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On Nano Generalized Alpha Generalized Closed Sets in Nano Topological Spaces

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Abstract

In this paper we present another class of N-CS called $Ng\alpha g$-CS and study their fundamental properties in nano topological spaces. We also present $Ng\alpha g$-continuous maps with some of its properties.

Keywords: $Ng\alpha g$-CS, $Ng\alpha g$-continuous maps, $Ng\alpha g$-irresolute maps.

1 Introduction

M.L. Thivagar and C. Richard [4] presented nano topological space (or simply nts) as for a subset G of a universe which is characterized regarding lower and upper approximations of G. He has additionally characterized nano closed sets (in short N-CS), nano interior and nano closure of a set. In 2014, Ng-CS was presented by K. Bhuvaneswari and K.M. Gnanapriya [1]. R.T. Nachiyar and K. Bhuvaneswari [6] presented the idea of Nαg-CS and Ngα-CS in nts. The purpose of this paper is to present the concept of Ngαg-CS and study their essential properties in nts. We likewise present Ngαg-continuous maps by utilizing Ngαg-CS and concentrate some of their principal properties.

2 Preliminaries

Throughout this paper, $(\mathcal{U}, \tau_{\mathcal{R}}(G)), (\mathcal{V}, \sigma_{\mathcal{R}}(H))$ and $(\mathcal{W}, \rho_{\mathcal{R}}(I))$ (or simply $\mathcal{U}, \mathcal{V}$ and $\mathcal{W}$) always mean nts on which no separation axioms are expected unless generally specified. For a set $\mathcal{C}$ in a nts $(\mathcal{U}, \tau_{\mathcal{R}}(G))$, N$cl(\mathcal{C})$, N$int(\mathcal{C})$ and $\mathcal{C}^c = \mathcal{U} - \mathcal{C}$ denote the nano closure of $\mathcal{C}$, the nano interior of $\mathcal{C}$ and the nano complement of $\mathcal{C}$ respectively.

Definition 2.1 [8]: *Let $\mathcal{U}$ be a non-empty finite set of objects called the universe and $\mathcal{R}$ be an equivalence relation on $\mathcal{U}$ named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair $(\mathcal{U}, \mathcal{R})$ is said to be the approximation space.*

Remark 2.2 [8]: *Let $(\mathcal{U}, \mathcal{R})$ be an approximation space and $G \subseteq \mathcal{U}$. Then:*

(i) The lower approximation of G with respect to $\mathcal{R}$ is the set of all objects, which can be for certain classified as G with respect to $\mathcal{R}$ and it is denoted by $L_{\mathcal{R}}(G)$. That is, $L_{\mathcal{R}}(G) = \bigcup\{\mathcal{R}(a): \mathcal{R}(a) \subseteq G, a \in \mathcal{U}\}$, where $\mathcal{R}(a)$ denotes the equivalence class determined by a.

(ii) The upper approximation of G with respect to $\mathcal{R}$ is the set of all objects, which can be possibly classified as G with respect to $\mathcal{R}$ and it is denoted by $U_{\mathcal{R}}(G)$. That is, $U_{\mathcal{R}}(G) = \bigcup\{\mathcal{R}(a): \mathcal{R}(a) \cap G \neq \phi, a \in \mathcal{U}\}$.

(iii) The boundary region of G with respect to $\mathcal{R}$ is the set of all objects, which can be classified neither as G nor as not G with respect to $\mathcal{R}$ and it is denoted by $B_{\mathcal{R}}(G)$. That is, $B_{\mathcal{R}}(G) = U_{\mathcal{R}}(G) - L_{\mathcal{R}}(G)$.

Proposition 2.3 [3]: *If $(\mathcal{U}, \mathcal{R})$ is an approximation space and $G, H \subseteq \mathcal{U}$. Then:*

(i) $L_{\mathcal{R}}(G) \subseteq G \subseteq U_{\mathcal{R}}(G)$.

(ii) $L_{\mathcal{R}}(\phi) = U_{\mathcal{R}}(\phi) = \phi$ *and* $L_{\mathcal{R}}(\mathcal{U}) = U_{\mathcal{R}}(\mathcal{U}) = \mathcal{U}$.

(iii) $U_{\mathcal{R}}(G \cup H) = U_{\mathcal{R}}(G) \cup U_{\mathcal{R}}(H)$.

(iv) $U_{\mathcal{R}}(G \cap H) \subseteq U_{\mathcal{R}}(G) \cap U_{\mathcal{R}}(H)$.

(v) $L_{\mathcal{R}}(G \cup H) \supseteq L_{\mathcal{R}}(G) \cup L_{\mathcal{R}}(H)$.

(vi) $L_{\mathcal{R}}(G \cap H) = L_{\mathcal{R}}(G) \cap L_{\mathcal{R}}(H)$.

(vii) $L_{\mathcal{R}}(G) \subseteq L_{\mathcal{R}}(H)$ *and* $U_{\mathcal{R}}(G) \subseteq U_{\mathcal{R}}(H)$ *whenever* $G \subseteq H$.

(viii) $U_{\mathcal{R}}(G^c) = (L_{\mathcal{R}}(G))^c$ *and* $L_{\mathcal{R}}(G^c) = (U_{\mathcal{R}}(G))^c$.

(ix) $U_{\mathcal{R}}U_{\mathcal{R}}(G) = L_{\mathcal{R}}U_{\mathcal{R}}(G) = U_{\mathcal{R}}(G)$.

(x) $L_{\mathcal{R}}L_{\mathcal{R}}(G) = U_{\mathcal{R}}L_{\mathcal{R}}(G) = L_{\mathcal{R}}(G)$.

