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Some Properties of Two-Fuzzy Metric Spaces

Noori F. AL-Mayahi[1] and Layth S. Ibrahim[2]

[1, 2] Department of Mathematics
College of Computer Science and Mathematics
University of AL-Qadissiya
[1] E-mail: afm2005@yahoo.com
[2] E-mail: liath20022002@yahoo.com

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Abstract

In this paper we have provided the set $\boldsymbol{F}(\boldsymbol{X})$ be the set of all fuzzy set is bounded functions and we introduced of the define a two-fuzzy metric and we've made some properties of these sets by studying the open and closed balls, as well as studied property fuzzy convergence and fuzzy closure set in the two-fuzzy metric space.

Keywords: *Fuzzy set, Fuzzy matric space, Two-fuzzy matric space, t-norm.*

1 Introduction

Since the introduction of the concept of fuzzy sets by Zadeh [6] in 1965, many authors have introduced the concept of fuzzy metric space in different ways [4]. George and Veeramani [3]. They showed also that every metric induces a fuzzy metric. The fuzzy version of Banach contraction principle was given by Grabiec [4] in 1988 and in [1] Amin Ahmed, Deepak Singh introduce the definition two-fuzzy matric space.

The purpose of this paper is to clarify some properties of two-fuzzy metric space through the set $F(X)$ which we will study in this paper the properties of open and closed balls as well as the fuzzy convergence study and fuzzy closure set in two-fuzzy metric space .

2 Preliminaries

Definition 2.1. [1]: *A binary operation* $*: [0,1] \times [0,1] \to [0,1]$ *is called a* $continuoust - normif$ $([0,1],*)$ *is an abelian topological monoid with unit 1 such that* $a * b \leq c * d$ $whenevera \leq candb \leq d$ *for all* $a, b, candd \in [0,1]$.

Definition 2.2. [3]: *The triple* $(X, M ,*)$ *is called a fuzzy metric space, if X is an arbitrary set, *is a continuous t-norm and M is a fuzzy set in* $X^2 \times [0,\infty)$*satisfying the following conditions:*

For all $x , y , z \in X , s, t > 0$

[FM-1] $M\ (x, y, 0) > 0$;

[FM-2] $M(x, y, t) = 1 , forallt > 0\ ifandonlyifx = y$;

[FM-3]

[FM-4] $M(x, y, t) * M(y, z, s) > M(x, z, t + s)$;

[FM-5] $M(x, y, .): [0, \infty) \to [0,1] isleftcontinuous$;

[FM-6]

Note that *M* (*x*, *y*, *t*) can be thought of as the degree of nearness between x and y with respect to t. We identify $x = y$ with$M\ (x, y, t) = 1$ for all$t > 0$. the following example shows that every metric space induces a fuzzy metric space.

Definition 2.3. [4]: *Let* $(X, M,*)$*be a fuzzy metric space. A sequence in X is said to be a convergent to a point*$x \in Xif$

$$\lim_{n\to\infty} M(x_n, x, t) = 1$$

for allt> 0. further the sequence$\{x_n\}$*inX Said to be aCauchysequence in X? If*

$$\lim_{n\to\infty} M\left(x_n, x_{n+p}, t\right) = 1\ forallt > 0\ andp > 0$$

The space is said to be complete if every Cauchy sequence in X converges to a point of$X$.

Definition 2.4. [1]: *A function M is continuous in fuzzy metric space if and only if whenever,* $\{x_n\} \to x, \{y_n\} \to y$, *then* $lim_{n\to\infty} M(x_n, y_n, t) = M(x, y, t)$ *for each t> 0.*

Definition 2.5. [1]: *A binary operation* $[0,1] \times [0,1] \times [0,1] \to [0,1]$ *is called a continuous* $t - norm$ *if*$([0,1],*)$ *is an abelian topological monoid with unit 1 such that*
$a * b * c \leq d * e * f,$

whenever $a \leq d , b \leq e and c \leq f for all a, b, c, d and$

Definition 2.6. [1]: *A map:* $[0,1] \times [0,1] \times [0,1] \to [0,1]$ *is a t-norm if it satisfies the following conditions:*

