_2))$ is pairwise τ_2-$\tau_1(\tau_2)$ fuzzy semi pre-T_1. This proves the theorem.

Theorem 3.11 *Let τ_1 and τ_2 be two fuzzy topologies on a set X such that τ_1 is fuzzy regular and $\tau_1 \subseteq \tau_2$. Let $\tau_1(\tau_2)$ be the mixed fuzzy topology and let $(X, \tau_1(\tau_2), \tau_1)$ be a fuzzy topological space. If $(X, \tau_1(\tau_2))$ is a fuzzy bitopological space which is fuzzy semi pre-T_1 then $(X, \tau_1(\tau_2), \tau_1)$ is pairwise $\tau_1(\tau_2)$-τ_1 fuzzy semi pre-T_1.*

Proof: Suppose $(X, \tau_1(\tau_2))$ is fuzzy semi pre-T_1 space. Thus, for any two distinct fuzzy points x_{λ_1} and x_{λ_2} of X, there exists $\tau_1(\tau_2)$ fuzzy semi preopen sets ν_1 and ν_2 such that $x_{\lambda_1} \in \nu_1$ and $x_{\lambda_2} \notin \nu_1$ also $x_{\lambda_1} \notin \nu_2$ and $x_{\lambda_2} \in \nu_2$. But by Theorem 3.7, ν_1 and ν_2 are pairwise $\tau_1(\tau_2)$-τ_1 fuzzy semi preopen. Thus, $(X, \tau_1(\tau_2), \tau_1)$ is pairwise $\tau_1(\tau_2)$-τ_1 fuzzy semi pre-T_1. This proves the theorem.

Theorem 3.12 *Let τ_1 and τ_2 be two fuzzy topologies on a set X such that $\tau_1 \subseteq \tau_2$. Let (X, τ_2, τ_1) be a fuzzy bitopological space. If (X, τ_2) is a fuzzy topological space which is fuzzy semi pre-T_1 then (X, τ_2, τ_1) is pairwise $\tau_1(\tau_2)$-τ_1 fuzzy semi pre-T_1.*

Proof. Since (X, τ_2) is fuzzy semi pre-T_1, for any two distinct fuzzy points x_{λ_1} and x_{λ_2} of X, there exists τ_2 fuzzy semi preopen sets ν_1 and ν_2 such that $x_{\lambda_1} \in \nu_1$ and $x_{\lambda_2} \notin \nu_1$ and also $x_{\lambda_1} \notin \nu_2$ and $x_{\lambda_2} \in \nu_2$. Now, the fuzzy sets which are τ_2-fuzzy semi preopen are pairwise τ_2-τ_1 fuzzy semi preopen, by Theorem 3.8 Hence, (X, τ_2, τ_1) is pairwise τ_2-τ_1 fuzzy semi pre-T_1. This proves the theorem.

Theorem 3.13 Let τ_1 and τ_2 be two fuzzy topologies on a set X such that $\tau_2 \subseteq \tau_1$. Let (X, τ_1, τ_2) be the fuzzy bitopological space. If (X, τ_1) is a fuzzy topological space which is fuzzy semi pre-T_1 then (X, τ_1, τ_2) is pairwise τ_1-τ_2 fuzzy semi pre-T_1.

Proof: Since (X, τ_1) is fuzzy semi pre-T_1, for any two distinct fuzzy points x_{λ_1} and x_{λ_2} of X, there exists τ_1 fuzzy semi preopen sets ν_1 and ν_2 such that $x_{\lambda_1} \in \nu_1$ and $x_{\lambda_2} \notin \nu_1$ and also $x_{\lambda_1} \notin \nu_2$ and $x_{\lambda_2} \in \nu_2$. But, fuzzy sets ν_1 and ν_2 are pairwise τ_1-τ_2 fuzzy semi preopen, by Theorem 3.9 Hence, (X, τ_1, τ_2) is pairwise τ_1-τ_2 fuzzy semi pre-T_1. This proves the theorem.