Definition 2.4 [4]: *Let* $\mathcal{U}$ *be the universe,* $\mathcal{R}$ *be an equivalence relation on* $\mathcal{U}$ *and* $\tau_{\mathcal{R}}(G) = \{\phi, \mathcal{U}, L_{\mathcal{R}}(G), U_{\mathcal{R}}(G), B_{\mathcal{R}}(G)\}$ *where* $G \subseteq \mathcal{U}$. *Then by proposition (2.3),* $\tau_{\mathcal{R}}(G)$ *satisfies the following axioms:*

(i) $\phi, \mathcal{U} \in \tau_{\mathcal{R}}(G)$.
(ii) *The union of the elements of any subcollection of* $\tau_{\mathcal{R}}(G)$ *is in* $\tau_{\mathcal{R}}(G)$.
(iii) *The intersection of the elements of any finite sub collection of* $\tau_{\mathcal{R}}(G)$ *is in* $\tau_{\mathcal{R}}(G)$.

That is, $\tau_{\mathcal{R}}(G)$ *is a topology on* $\mathcal{U}$ *called the nano topology on* $\mathcal{U}$ *with respect to* G *and the pair* $(\mathcal{U}, \tau_{\mathcal{R}}(G))$ *is called a nano topological space (or simply* nts*). The elements of* $\tau_{\mathcal{R}}(G)$ *are called nano open sets (in short* N-OS*).*

Remark 2.5 [4]: *Let* $(\mathcal{U}, \tau_{\mathcal{R}}(G))$ *be a* nts *with respect to* G *where* $G \subseteq \mathcal{U}$ *and* $\mathcal{R}$ *be an equivalence relation on* $\mathcal{U}$. *Then* $\mathcal{U}/\mathcal{R}$ *denotes the family of equivalence classes of* $\mathcal{U}$ *by* $\mathcal{R}$.

Definition 2.6 [4]: *A subset* $\mathcal{C}$ *of a* nts $(\mathcal{U}, \tau_{\mathcal{R}}(G))$ *is said to be a nano* α*-open set (in short* $N\alpha$-OS*) if* $\mathcal{C} \subseteq Nint(Ncl(Nint(\mathcal{C})))$ *and a nano* α*-closed set (in short* $N\alpha$-CS*) if* $Ncl(Nint(Ncl(\mathcal{C}))) \subseteq \mathcal{C}$. *The nano* α*-closure of a set* $\mathcal{C}$ *of a* nts $(\mathcal{U}, \tau_{\mathcal{R}}(G))$ *is the intersection of all* $N\alpha$-CS *that contain* $\mathcal{C}$ *and is denoted by* $N\alpha cl(\mathcal{C})$.

Definition 2.7 [1]: *A subset* $\mathcal{C}$ *of a* nts $(\mathcal{U}, \tau_{\mathcal{R}}(G))$ *is said to be a nano generalized closed set (in short* Ng-CS*) if* $Ncl(\mathcal{C}) \subseteq \mathcal{M}$ *whenever* $\mathcal{C} \subseteq \mathcal{M}$ *and* $\mathcal{M}$ *is a* N-OS *in* $(\mathcal{U}, \tau_{\mathcal{R}}(G))$. *The complement of a* Ng-CS *is a* Ng-OS *in* $(\mathcal{U}, \tau_{\mathcal{R}}(G))$.

Definition 2.8 [6]: *A subset* $\mathcal{C}$ *of a* nts $(\mathcal{U}, \tau_{\mathcal{R}}(G))$ *is said to be a nano* αg*-closed set (in short* $N\alpha g$-CS*) if* $N\alpha cl(\mathcal{C}) \subseteq \mathcal{M}$ *whenever* $\mathcal{C} \subseteq \mathcal{M}$ *and* $\mathcal{M}$ *is a* N-OS *in* $(\mathcal{U}, \tau_{\mathcal{R}}(G))$. *The complement of a* $N\alpha g$-CS *is a* $N\alpha g$-OS *in* $(\mathcal{U}, \tau_{\mathcal{R}}(G))$.

Definition 2.9 [6]: *A subset* $\mathcal{C}$ *of a* nts $(\mathcal{U}, \tau_{\mathcal{R}}(G))$ *is said to be a nano* $g\alpha$*-closed set (in short* $Ng\alpha$-CS*) if* $N\alpha cl(\mathcal{C}) \subseteq \mathcal{M}$ *whenever* $\mathcal{C} \subseteq \mathcal{M}$ *and* $\mathcal{M}$ *is a* $N\alpha$-OS *in* $(\mathcal{U}, \tau_{\mathcal{R}}(G))$. *The complement of a* $Ng\alpha$-CS *is a* $Ng\alpha$-OS *in* $(\mathcal{U}, \tau_{\mathcal{R}}(G))$.

Theorem 2.10 [4, 6]: *In a* nts $(\mathcal{U}, \tau_{\mathcal{R}}(G))$, *then the following statements hold and the contrary of each statement is not true:*

(i) Every N-OS *(resp.* N-CS*) is a* $N\alpha$-OS *(resp.* $N\alpha$-CS*).*

(ii) Every N-OS (resp. N-CS) is a Ng-OS (resp. Ng-CS).

(iii) Every Ng-OS (resp. Ng-CS) is a $N\alpha g$-OS (resp. $N\alpha g$-CS).

(iv) Every $N\alpha$-OS (resp. $N\alpha$-CS) is a $Ng\alpha$-OS (resp. $Ng\alpha$-CS).

(v) Every $Ng\alpha$-OS (resp. $Ng\alpha$-CS) is a $N\alpha g$-OS (resp. $N\alpha g$-CS).

Definition 2.11: *Let $(\mathcal{U}, \tau_{\mathcal{R}}(G))$ and $(\mathcal{V}, \sigma_{\mathcal{R}}(H))$ be nts. Then the map $h: (\mathcal{U}, \tau_{\mathcal{R}}(G)) \longrightarrow (\mathcal{V}, \sigma_{\mathcal{R}}(H))$ is called:*

(i) nano continuous (in short N-continuous) [5] if $h^{-1}(\mathcal{K})$ is a N-OS (resp. N-CS) in $(\mathcal{U}, \tau_{\mathcal{R}}(G))$, for each N-OS (resp. N-CS) $\mathcal{K}$ in $(\mathcal{V}, \sigma_{\mathcal{R}}(H))$.