(T1) $*(a ,1 ,1) = a \quad , \quad *(0 ,0 ,0) = 0$

(T2) $* (a , b , c) =* (a , c , b) =* (b , a, c)$

(T3) $* (a_1, b_1, c_1) \geq * (a_2, b_2, c_2) for\ a_1 \geq a_2, b_1 \geq b_2 , c_1 \geq c_2$

(T4) $* (* (a, b, c), d, e) =* (a ,* (b , c, d), e) = (a , b ,* (c , d , e))$

Definition 2.7. [1]: *The triplet* $(X, M,*)$ *is a fuzzy two-metric space if X is an arbitrary set, * is a continuous t-norm, and M is a fuzzy set in*$X^3 \times [0, \infty)$ *satisfying the following conditions:*

FM1.$M (x, y, a, 0) = 0$

FM2.$M (x, y, a, t) = 1 for all t > 0$ if and only if at least two of them are equal.

FM3.$M (x, y, a, t) = M (y, a, x, t) = M (a, y, x, t)$ (symmetric)

FM4.$M(x, y, a, r + s + t) \geq M(x, y, z, r) * M(x, z, a, s) * M(z, y, a, t)$ for all $x, y, z, a \in X and r, s, t > 0$

FM5.$M(x, y, a, .): [0, \infty) \to [0,1]$is left continuous for all$x, y, z, a \in X$

FM6.$\lim_{n\to\infty} M(x, y, a, t) = 1 for all x, y, a \in X, t > 0$

Example 2.8. [1]: Let X be the set {1, 2, 3, 4} with two-metric defined by

$$d(x , y , z) = \begin{cases} 0 & if x = y , y = z , z = x and \{x , y , z\} = \{1 ,2 ,3\} \\ & \frac{1}{2} otherwise \end{cases}$$

For each $t \in [0,\infty)$ is two-matric space.

$$M(x\,,y\,,z) = \begin{cases} 0 & if t = 0 \\ \frac{t}{t+d(x\,,y\,,z)} & if t > 0 \; where x\,,y\,,z \in X \end{cases}$$

Then $M\ (X,M,*)$ is a fuzzy two-metric space.

Remark 2.9. [1]: From example 1, it is clear that every two-metric space induces a Fuzzy two-metric space by the relation M (x, y, z, t) =$\frac{t}{t+d(x\,,y\,,z)}$ such a fuzzy two-metric space is known as induced fuzzy two-metric space.

Example 2.10. [5]: A sequence is convergent to $x \in X$ if

$$\lim_{m,n\to\infty} M(x_n, x_m, a, t) = 1 \qquad , for each t > 0.$$

Definition 2.11. [1]: *A fuzzy two-metric space $(X, M,*)$ is called Cauchy if*

$$lim_{m,n\to\infty} M(x_n, x_m, a, t) = 1 \; for every t > 0$$

and M (X, M,)is called complete if every Cauchy sequence in X convergence in X.*

Remark 2.12 [5]: A fuzzy two-metric on a set X is said to be continuous on X if it is sequentially continuous in two of its arguments. A fuzzy two-metric is a non-negative real valued function, that it is continuous in any one of its arguments and that if it is continuous in two of its arguments then it is continuous in all the three arguments.

Definition 2.13. [5]: *A mapping f from a fuzzy two-metric space (X, M ,*) into itself is said to be continuous at x if for every sequence in X such that*

$$\lim_{n\to\infty} M(x_n, x, a, t) = 1 \; for each t > 0 \; lim_{n\to\infty} M(fx_n, fx, a, t) = 1$$

Theorem 2.14. [2]: *Let (M_1, d) and (M_2, p)be complete metric spaces. Further, Let A,B be mappings from X to Y and S,T be mappings from Y to X satisfying;*

$$d(SAu, TBu') \le cmax\{d(u, u'), d(u, SAu), d(u', TBu'), p(Au, Bu')\}$$
$$p(BSv, ATv') \le cmax\{p(v, v'), p(v, BSv), p(v', ATv'), d(Sv, Tv')\}.$$

For all $u, u' \in M_1$ and $v, v' \in M_2$, where $0 \le c \le 1$, if one of the mapping A,B,S and T is continuous, then SA and TB have a unique common fixed point $z in X$and BS and AT have common fixed point $w \in Y$.
Further $Az = Bz = \omega and S\omega = T\omega = z$

3 Main Result

Let X be a non-empty set, and $F(X)$ be the set of all fuzzy sets in X. If f ∈ F(X) then $f = \{(x,\alpha): x \in X and \alpha \in (0,1]\}$.