Simiar result can be obtained for pairwise fuzzy semi pre-T_2 spaces also

Theorem 3.14 *Let τ_1 and τ_2 be two fuzzy topologies and let $\tau_1(\tau_2)$ be the mixed fuzzy topology. Let $(X, \tau_2, \tau_1(\tau_2))$ be a fuzzy bitopological space. If (X, τ_2) is a fuzzy topological space which is fuzzy semi pre-T_2 then $(X, \tau_2, \tau_1(\tau_2))$ is pairwise τ_2-$\tau_1(\tau_2)$ fuzzy semi pre-T_2.*

Proof: Suppose (X, τ_2) is fuzzy topological space which is fuzzy semi pre-T_2. Thus, for any pair of distinct fuzzy points x_{λ_1} and x_{λ_2} of X, there exists τ_2 fuzzy semi preopen sets ν_1 and ν_2 such that $x_{\lambda_1} \in \nu_1$ and $x_{\lambda_2} \notin \nu_2$.

Now, any fuzzy set which is τ_2 fuzzy semi preopen is pairwise τ_2-$\tau_1(\tau_2)$ fuzzy semi preopen, by Theorem 3.6 Thus, $(X, \tau_2, \tau_1(\tau_2))$ is pairwise τ_2-$\tau_1(\tau_2)$ fuzzy semi pre-T_2. This proves the theorem.

Theorem 3.15 *Let τ_1 and τ_2 be two fuzzy topologies on a set X such that τ_1 is fuzzy regular and $\tau_1 \subseteq \tau_2$. Let $\tau_1(\tau_2)$ be the mixed fuzzy topology and let $(X, \tau_1(\tau_2), \tau_1)$ be the fuzzy bitopological space. If $(X, \tau_1(\tau_2))$ is a fuzzy topological space which is fuzzy semi pre-T_2 then $(X, \tau_1(\tau_2), \tau_1)$ is pairwise $\tau_1(\tau_2)$-τ_1 fuzzy semi pre-T_2.*

Proof: Suppose $(X, \tau_1(\tau_2))$ is fuzzy semi pre-T_1 space. Thus, for any pair of distinct fuzzy points x_{λ_1} and x_{λ_2} of X, there exists $\tau_1(\tau_2)$ fuzzy semi preopen sets ν_1 and ν_2 such that $x_{\lambda_1} \in \nu_1$ and $x_{\lambda_2} \notin \nu_2$. But by Theorem 3.7, ν_1 and ν_2 are pairwise $\tau_1(\tau_2)$-τ_1 fuzzy semi preopen. Thus, $(X, \tau_1(\tau_2), \tau_1)$ is pairwise $\tau_1(\tau_2)$-τ_1 fuzzy semi pre-T_2. This proves the theorem.

Theorem 3.16 Let τ_1 and τ_2 be two fuzzy topologies on a set X such that $\tau_1 \subseteq \tau_2$. Let (X, τ_2, τ_1) be a fuzzy bitopological space. If (X, τ_2) is a fuzzy topological space which is fuzzy semi pre-T_2 then (X, τ_2, τ_1) is pairwise τ_2-τ_1 fuzzy semi pre-T_2.

Proof: Since (X, τ_2) is fuzzy semi pre-T_1, for any two distinct fuzzy points x_{λ_1} and x_{λ_2} of X, there exists τ_2 fuzzy semi preopen sets ν_1 and ν_2 such that $x_{\lambda_1} \in \nu_1$ and $x_{\lambda_2} \notin \nu_2$. Now, the fuzzy sets which are τ_2-fuzzy semi preopen are pairwise τ_2-τ_1 fuzzy semi preopen, by Theorem 3.8 Hence, (X, τ_2, τ_1) is pairwise τ_2-τ_1 fuzzy semi pre-T_2. This proves the theorem.

Theorem 3.17 *Let τ_1 and τ_2 be two fuzzy topologies on a set X such that $\tau_2 \subseteq \tau_1$. Let (X, τ_1, τ_2) be the fuzzy bitopological space. If (X, τ_1) is a fuzzy topological space which is fuzzy semi pre-T_2 then (X, τ_1, τ_2) is pairwise τ_1-τ_2 fuzzy semi pre-T_2.*

Proof: Since (X, τ_1) is fuzzy semi pre-T_2, for any pair of distinct fuzzy points x_{λ_1} and x_{λ_2} of X, there exists τ_1 fuzzy semi preopen sets ν_1 and ν_2 such that $x_{\lambda_1} \in \nu_1$ and $x_{\lambda_2} \notin \nu_2$. But, fuzzy sets ν_1 and ν_2 are pairwise τ_1-τ_2 fuzzy semi preopen, by Theorem 3.9 Hence, (X, τ_1, τ_2) is pairwise τ_1-τ_2 fuzzy semi pre-T_2. This proves the theorem.