(ii) nano α-continuous (in short $N\alpha$-continuous) [7] if $h^{-1}(\mathcal{K})$ is a $N\alpha$-OS (resp. $N\alpha$-CS) in $(\mathcal{U}, \tau_{\mathcal{R}}(G))$, for each N-OS (resp. N-CS) $\mathcal{K}$ in $(\mathcal{V}, \sigma_{\mathcal{R}}(H))$.

(iii) nano g-continuous (in short Ng-continuous) [2] if $h^{-1}(\mathcal{K})$ is a Ng-OS (resp. Ng-CS) in $(\mathcal{U}, \tau_{\mathcal{R}}(G))$, for each N-OS (resp. N-CS) $\mathcal{K}$ in $(\mathcal{V}, \sigma_{\mathcal{R}}(H))$.

(iv) nano αg-continuous (in short $N\alpha g$-continuous) [7] if $h^{-1}(\mathcal{K})$ is a $N\alpha g$-OS (resp. $N\alpha g$-CS) in $(\mathcal{U}, \tau_{\mathcal{R}}(G))$, for each N-OS (resp. N-CS) $\mathcal{K}$ in $(\mathcal{V}, \sigma_{\mathcal{R}}(H))$.

(v) nano $g\alpha$-continuous (in short $Ng\alpha$-continuous) [7] if $h^{-1}(\mathcal{K})$ is a $Ng\alpha$-OS (resp. $Ng\alpha$-CS) in $(\mathcal{U}, \tau_{\mathcal{R}}(G))$, for each N-OS (resp. N-CS) $\mathcal{K}$ in $(\mathcal{V}, \sigma_{\mathcal{R}}(H))$.

Theorem 2.12 [2, 7]: *Let $h: (\mathcal{U}, \tau_{\mathcal{R}}(G)) \longrightarrow (\mathcal{V}, \sigma_{\mathcal{R}}(H))$ be a map. Then the following statements hold and the contrary of each statement is not true:*

(i) Every N-continuous map is a $N\alpha$-continuous.
(ii) Every N-continuous map is a Ng-continuous.
(iii) Every Ng-continuous map is a $N\alpha g$-continuous.
(iv) Every $N\alpha$-continuous map is a $Ng\alpha$-continuous.
(v) Every $Ng\alpha$-continuous map is a $N\alpha g$-continuous.

3 Nano Generalized αg-Closed Sets

In this section we present and study the nano generalized $\boldsymbol{\alpha}$**g**-closed sets and some of its properties.

Definition 3.1: *A subset $\mathcal{C}$ of a nts $(\mathcal{U}, \tau_{\mathcal{R}}(G))$ is said to be a nano generalized αg-closed set (in short $Ng\alpha g$-CS) if $Ncl(\mathcal{C}) \subseteq \mathcal{M}$ whenever $\mathcal{C} \subseteq \mathcal{M}$ and $\mathcal{M}$ is a $N\alpha g$-OS in $(\mathcal{U}, \tau_{\mathcal{R}}(G))$. The family of all $Ng\alpha g$-CS of a nts $(\mathcal{U}, \tau_{\mathcal{R}}(G))$ is denoted by $Ng\alpha g$-$C(\mathcal{U}, G)$.*

Definition 3.2: *The intersection of all $Ng\alpha g$-CS in a nts $(\mathcal{U}, \tau_{\mathcal{R}}(G))$ containing $\mathcal{C}$ is called nano $g\alpha g$-closure of $\mathcal{C}$ and is denoted by $Ng\alpha g$-$cl(\mathcal{C})$, $Ng\alpha g$-$cl(\mathcal{C}) = \bigcap\{\mathcal{D}: \mathcal{C} \subseteq \mathcal{D}, \mathcal{D}$ is a $Ng\alpha g$-$CS\}$.*

Theorem 3.3: *In a nts $(\mathcal{U}, \tau_{\mathcal{R}}(G))$, the following statements are true:*

(i) Every N-CS is a $Ng\alpha g$-CS.

(ii) Every $Ng\alpha g$-CS is a Ng-CS.

(iii) Every $Ng\alpha g$-CS is a $N\alpha g$-CS.

(iv) Every $Ng\alpha g$-CS is a $Ng\alpha$-CS.

Proof:

(i)Let $\mathcal{C}$ be a N-CS in a nts $(\mathcal{U}, \tau_{\mathcal{R}}(G))$ and let $\mathcal{M}$ be any Nαg-OS containing $\mathcal{C}$. Then N$cl(\mathcal{C}) = \mathcal{C} \subseteq \mathcal{M}$. Hence, $\mathcal{C}$ is a Ngαg-CS.

(ii) Let $\mathcal{C}$ be a Ngαg-CS in a nts $(\mathcal{U}, \tau_{\mathcal{R}}(G))$ and let $\mathcal{M}$ be any N-OS containing $\mathcal{C}$. By theorem (2.10); $\mathcal{M}$ is a Nαg-OS in $(\mathcal{U}, \tau_{\mathcal{R}}(G))$. Since $\mathcal{C}$ is a Ngαg-CS, we have N$cl(\mathcal{C}) \subseteq \mathcal{M}$. Hence, $\mathcal{C}$ is a Ng-CS.

(iii) Let $\mathcal{C}$ be a Ngαg-CS in a nts $(\mathcal{U}, \tau_{\mathcal{R}}(G))$ and let $\mathcal{M}$ be any N-OS containing $\mathcal{C}$. By theorem (2.10); $\mathcal{M}$ is a Nαg-OS in $(\mathcal{U}, \tau_{\mathcal{R}}(G))$. Since $\mathcal{C}$ is a Ngαg-CS, we have N$\alpha cl(\mathcal{C}) \subseteq$ N$cl(\mathcal{C}) \subseteq \mathcal{M}$. Hence, $\mathcal{C}$ is a Nαg-CS.