Clearly f is a bounded function for$|f(x)| \leq 1$. Let K be the space of real numbers, then$F(X)$ is a vector space over the field K where the addition and scalar multiplication are defined by

$$f + g = \{(x,\alpha) + (y,\beta)\} = \{x + y , \alpha \Lambda \beta): (x,\alpha) \in f , (y,\beta) \in g\}$$

And $kf = \{k(x,\alpha): (x,\alpha) \in f , where k \in K\}$

The vector space F(X) is said to be matric space if for every f ∈ F(X), A function $d: F(X) \times F(X) \to R$ is called a metric function (distance function) on F(X) if satisfies the following axioms:

(1) $d(f,g) \geq 0 \quad for all f,g \in F(X)$
(2) $d(f,g) = 0 \quad iff f = g for all f,g \in F(X)$
(3) $d(f,g) = d(f,g) for all f,g \in F(X)$
(4) $d(f,g) \leq d(f,h) + d(h,g) for all f,g,h \in F(X)$
then $(F(X),d)$ is a matric space.

Definition 3.1: *Let F(X) be a linear space over the real field K. A fuzzy subset Mof* $F(X) \times F(X) \times R$ *. (R, the set of real numbers) is called a 2-fuzzy 2-matric function on X (or fuzzy two-matric function on F(X)) if and only if,*

(N1)$for all t \in R with t \leq 0 , M(f_1,f_2,t) = 0,$

(N2) for all $t \in R with t \geq 0, M(f_1,f_2,t) = 1$, if and only if $f_1 and f_2$ are linearly dependent,

(N3)$M(f_1,f_2,t) > 0,$

(N4)$for all t \in R, with t \geq 0, M(f_1,f_2,t) = M(f_2,f_1,t)$

(N5)$for all s,t \in R, M(f_1,f_2,t+s) \geq M(f_1,f_3,s) * M(f_3,f_2,s),$

(N6) $M(f_1,f_2,\bullet) : (0,\infty) \to [0,1]$ $is continuous,$

(N7) $\lim_{t\to\infty} M(f_1,f_2,t) = 1$

Then $(F(X),M)$ is a fuzzy two-matric space or (X,M) is a two-fuzzy two-matric space for all $f_2,f_2,f_3 \in F(X)$.

Definition 3.2: *Let* $(F(X), M, *)$ *be a two-fuzzy metric space. We define the open ball*
$B(f,r,t)$ *with center* $f \in F(X)$ *and radius r,* $0 < r < 1$, $t > 0$, *as*

$$B(f,r,t) = \{g \in F(X) : M(f,g,t) > 1 - r\}.$$

Remark 3.3: Let $(F(X), M, *)$ be two-fuzzy metric space and let $f, g \in F(X), t > 0$

$0 < r < 1$. *Then if* $M(f,g,t) > 1 - r$ *we can find* t_0 *with* $0 < t_0 < t$ *such that* $M(f,g,t_0) > 1 - r$

Theorem 3.4: *Let* $B(f,r_1,t)$ *and* $B(f,r_2,t)$ *be open balls with the same center* $x \in F(X)$ *and* $t > 0$ *with radius* $0 < r_1 < 1$ *and* $0 < r_2 < 1$, *respectively. Then we either have*

$$B(f,r_1,t) \subset B(f,r_2,t)$$

or

$$B(f,r_2,t) \subset B(f,r_1,t)$$

Proof: Let $f \in F(X)$ *and* $t > 0$. Consider the open balls

$B(f,r_1,t)$ *and* $B(f,r_2,t)$, *with*

$$0 < r_1 < 1,$$
$$0 < r_1 < 1,$$

If $r_1 = r_2$, then the proposition holds. Next, we assume that $r_1 \neq r_2$ .We may assume, without loss of generality, that $0 < r_1 < r_2 < 1$. Then $1 - r_2 < 1 - r_1$. Now, let $a \in B(f, r_1, t)$. It follows that

$$M(a, g, t) > 1 - r_1 > 1 - r_2$$

Hence, $a \in B(f,r_2,t)$. This shows that $B(f,r_1,t) \subseteq B(f,r_2,t)$. By assuming that $0 < r_2 < r_1 < 1$, we can similarly show

$$B(f,r_2,t) \subseteq B(f,r_1,t).$$

Definition 3.5: *A subset A of a two- fuzzy metric space* $(F(X), M, *)$ *is said to be open if given any point* $a \in A$, *there exists* $0 < r < 1$, *and* $t > 0$, *such that* $B(a,r,t) \subseteq A$.