4 Pairwise Fuzzy Semi Pre-Continuous Functions

Definition 4.1 *A mapping $f : (X,\ \tau_1,\ \tau_2) \to (Y,\ \tau_1^*,\ \tau_2^*)$ is called pairwise fuzzy semi pre-continuous if V is pairwise τ_1^*-τ_2 fuzzy semi preopen implies that $f^{-1}(V)$ is pairwise τ_1-τ_2 fuzzy semi preopen.*

Theorem 4.2 *If a mapping $f : (X,\ \tau_1,\ \tau_2) \to (Y,\ \tau_1^*,\ \tau_2^*)$ is pairwise fuzzy semi pre-continuous then* *(i) every pairwise τ_1^*-τ_2^* fuzzy semi preopen set is pairwise τ_1^*-$\tau_1^*(\tau_2^*)$ fuzzy semi preopen.*

(ii) $f : (X,\ \tau_1,\ \tau_1(\tau_2)) \to (Y,\ \tau_1^*,\ \tau_2^*)$ is pairwise fuzzy semi pre-continuous.

Proof: Suppose $f : (X,\ \tau_1,\ \tau_2) \to (Y,\ \tau_1^*,\ \tau_2^*)$ is called pairwise fuzzy semi pre-continuous. Let V be pairwise τ_1^*-τ_2^* fuzzy semi preopen. Then $f^{-1}(V)$ is pairwise τ_1-τ_2 fuzzy semi preopen. So,

$$V \subseteq Cl_{\tau_2^*}(Int_{\tau_1^*}(Cl_{\tau_2^*}V)) \tag{2}$$

implies

$$f^{-1}(V) \subseteq Cl_{\tau_2}(Int_{\tau_1}Cl_{\tau_2}f^{-1}(V)) \tag{3}$$

Since $\tau_1^*(\tau_2^*) \subseteq \tau_2^*$, by Theorem 2.11, we have

$$Cl_{\tau_2^*}V \subseteq Cl_{\tau_1^*\tau_2^*}V \tag{4}$$

Also $\tau_1(\tau_2) \subseteq \tau_2$, by Theorem 2.11 Thus,

$$Cl_{\tau_2^*}(f^{-1}(V)) \subseteq Cl_{\tau_1(\tau_2)}(f^{-1}(V)) \tag{5}$$

Combining (2) and (4) we have $V \subseteq Cl_{\tau_2^*}(Int_{\tau_1^*}(Cl_{\tau_1^*(\tau_2^*)}V))$. This proves (i). Also from (3) and (5) we have $f^{-1}(V) \subseteq Cl_{\tau_2}(Int_{\tau_1}(Cl_{\tau_2}f^{-1}(V))) \subseteq cl_{\tau_2}(int_{\tau_1}(Cl_{\tau_1(\tau_2)}f^{-1}(V)))$. Hence, if V is pairwise τ_1^*-τ_2^* fuzzy semi preopen then $f^{-1}(V)$ is pairwise τ_1-$\tau_1(\tau_2)$ fuzzy semi preopen. Therefore, $f : (X,\ \tau_1,\ \tau_1(\tau_2)) \to (Y,\ \tau_1^*,\ \tau_2^*)$ is pairwise fuzzy semi pre continuous. This proves the theorem.

Theorem 4.3 *Let τ_1 and τ_2 be fuzzy topologies on X and let τ_1^* and τ_2^* be fuzzy topologies on Y satisfying the conditions*

(i) τ_1 is fuzzy regular and $\tau_1 \subseteq \tau_2$.

(ii) τ_1^* is fuzzy regular and $\tau_1^* \subseteq \tau_2^*$.

If a mapping $f : (X,\ \tau_2,\ \tau_1(\tau_2)) \to (Y,\ \tau_2^*,\ \tau_1^*(\tau_2^*))$ is pairwise fuzzy semi precontinuous, then

(i) every pairwise τ_2^*-$\tau_1^*(\tau_2^*)$ fuzzy semi preopen set is pairwise τ_2^*-τ_1^* fuzzy semi preopen.