(iv) Let $\mathcal{C}$ be a Ngαg-CS in a nts $(\mathcal{U}, \tau_{\mathcal{R}}(G))$ and let $\mathcal{M}$ be any Nα-OS containing $\mathcal{C}$. By theorem (2.10); $\mathcal{M}$ is a Nαg-OS in $(\mathcal{U}, \tau_{\mathcal{R}}(G))$. Since $\mathcal{C}$ is a Ngαg-CS, we have N$\alpha cl(\mathcal{C}) \subseteq$ N$cl(\mathcal{C}) \subseteq \mathcal{M}$. Hence, $\mathcal{C}$ is a Ngα-CS.

The contrary of the above theorem need not be true as appeared in the following examples.

Example 3.4: *Let $\mathcal{U} = \{p, q, r, s\}$ with $\mathcal{U}/\mathcal{R} = \{\{p\}, \{r\}, \{q, s\}\}$ and $G = \{p, q\}$. Let $\tau_{\mathcal{R}}(G) = \{\phi, \{p\}, \{q, s\}, \{p, q, s\}, \mathcal{U}\}$ be a nts. Then the set $\{p, q, r\}$ is a $Ng\alpha g$-CS but not N-CS.*

Example 3.5: *Let $\mathcal{U} = \{p, q, r, s, t\}$ with $\mathcal{U}/\mathcal{R} = \{\{s\}, \{p, q\}, \{r, t\}\}$ and $G = \{p, s\}$. Let $\tau_{\mathcal{R}}(G) = \{\phi, \{s\}, \{p, q\}, \{p, q, s\}, \mathcal{U}\}$ be a nts. Then the set $\{p, r, s\}$ is a Ng-CS but not $Ng\alpha g$-CS.*

Example 3.6: *Let $\mathcal{U} = \{t, u, v, w\}$ with $\mathcal{U}/\mathcal{R} = \{\{t\}, \{v\}, \{u, w\}\}$ and $G = \{t, u\}$. Let $\tau_{\mathcal{R}}(G) = \{\phi, \{t\}, \{u, w\}, \{t, u, w\}, \mathcal{U}\}$ be a nts. Then the set $\{t, v\}$ is a $Ng\alpha$-CS and hence $N\alpha g$-CS but not $Ng\alpha g$-CS.*

Definition 3.7: *A subset $\mathcal{C}$ of a nts $(\mathcal{U}, \tau_{\mathcal{R}}(G))$ is said to be a nano generalized αg-open set (in short $Ng\alpha g$-OS) iff $\mathcal{U} - \mathcal{C}$ is a $Ng\alpha g$-CS. The family of all $Ng\alpha g$-OS of a nts $(\mathcal{U}, \tau_{\mathcal{R}}(G))$ is denoted by $Ng\alpha g$-$O(\mathcal{U}, G)$.*

Definition 3.8: *The union of all $Ng\alpha g$-OS in a nts $(\mathcal{U}, \tau_{\mathcal{R}}(G))$ contained in $\mathcal{C}$ is called nano $g\alpha g$-interior of $\mathcal{C}$ and is denoted by $Ng\alpha g$-$int(\mathcal{C})$, $Ng\alpha g$-$int(\mathcal{C}) = \bigcup\{\mathcal{D}: \mathcal{C} \supseteq \mathcal{D}, \mathcal{D}$ is a $Ng\alpha g$-$OS\}$.*

Proposition 3.9: *Let $\mathcal{C}$ be any set in a nts $(\mathcal{U}, \tau_{\mathcal{R}}(G))$. Then the following properties hold:*

(i) $Ng\alpha g$-$int(\mathcal{C}) = \mathcal{C}$ iff $\mathcal{C}$ is a $Ng\alpha g$-OS.

(ii) $Ng\alpha g$-$cl(\mathcal{C}) = \mathcal{C}$ iff $\mathcal{C}$ is a $Ng\alpha g$-CS.

(iii) $Ng\alpha g$-$int(\mathcal{C})$ is the largest $Ng\alpha g$-OS contained in $\mathcal{C}$.

(iv) $Ng\alpha g$-$cl(\mathcal{C})$ is the smallest $Ng\alpha g$-CS containing $\mathcal{C}$.

Proof: (i), (ii), (iii) and (iv) are obvious.

Proposition 3.10: *Let $\mathcal{C}$ be any set in a nts $(\mathcal{U}, \tau_{\mathcal{R}}(G))$. Then the following properties hold:*

(i) $Ng\alpha g$-$int(\mathcal{U} - \mathcal{C}) = \mathcal{U} - (Ng\alpha g$-$cl(\mathcal{C}))$,

(ii) $Ng\alpha g$-$cl(\mathcal{U} - \mathcal{C}) = \mathcal{U} - (Ng\alpha g$-$int(\mathcal{C}))$.

Proof:

(i) By definition, Ngαg-$cl(\mathcal{C}) = \bigcap\{\mathcal{D}: \mathcal{C} \subseteq \mathcal{D}, \mathcal{D}$ is a Ngαg-CS$\}$

$$\begin{aligned}\mathcal{U} - (\text{Ng}\alpha\text{g-}cl(\mathcal{C})) &= \mathcal{U} - \bigcap\{\mathcal{D}: \mathcal{C} \subseteq \mathcal{D}, \mathcal{D} \text{ is a Ng}\alpha\text{g-CS}\} \\ &= \bigcup\{\mathcal{U} - \mathcal{D}: \mathcal{C} \subseteq \mathcal{D}, \mathcal{D} \text{ is a Ng}\alpha\text{g-CS}\} \\ &= \bigcup\{\mathcal{M}: \mathcal{U} - \mathcal{C} \supseteq \mathcal{M}, \mathcal{M} \text{ is a Ng}\alpha\text{g-OS}\} \\ &= \text{Ng}\alpha\text{g-}int(\mathcal{U} - \mathcal{C}).\end{aligned}$$

(ii) The proof is similar to (i).