Theorem 3.6: *Every open ball in a two-fuzzy metric space* $(F(X), M, *)$ *is an open set.*

Proof: Consider an open ball $B(f, r, t)$. Now $y \in B(x,r,t)$ implies that

$$M(f,g,t) > 1 - r.$$

Since$M(f,g,t) > 1 - r$, by Remark(3.3) we can find at_0 , $0 < t_0 < t$,

Such that $M(f,g,t_0) > 1 - r$.

Let $r_0 = M(f,g,t_0) > 1 - r$. Since$r_0 > 1 - r$, we can find an s, $0 < s < 1$,

Such that $r_0 > 1 - s > 1 - r$.

Now for a given r_0 and s such that $r_0 > 1 - s$we can find r_1 , $0 < r_1 < 1$,

Such that $r_0 * r_1 > 1 - s$.

Now consider the ball $B(g, 1 - r_1, t - t_0)$. We claim

$B(g, 1 - r_1, t - t_0) \subset B(f,r,t)$.

Now $z \in B(g, 1 - r_1, t - t_0)$implies that$M(g,h,t - t_0) > r_1$.

Therefore

$$M(f,h,t) > M(f,g,t_0) * M(g,h,t - t_0)$$

$$> r_0 * r_1$$
$$> 1 - s$$
$$> 1 - r.$$

Therefore

$$h \in B(f,r,t)$$

And hence

$$B(g, 1 - r_1, t - t_0) \subset B(f,r,t).$$

Definition 3.7: Let $(F(X), M,*)$ be a two-fuzzy *metric space. Then we define a closed ball with the center* $f \in F(X)$*and the radius*r, $0 < r < 1$, $t > 0$, *as*

$$B[f,r,t] = \{g \in F(X) : M(f,g,t) > 1 - r\}.$$

Lemma 3.8: *Every closed ball in a two- fuzzy metric space*$(F(X), M,*)$ *is a closed set.*

Proof: Let$g \in \overline{B[f\ ,r\ ,t]}$.Since X is first countable, there exists a sequence$\{g_n\}$in $B[f,r,t]$such that the sequence$\{g_n\}$converges to g. Therefore

$M(g_n, g, t)$ converges to 1 for all t. For a given$\varepsilon > 0$,

$$M(f,g,t + \varepsilon) > M(f,g\_n,t) * M(g\_n,g,\varepsilon).$$

Hence

$$M(f,g,t + \varepsilon) > \lim_{n} M(f,g_n,t) * \lim_{n} M(g_n,g,\varepsilon)$$

$$\geq (1 - r) * 1$$
$$= 1 - r$$

(If $M(f,g_n,t)$ is bounded, the sequence $\{g_n\}$ has a subsequence, which we again denote by $\{g_n\}$ for which $\lim_n M(f,g_n,t)$ exists.) In particular for $n \in N$,

take $\varepsilon = \frac{1}{n}$. Then

$$M\left(f,g,t + \frac{1}{n}\right) > 1 - r.$$

Hence

$$M(f,g,t) = \lim_{n} M\left(f,g,t + \frac{1}{n}\right) \geq 1 - r$$

Thus $g \in B[f,r,t]$ is closed set.

Theorem 3.9: *Let $(f(X),M,*)$ be a complete two-fuzzy metric space. Then the intersection of a countable number of dense open sets is dense.*

Proof: Let $F(X)$ be the given complete two- fuzzy metric space. Let B_0 be a nonempty open set. Let $D_1, D_2, D_3, \ldots$ be dense open sets in F(X). Since D_1 is dense in $F(X)$, $B_0 \cap D_1 \neq \emptyset$. Let

$$f_1 \in B_0 \cap D_1.$$

Since $B_0 \cap D_1$ is open, there exists $0 < r_1 < 1$, $t > 0$, such that

$B(f_1,r_1,t_1) \subset B_0 \cap D_1$.