(ii) $f : (X,\ \tau_2,\ \tau_1) \to (Y,\ \tau_2^*,\ \tau_1^*(\tau_2^*))$ is pairwise fuzzy semi pre continuous.

Proof: Suppose $f : (X,\ \tau_2,\ \tau_1(\tau_2)) \to (Y,\ \tau_2^*,\ \tau_1^*(\tau_2^*))$ be pairwise fuzzy semi pre continuous. Let V be pairwise τ_2^*-$\tau_1^*(\tau_2^*)$ fuzzy semi preopen. Then $f^{-1}(V)$ is pairwise τ_2-$\tau_1(\tau_2)$ fuzzy semi preopen. So,

$$V \subseteq Cl_{\tau_1^*(\tau_2^*)}(Int_{\tau_2^*}(Cl_{\tau_1^*(\tau_2^*)}V)) \tag{6}$$

implies

$$f^{-1}(V) \subseteq Cl_{\tau_1^*(\tau_2^*)}(Int_{\tau_2}(Cl_{\tau_1(\tau_2)}f^{-1}(V))) \tag{7}$$

But we have $\tau_1^* \subseteq \tau_1^*(\tau_2^*)$ by Proposition 2.12 So,

$$Cl_{\tau_1^*(\tau_2^*)}V \subseteq Cl_{\tau_1^*}V \tag{8}$$

Again $\tau_1 \subseteq \tau_1(\tau_2)$ by Proposition 2.12 So,

$$Cl_{\tau_1(\tau_2)}f^{-1}(V) \subseteq Cl_{\tau_1}f^{-1}(V) \tag{9}$$

Combining (6) and (8) we have $V \subseteq Cl_{\tau_1^*(\tau_2^*)}(Int_{\tau_2^*}(Cl_{\tau_1^*(\tau_2^*)}V)) \subseteq cl_{\tau_1^*}(Int_{\tau_2^*}(Cl_{\tau_1^*}V))$. This proves (i). Also combining (7) and (9) we have
$f^{-1}(V) \subseteq Cl_{\tau_1(\tau_2)}(Int_{\tau_2}(Cl_{\tau_1(\tau_2)}f^{-1}(V)) \subseteq Cl_{\tau_1}Int_{\tau_2}(Cl_{\tau_1}f^{-1}(V))$.

Hence, if V is pairwise τ_2^*-$\tau_1^*(\tau_2^*)$ fuzzy semi preopen then $f^{-1}(V)$ is pairwise τ_2-τ_1 fuzzy semi preopen. Therefore, $f : (X,\ \tau_2,\ \tau_1) \to (Y,\ \tau_2^*,\ \tau_1^*(\tau_2^*))$ is pairwise fuzzy semi pre continuous. This proves the theorem.

Theorem 4.4 *Let τ_1 and τ_2 be fuzzy topologies on X and let τ_1^* and τ_2^* be fuzzy topologies on Y such that $\tau_1 \subseteq \tau_2$ and $\tau_1^* \subseteq \tau_2^*$. If a mapping $f : (X,\ \tau_1(\tau_2),\ \tau_2) \to (Y,\ \tau_1^*(\tau_2^*),\ \tau_2^*)$ is pairwise fuzzy semi pre continuous, then*

(i) every pairwise $\tau_1^*(\tau_2^*)$-τ_2^* fuzzy semi preopen set is pairwise $\tau_1^*(\tau_2^*)$-τ_1^* fuzzy semi preopen.

(ii) $f : (X,\ \tau_1(\tau_2),\ \tau_1) \to (Y,\ \tau_1^*(\tau_2^*),\ \tau_2^*)$ is pairwise fuzzy semi pre continuous.