Theorem 3.11: *Let $(\mathcal{U}, \tau_{\mathcal{R}}(G))$ be a nts. If $\mathcal{C}$ is a N-OS, then it is a $Ng\alpha g$-OS in $(\mathcal{U}, \tau_{\mathcal{R}}(G))$.*

Proof: Let $\mathcal{C}$ be a N-OS in a nts $(\mathcal{U}, \tau_{\mathcal{R}}(G))$, then $\mathcal{U} - \mathcal{C}$ is a N-CS in $(\mathcal{U}, \tau_{\mathcal{R}}(G))$. By theorem (3.3) part (i); $\mathcal{U} - \mathcal{C}$ is a Ngαg-CS. Hence, $\mathcal{C}$ is a Ngαg-OS in $(\mathcal{U}, \tau_{\mathcal{R}}(G))$.

Theorem 3.12: *Let $(\mathcal{U}, \tau_{\mathcal{R}}(G))$ be a nts. If $\mathcal{C}$ is a $Ng\alpha g$-OS, then it is a Ng-OS in $(\mathcal{U}, \tau_{\mathcal{R}}(G))$.*

Proof: Let $\mathcal{C}$ be a Ngαg-OS in a nts $(\mathcal{U}, \tau_{\mathcal{R}}(G))$, then $\mathcal{U} - \mathcal{C}$ is a Ngαg-CS in $(\mathcal{U}, \tau_{\mathcal{R}}(G))$. By theorem (3.3) part (ii); $\mathcal{U} - \mathcal{C}$ is a Ng-CS. Hence, $\mathcal{C}$ is a Ng-OS in $(\mathcal{U}, \tau_{\mathcal{R}}(G))$.

Lemma 3.13: *Let $(\mathcal{U}, \tau_{\mathcal{R}}(G))$ be a nts. If $\mathcal{C}$ is a $Ng\alpha g$-OS, then it is a $N\alpha g$-OS (resp. $Ng\alpha$-OS) in $(\mathcal{U}, \tau_{\mathcal{R}}(G))$.*

Proof: Similar to above theorem.

Proposition 3.14: *If $\mathcal{C}$ and $\mathcal{D}$ are $Ng\alpha g$-CS in a nts $(\mathcal{U}, \tau_{\mathcal{R}}(G))$, then $\mathcal{C} \cup \mathcal{D}$ is a $Ng\alpha g$-CS.*

Proof: Let $\mathcal{C}$ and $\mathcal{D}$ be Ngαg-CS in a nts $\left(\mathcal{U}, \tau_{\mathcal{R}}(G)\right)$ and let $\mathcal{M}$ be any Nαg-OS containing $\mathcal{C}$ and $\mathcal{D}$. Then $\mathcal{C} \cup \mathcal{D} \subseteq \mathcal{M}$. Then $\mathcal{C} \subseteq \mathcal{M}$ and $\mathcal{D} \subseteq \mathcal{M}$. Since $\mathcal{C}$ and $\mathcal{D}$ are Ngαg-CS, N$cl(\mathcal{C}) \subseteq \mathcal{M}$ and N$cl(\mathcal{D}) \subseteq \mathcal{M}$.

Now, N$cl(\mathcal{C} \cup \mathcal{D}) =$ N$cl(\mathcal{C}) \cup$N$cl(\mathcal{D}) \subseteq \mathcal{M}$ and so N$cl(\mathcal{C} \cup \mathcal{D}) \subseteq \mathcal{M}$.

Hence, $\mathcal{C} \cup \mathcal{D}$ is a Ngαg-CS.

Proposition 3.15: *If $\mathcal{C}$ and $\mathcal{D}$ are $Ng\alpha g$-OS in a nts $(\mathcal{U}, \tau_{\mathcal{R}}(G))$, then $\mathcal{C} \cap \mathcal{D}$ is a $Ng\alpha g$-OS.*

Proof: Let $\mathcal{C}$ and $\mathcal{D}$ be Ngαg-OS in a nts $(\mathcal{U}, \tau_{\mathcal{R}}(G))$. Then $\mathcal{U} - \mathcal{C}$ and $\mathcal{U} - \mathcal{D}$ are Ngαg-CS. By proposition (3.14); $(\mathcal{U} - \mathcal{C}) \cup (\mathcal{U} - \mathcal{D})$ is a Ngαg-CS. Since $(\mathcal{U} - \mathcal{C}) \cup (\mathcal{U} - \mathcal{D}) = \mathcal{U} - (\mathcal{C} \cap \mathcal{D})$. Hence, $\mathcal{C} \cap \mathcal{D}$ is a Ngαg-OS.

Proposition 3.16: *If a set $\mathcal{C}$ is $Ng\alpha g$-CS in a nts $(\mathcal{U}, \tau_{\mathcal{R}}(G))$, then $Ncl(\mathcal{C}) - \mathcal{C}$ contains no non-empty N-CS in $(\mathcal{U}, \tau_{\mathcal{R}}(G))$.*

Proof: Let $\mathcal{C}$ be a Ngαg-CS in a nts $(\mathcal{U}, \tau_{\mathcal{R}}(G))$ and let $\mathcal{F}$ be any N-CS in $(\mathcal{U}, \tau_{\mathcal{R}}(G))$ such that $\mathcal{F} \subseteq$ N$cl(\mathcal{C}) - \mathcal{C}$. Since $\mathcal{C}$ is a Ngαg-CS, we have N$cl(\mathcal{C}) \subseteq \mathcal{U} - \mathcal{F}$. This implies $\mathcal{F} \subseteq \mathcal{U} -$ N$cl(\mathcal{C})$. Then $\mathcal{F} \subseteq$ N$cl(\mathcal{C}) \cap (\mathcal{U} -$ N$cl(\mathcal{C})) = \phi$. Thus, $\mathcal{F} = \phi$. Hence, N$cl(\mathcal{C}) - \mathcal{C}$ contains no non-empty N-CS in $(\mathcal{U}, \tau_{\mathcal{R}}(G))$.