Choose $\bar{r}_1 < r_1$ and $\bar{t}_1 = min\{t_1, 1\}$ such that

$B(f_1,\bar{r}_1,\bar{t}_1) \subset B_0 \cap D_1$.

Let $B_1 = B(f_1,\bar{r}_1,\bar{t}_1)$.

Since D_2 is dense in F(X), $B_1 \cap D_2 \neq \emptyset$. Let $f_2 \in B_1 \cap D_2$. Since $B_1 \cap D_2$ is open, there exists $0 < r_2 < \frac{1}{2}$ and $t_2 > 0$ such that

$B(f_2,r_2,t_2) \subset B_1 \cap D_2$.

Choose $\bar{r}_2 < r_2$ and $\bar{t}_2 = min\{t_2, \frac{1}{2}\}$ such that

$B[f_2,\bar{r}_2,\bar{t}_2] \subset B_1 \cap D_2$..

Let $B_2 = B(f_2, \bar{r}_2, \bar{t}_2)$. Similarly proceeding by induction we can find an

$$f_n \in B_{n-1} \cap D_n$$

Since $B_{n-1} \cap D_n$is open, there exists$0 < r_n < \frac{1}{n}$and$t_n > 0$ such that

$$B(f_n, r_n, t_n) \subset B_{n-1} \cap D_n$$

Choose $\bar{r}_n < r_n and \bar{t}_n = min\{t_n, \frac{1}{n}\}$ } such that

$$B\ [f_n, \bar{r}_n, \bar{t}_n] \subset B_{n-1} \cap D_n$$

Let $B_n = B(f_n, \bar{r}_n, \bar{t}_n)$.). Now we claim that$\{f_n\}$is a Cauchy sequence. For a given

$t > 0, \ \varepsilon > 0$

Choose n_0 such that $\frac{1}{\mathrm{n}_0} < \mathrm{t}$ and $\frac{1}{\mathrm{n}_0} < \varepsilon$. Then for $n > n_0, m > n$.

$$M(f_n, f_m, t) > M(f_n, f_m, \frac{1}{n})$$

$$> 1 - (\frac{1}{n})$$

$$\geq 1 - \varepsilon$$

Therefore$\{\mathrm{f_n}\}$ is a Cauchy sequence. SinceF(X) is complete, the sequence$\{f_n\}$ converges to$f$inF(X) . But

$$f_k \in B\ [f_n, \bar{r}_n, \bar{t}_n]$$

for allk $\geq$ nand by the previous result$B\ [f_n, \bar{r}_n, \bar{t}_n]$ is a closed set. Hence

$$f \in B\ [f_n, \bar{r}_n, \bar{t}_n] \subset B_{n-1} \cap D_n$$

for all n. Therefore

$$B_0 \cap (\cap_{n-1}^{\infty} D_n) \neq \emptyset.$$

Hence$\cap_{n-1}^{\infty} D_n$is dense in F(X).

Definition 3.10: *Let $(F(X), M, *)$ be a two-fuzzy metric space. Then*

(a) *A sequence $\{f_n\}$ in $F(X)$ is said to be fuzzy convergent to x in $F(X)$ if for each $\varepsilon \in (0,1)$and each $t > 0$, there exist $n_0 \in Z^+$ such that $M\ (f_n, f, t) > 1 - \varepsilon$ for all $n \geq n_0$(or equivalent $lim_{n\to\infty} M(f_n, f, t) =$*

1).

(b) *A sequence* $\{f_n\}$ *in* X *is said to be 2-fuzzy Cauchy sequence if for each* $\varepsilon \in (0,1)$*and each* $t > 0$, *there exist* $n_0 \in Z^+$ *such that* $M(f_n, f_m, t) > 1 - \varepsilon$*for all* $n, m \geq n_0$*(or equivalent* $lim_{n,m\to\infty} M(f_n, f_m t) = 1$ *).*

(c) *A fuzzy metric space in which every fuzzy Cauchy sequence is fuzzy convergent is said to be complete.*

Theorem 3.11:

(i) *Every fuzzy convergent sequence is two-fuzzy Cauchy sequence in two-fuzzy metric space* $(F(X), M, *)$.