Proof: Suppose $f : (X,\ \tau_1(\tau_2),\ \tau_2) \to (Y,,\ \tau_1^*(\tau_2^*),\ \tau_2^*)$ is pairwise fuzzy semi pre continuous. Let V be pairwise $\tau_1^*(\tau_2^*)$-τ_2^* fuzzy semi preopen. So, $f^{-1}(V)$ is pairwise $\tau_1(\tau_2)$-τ_2 fuzzy semi preopen. Thus,

$$V \subseteq Cl_{\tau_2^*}Int_{\tau_1^*(\tau_2^*)}(Cl_{\tau_2^*}V) \tag{10}$$

implies

$$f^{-1}(V) \subseteq Cl_{\tau_2^*}(Int_{\tau_1(\tau_2)}(Cl_{\tau_2}f^{-1}(V))) \tag{11}$$

Since $\tau_1^* \subseteq \tau_2^*$, we have

$$Cl_{\tau_2^*}V \subseteq Cl_{\tau_1^*}V \tag{12}$$

Similarly $\tau_1 \subseteq \tau_2$ implies

$$Cl_{\tau_2} f^{-1}(V) \subseteq Cl_{\tau_1} f^{-1}(V) \tag{13}$$

Combining (10) and (12) we get, $V \subseteq Cl_{\tau_2^*}(Int_{\tau_1^*(\tau_2^*)}(Cl_{\tau_2^*} V))$. This proves (i). Again combining (11) and (13) we get $f^{-1}(V) \subseteq Cl_{\tau_1}(Int_{\tau_1(\tau_2)}(Cl_{\tau_1} f^{-1}(V)))$. Thus, $f : (X,\ \tau_1(\tau_2),\ \tau_1) \to (Y,\ \tau_1^*(\tau_2^*),\ \tau_2^*)$ is pairwise fuzzy semi pre continuous. This completes the proof.

Theorem 4.5 *Let τ_1 and τ_2 be fuzzy topologies on X and let τ_1^* and τ_2^* be fuzzy topologies on Y such that $\tau_2 \subseteq \tau_1$ and $\tau_2^* \subseteq \tau_1^*$. If a mapping $f : (X,\ \tau_1(\tau_2),\ \tau_1) \to (Y,\ \tau_1^*(\tau_2^*),\ \tau_1^*)$ is pairwise fuzzy semi pre continuous, then*

(i) every pairwise $\tau_1^*(\tau_2^*)$-τ_1^* fuzzy semi preopen set is pairwise $\tau_1^*(\tau_2^*)$-τ_2^* fuzzy semi preopen.

(ii) $f : (X,\ \tau_1(\tau_2), \tau_2) \to (Y, \tau_1^*(\tau_2^*), \tau_2^*)$ is pairwise fuzzy semi pre continuous.

Proof: Suppose $f : (X,\ \tau_1(\tau_2), \tau_1) \to (Y, \tau_1^*(\tau_2^*), \tau_2^*)$ is pairwise fuzzy semi pre continuous. Let V be pairwise $\tau_1^*(\tau_2^*)$-τ_1^* fuzzy semi preopen. So, $f^{-1}(V)$ is pairwise $\tau_1(\tau_2)$-τ_1 fuzzy semi preopen. Thus,

$$V \subseteq Cl_{\tau_1^*}(Int_{\tau_1^*(\tau_2^*)}(Cl_{\tau_1^*} V)) \tag{14}$$

implies

$$f^{-1}(V) \subseteq Cl_{\tau_1^*}(Int_{\tau_1(\tau_2)}(Cl_{\tau_1} f^{-1}(V))) \tag{15}$$

Since $\tau_2^* \subseteq \tau_1^*$, we have

$$Cl_{\tau_1^*} V \subseteq Cl_{\tau_2^*} V \tag{16}$$

Similarly $\tau_2 \subseteq \tau_1$ implies

$$Cl_{\tau_1} f^{-1}(V) \subseteq Cl_{\tau_2} f^{-1}(V) \tag{17}$$

Combining (14) and (16) we get $V \subseteq Cl_{\tau_2^*}(Int_{\tau_1^*(\tau_2^*)}(Cl_{\tau_2^*} V))$. This proves (i). Again combining (15) and (17) we have $f^{-1}(V) \subseteq Cl_{\tau_2^*}(Int_{\tau_1(\tau_2)}(Cl_{\tau_2} f^{-1} V))$. Thus, $f : (X, \tau_1(\tau_2) - \tau_2) \to (Y, \tau_1^*(\tau_2^*) - \tau_2^*)$ is pairwise fuzzy semi pre continuous. This completes the proof.

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