Proposition 3.17: *A set $\mathcal{C}$ is $Ng\alpha g$-CS in a nts $(\mathcal{U}, \tau_{\mathcal{R}}(G))$ iff $Ncl(\mathcal{C}) - \mathcal{C}$ contains no non-empty $N\alpha g$-CS in $(\mathcal{U}, \tau_{\mathcal{R}}(G))$.*

Proof: Let $\mathcal{C}$ be a Ngαg-CS in a nts $(\mathcal{U}, \tau_{\mathcal{R}}(G))$ and let $\mathcal{S}$ be any Nαg-CS in $(\mathcal{U}, \tau_{\mathcal{R}}(G))$ such that $\mathcal{S} \subseteq$ N$cl(\mathcal{C}) - \mathcal{C}$. Since $\mathcal{C}$ is a Ngαg-CS, we have N$cl(\mathcal{C}) \subseteq \mathcal{U} - \mathcal{S}$. This implies $\mathcal{S} \subseteq \mathcal{U} -$ N$cl(\mathcal{C})$. Then $\mathcal{S} \subseteq$ N$cl(\mathcal{C}) \cap (\mathcal{U} -$ N$cl(\mathcal{C})) = \phi$. Thus, $\mathcal{S}$ is empty.

Conversely, suppose that N$cl(\mathcal{C}) - \mathcal{C}$ contains no non-empty Nαg-CS in $(\mathcal{U}, \tau_{\mathcal{R}}(G))$. Let $\mathcal{C} \subseteq \mathcal{M}$ and $\mathcal{M}$ is Nαg-OS. If N$cl(\mathcal{C}) \subseteq \mathcal{M}$ then N$cl(\mathcal{C}) \cap (\mathcal{U} - \mathcal{M})$ is non-empty. Since N$cl(\mathcal{C})$ is N-CS and $\mathcal{U} - \mathcal{M}$ is Nαg-CS, we have N$cl(\mathcal{C}) \cap (\mathcal{U} - \mathcal{M})$ is non-empty Nαg-CS of N$cl(\mathcal{C}) - \mathcal{C}$ which is a contradiction. Therefore N$cl(\mathcal{C}) \nsubseteq \mathcal{M}$. Hence, $\mathcal{C}$ is a Ngαg-CS.

Theorem 3.18: *If $\mathcal{C}$ is a $N\alpha g$-OS and a $Ng\alpha g$-CS in a nts $(\mathcal{U}, \tau_{\mathcal{R}}(G))$, then $\mathcal{C}$ is a N-CS in $(\mathcal{U}, \tau_{\mathcal{R}}(G))$.*

Proof: Suppose that $\mathcal{C}$ is a Nαg-OS and a Ngαg-CS in a nts $(\mathcal{U}, \tau_{\mathcal{R}}(G))$, then N$cl(\mathcal{C}) \subseteq \mathcal{C}$ and since $\mathcal{C} \subseteq$ N$cl(\mathcal{C})$. Thus, N$cl(\mathcal{C}) = \mathcal{C}$. Hence, $\mathcal{C}$ is a N-CS.

Theorem 3.19: *If $\mathcal{C}$ is a $Ng\alpha g$-CS in a nts $(\mathcal{U}, \tau_{\mathcal{R}}(G))$ and $\mathcal{C} \subseteq \mathcal{D} \subseteq Ncl(\mathcal{C})$, then $\mathcal{D}$ is a $Ng\alpha g$-CS in $(\mathcal{U}, \tau_{\mathcal{R}}(G))$.*

Proof: Suppose that $\mathcal{C}$ is a Ngαg-CS in a nts $(\mathcal{U}, \tau_{\mathcal{R}}(G))$. Let $\mathcal{M}$ be a Nαg-OS in $(\mathcal{U}, \tau_{\mathcal{R}}(G))$ such that $\mathcal{D} \subseteq \mathcal{M}$. Then $\mathcal{C} \subseteq \mathcal{M}$. Since $\mathcal{C}$ is a Ngαg-CS, it follows that N$cl(\mathcal{C}) \subseteq \mathcal{M}$. Now, $\mathcal{D} \subseteq$ N$cl(\mathcal{C})$ implies N$cl(\mathcal{D}) \subseteq$ N$cl($N$cl(\mathcal{C})) =$ N$cl(\mathcal{C})$. Thus, N$cl(\mathcal{D}) \subseteq \mathcal{M}$. Hence, $\mathcal{D}$ is a Ngαg-CS.

Theorem 3.20: *If $\mathcal{C}$ is a $Ng\alpha g$-OS in a nts $(\mathcal{U}, \tau_{\mathcal{R}}(G))$ and $Nint(\mathcal{C}) \subseteq \mathcal{D} \subseteq \mathcal{C}$, then $\mathcal{D}$ is a $Ng\alpha g$-OS in $(\mathcal{U}, \tau_{\mathcal{R}}(G))$.*

Proof: Suppose that $\mathcal{C}$ is a Ngαg-OS in a nts $(\mathcal{U}, \tau_{\mathcal{R}}(G))$ and N$int(\mathcal{C}) \subseteq \mathcal{D} \subseteq \mathcal{C}$. Then $\mathcal{U} - \mathcal{C}$ is a Ngαg-CS and $\mathcal{U} - \mathcal{C} \subseteq \mathcal{U} - \mathcal{D} \subseteq$ N$cl(\mathcal{U} - \mathcal{C})$. Then $\mathcal{U} - \mathcal{D}$ is a Ngαg-CS by theorem (3.19). Hence, $\mathcal{D}$ is a Ngαg-OS.