(ii) *Every sequence in*$F(X)$ *has a unique fuzzy limit.*

Proof:

(i): Let $\{f_n\}$ be a sequence in $F(X)$ such that for each $t > s > 0$

$$\lim_{n\to\infty} M(f_n, f, t) = 1$$

$$M(f_n, f_m, t) \geq M(f_n, f, t - s) * M(f_m, f, s)$$

Taking limit as $n, m \to \infty$

$$\lim_{n,m\to\infty} M(f_n, f_m, t) \geq \lim_{n\to\infty} M(f_n, f, t - s) * \lim_{m\to\infty} M(f_m, f, s) = 1 * 1 = 1$$

$$\Longrightarrow but \lim_{n,m\to\infty} M(f_n, f_m, t) \leq 1$$

then

$$\lim_{n,m\to\infty} M(f_n, f_m, t) = 1$$

$\Longrightarrow \{f_n\}$ is fuzzy Cauchy sequence in$X$.

(ii): Let $\{f_n\}$ be a sequence in $F(X)$ such that $f_n \to f$ and $f_n \to g$and $f \neq g$then for each $t > s > 0$

Then

$$\lim_{n\to\infty} M(f_n, f, s) = 1,$$

$$\lim_{n\to\infty} M(f_n, g, t - s) = 1$$

$$M(f, g, t) \geq M(f_n, f, s) * M(f_n, g, t - s)$$

Taking limit:

$$M(f, g, t) \geq \lim_{n\to\infty} M(f_n, f, s) * \lim_{n\to\infty} M(f_n, f, t - s)$$

$$M(f,g,t) \geq 1 * 1 = 1 \Longrightarrow but M(f,g,t) \leq 1$$

$$\Longrightarrow M(f,g,t) = 1.$$

Then by axiom (2)$f = g$.

Definition 3.12: *Let $(F(X), M, *)$ be a two-fuzzy metric space. A subset$A$ of $F(X)$ is said to be fuzzy closed if for any sequence $\{f_n\}$in A two-fuzzy convergence to$f \in A$ that is,*

$$\lim_{n\to\infty} M(f_n, f,, a, t) = 1$$

for all $t > 0, f \in A, a \in F(X)$.

Definition 3.13: *Let $(F(X), M, *)$ be a two-fuzzy metric space. A subset$\bar{A}$ of $F(X)$is said to be the two-fuzzy closure of A $(A \subset F(X))$ if for any $f \in \bar{A}$ $a \in F(X)$, there exists a sequence $\{f_n\}$in A such that $lim_{n\to\infty} M(f_n, f, a, t) = 1$ for all $t > 0$.*

Theorem 3.14: *Let A be a two-fuzzy subspace of complete two-fuzzy space $F(X)$ then A is complete two-fuzzy space if and only if it is two-fuzzy closed in $F(X)$.*

Proof: Suppose A is complete two-fuzzy space, let $f \in \bar{A}$, there is a sequence $\{f_n\}$ in A such that $f_n \to f$, hence $\{f_n\}$ is a two-fuzzy Cauchy sequence in A.

Since A is a complete two-fuzzy space $\Longrightarrow$there is $g \in A$ such that $f_n \to g$, but the two-fuzzy converge is unique $\Longrightarrow g = f \Longrightarrow f \in A \Longrightarrow \bar{A} \subseteq A$ then A is closed two-fuzzy subspace.

Conversely: Suppose that A is closed two-fuzzy subspace in $F(X)$.

Let $\{f_n\}$ be a two-fuzzy Cauchy sequence in A.Since $A \subset F(X) \Longrightarrow \{f_n\}$ is a two-fuzzy Cauchy sequence in $F(X)$. Since $F(X)$ is complete fuzzy space, there is $f \in F(X)$ such that$f_n \to f$.Since $f_n \in A \Longrightarrow f \in \bar{A}$. Since A is a closed two-fuzzy set in $F(X)$,

$\bar{A} = A \Longrightarrow f \in A \Longrightarrow \{f_n\}$ is two-fuzzy convergesequence in A, then A is complete two-fuzzy subspace.

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