Theorem 3.21: *A set $\mathcal{C}$ is $Ng\alpha g$-OS iff $\mathcal{P} \subseteq Nint(\mathcal{C})$ where $\mathcal{P}$ is a $Ng\alpha g$-CS and $\mathcal{P} \subseteq \mathcal{C}$.*

Proof: Suppose that $\mathcal{P} \subseteq$ N$int(\mathcal{C})$ where $\mathcal{P}$ is a Ngαg-CS and $\mathcal{P} \subseteq \mathcal{C}$. Then $\mathcal{U} - \mathcal{C} \subseteq \mathcal{U} - \mathcal{P}$ and $\mathcal{U} - \mathcal{P}$ is a Nαg-OS by lemma (3.13). Now, N$cl(\mathcal{U} - \mathcal{C}) = \mathcal{U} -$ N$int(\mathcal{C}) \subseteq \mathcal{U} - \mathcal{P}$. Then $\mathcal{U} - \mathcal{C}$ is a Ngαg-CS. Hence, $\mathcal{C}$ is a Ngαg-OS.

Conversely, let $\mathcal{C}$ be a Ngαg-OS and $\mathcal{P}$ be a Ngαg-CS and $\mathcal{P} \subseteq \mathcal{C}$. Then $\mathcal{U} - \mathcal{C} \subseteq \mathcal{U} - \mathcal{P}$. Since $\mathcal{U} - \mathcal{C}$ is a Ngαg-CS and $\mathcal{U} - \mathcal{P}$ is a Nαg-OS, we have N$cl(\mathcal{U} - \mathcal{C}) \subseteq \mathcal{U} - \mathcal{P}$. Then $\mathcal{P} \subseteq$ N$int(\mathcal{C})$.

Remark 3.22: *The following diagram shows the relation between the different types of N-CS:*

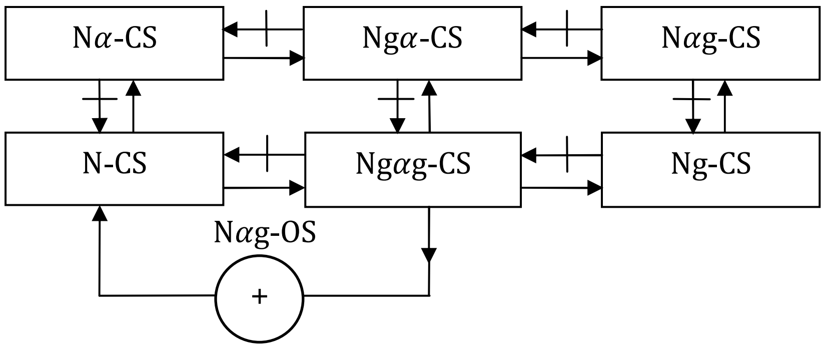

Diagram (3.1)

4 Nano Generalized αg-Continuous Maps

In this section we present the nano generalized $\boldsymbol{\alpha}$**g**-continuous maps and study some of their crucial properties.

Definition 4.1: *A map* $h: (\mathcal{U}, \tau_{\mathcal{R}}(G)) \longrightarrow (\mathcal{V}, \sigma_{\mathcal{R}}(H))$ *is said to be a nano generalized* αg*-continuous (in short* $Ng\alpha g$*-continuous) if* $h^{-1}(\mathcal{K})$ *is a* $Ng\alpha g$*-*CS *in* $(\mathcal{U}, \tau_{\mathcal{R}}(G))$ *for every* N*-*CS $\mathcal{K}$ *in* $(\mathcal{V}, \sigma_{\mathcal{R}}(H))$*.*

Theorem 4.2: *Let* $(\mathcal{U}, \tau_{\mathcal{R}}(G))$ *and* $(\mathcal{V}, \sigma_{\mathcal{R}}(H))$ *be* nts*, and* $h: (\mathcal{U}, \tau_{\mathcal{R}}(G)) \longrightarrow (\mathcal{V}, \sigma_{\mathcal{R}}(H))$ *be a map. Then* h *is a* $Ng\alpha g$*-continuous map iff* $h^{-1}(\mathcal{K})$ *is a* $Ng\alpha g$*-*OS *in* $(\mathcal{U}, \tau_{\mathcal{R}}(G))$*, for every* N*-*OS $\mathcal{K}$ *in* $(\mathcal{V}, \sigma_{\mathcal{R}}(H))$*.*

Proof: Let $\mathcal{K}$ be a N-OS in $(\mathcal{V}, \sigma_{\mathcal{R}}(H))$. Then $\mathcal{K}^c$ is a N-CS in $(\mathcal{V}, \sigma_{\mathcal{R}}(H))$, so $h^{-1}(\mathcal{K}^c) = (h^{-1}(\mathcal{K}))^c$ is a Ngαg-CS in $(\mathcal{U}, \tau_{\mathcal{R}}(G))$. Thus, $h^{-1}(\mathcal{K})$ is a Ngαg-OS in $(\mathcal{U}, \tau_{\mathcal{R}}(G))$. The proof of the opposite is obvious..

Proposition 4.3: *Every* $Ng\alpha g$*-continuous map is a* $N\alpha g$*-continuous.*

Proof: Let $h: (\mathcal{U}, \tau_{\mathcal{R}}(G)) \longrightarrow (\mathcal{V}, \sigma_{\mathcal{R}}(H))$ be a Ngαg-continuous map and let $\mathcal{K}$ be a N-CS in $(\mathcal{V}, \sigma_{\mathcal{R}}(H))$. Since h is a Ngαg-continuous, $h^{-1}(\mathcal{K})$ is a Ngαg-CS in $(\mathcal{U}, \tau_{\mathcal{R}}(G))$. By theorem (3.3) part (iii); $h^{-1}(\mathcal{K})$ is a Nαg-CS in $(\mathcal{U}, \tau_{\mathcal{R}}(G))$. Thus, h is a Nαg-continuous.

Proposition 4.4: *Every* $Ng\alpha g$*-continuous map is a* $Ng\alpha$*-continuous.*

Proof: Let $h: (\mathcal{U}, \tau_{\mathcal{R}}(G)) \longrightarrow (\mathcal{V}, \sigma_{\mathcal{R}}(H))$ be a Ngαg-continuous map and let $\mathcal{K}$ be a N-CS in $(\mathcal{V}, \sigma_{\mathcal{R}}(H))$. Since h is a Ngαg-continuous, $h^{-1}(\mathcal{K})$ is a Ngαg-CS in $(\mathcal{U}, \tau_{\mathcal{R}}(G))$. By theorem (3.3) part (iv); $h^{-1}(\mathcal{K})$ is a Ngα-CS in $(\mathcal{U}, \tau_{\mathcal{R}}(G))$. Thus, h is a Ngα-continuous.

The contrary of the above propositions need not be true as appeared in the following example.

Example 4.5: *Let $\mathcal{U}=\{p,q,r,s\}$ with $\mathcal{U}/\mathcal{R}=\{\{p\},\{q\},\{q,s\}\}$ and $G=\{p,q\}$. Let $\tau_{\mathcal{R}}(G)=\{\phi,\{p\},\{q,s\},\{p,q,s\},\mathcal{U}\}$ be a nts. Let $\mathcal{V}=\{t,u,v,w\}$ with $\mathcal{V}/\mathcal{R}=\{\{u\},\{w\},\{t,v\}\}$ and $H=\{t,u\}$. Let $\sigma_{\mathcal{R}}(H)=\{\phi,\{u\},\{t,v\},\{t,u,v\},\mathcal{V}\}$ be a nts. Define a map $h:(\mathcal{U},\tau_{\mathcal{R}}(G))\longrightarrow(\mathcal{V},\sigma_{\mathcal{R}}(H))$ as $h(p)=v$, $h(q)=t, h(r)=w, h(s)=u$. Then h is a $Ng\alpha$-continuous and hence $N\alpha g$-continuous but not $Ng\alpha g$-continuous.*

Theorem 4.6: *If $h:(\mathcal{U},\tau_{\mathcal{R}}(G))\longrightarrow(\mathcal{V},\sigma_{\mathcal{R}}(H))$ is a $Ng\alpha g$-continuous map then for each point c of $\mathcal{U}$ and $\mathcal{D}\in\sigma_{\mathcal{R}}(H)$ such that $h(c)\in\mathcal{D}$, there exists a $Ng\alpha g$-OS $\mathcal{C}$ of $(\mathcal{U},\tau_{\mathcal{R}}(G))$ such that $c\in\mathcal{C}$ and $h(\mathcal{C})\subseteq\mathcal{D}$.*

Proof: Let c be a point of $\mathcal{U}$ and $\mathcal{D}\in\sigma_{\mathcal{R}}(H)$ such that $h(c)\in\mathcal{D}$. Take $\mathcal{C}=h^{-1}(\mathcal{D})$. Since $\mathcal{V}-\mathcal{D}$ is a N-CS in $(\mathcal{V},\sigma_{\mathcal{R}}(H))$ and h is a Ngαg-continuous map, we have $h^{-1}(\mathcal{V}-\mathcal{D})=\mathcal{U}-h^{-1}(\mathcal{D})$ is a Ngαg-CS in $(\mathcal{U},\tau_{\mathcal{R}}(G))$. This gives $\mathcal{C}=h^{-1}(\mathcal{D})$ is a Ngαg-OS in $(\mathcal{U},\tau_{\mathcal{R}}(G))$ and $c\in\mathcal{C}$ and $h(\mathcal{C})=h(h^{-1}(\mathcal{D}))\subseteq\mathcal{D}$.

Definition 4.7: *A map $h:(\mathcal{U},\tau_{\mathcal{R}}(G))\longrightarrow(\mathcal{V},\sigma_{\mathcal{R}}(H))$ is said to be a nano generalized αg-irresolute (in short $Ng\alpha g$-irresolute) if $h^{-1}(\mathcal{K})$ is a $Ng\alpha g$-CS in $(\mathcal{U},\tau_{\mathcal{R}}(G))$ for every $Ng\alpha g$-CS $\mathcal{K}$ in $(\mathcal{V},\sigma_{\mathcal{R}}(H))$.*

Theorem 4.8: *Let $(\mathcal{U},\tau_{\mathcal{R}}(G))$ and $(\mathcal{V},\sigma_{\mathcal{R}}(H))$ be nts, and $h:(\mathcal{U},\tau_{\mathcal{R}}(G))\longrightarrow(\mathcal{V},\sigma_{\mathcal{R}}(H))$ be a map. Then h is a $Ng\alpha g$-irresolute map iff $h^{-1}(\mathcal{K})$ is a $Ng\alpha g$-OS in $(\mathcal{U},\tau_{\mathcal{R}}(G))$, for every $Ng\alpha g$-OS $\mathcal{K}$ in $(\mathcal{V},\sigma_{\mathcal{R}}(H))$.*

Proof: Let $\mathcal{K}$ be a Ngαg-OS in $(\mathcal{V},\sigma_{\mathcal{R}}(H))$. Then $\mathcal{V}-\mathcal{K}$ is a Ngαg-CS in $(\mathcal{V},\sigma_{\mathcal{R}}(H))$, so $h^{-1}(\mathcal{V}-\mathcal{K})=\mathcal{U}-h^{-1}(\mathcal{K})$ is a Ngαg-CS in $(\mathcal{U},\tau_{\mathcal{R}}(G))$. Thus, $h^{-1}(\mathcal{K})$ is a Ngαg-OS in $(\mathcal{U},\tau_{\mathcal{R}}(G))$. The proof of the opposite is obvious..

Proposition 4.9: *Every $Ng\alpha g$-irresolute map is a $Ng\alpha g$-continuous.*

Proof: Let $h:(\mathcal{U},\tau_{\mathcal{R}}(G))\longrightarrow(\mathcal{V},\sigma_{\mathcal{R}}(H))$ be a Ngαg-irresolute map and let $\mathcal{K}$ be a N-CS in $(\mathcal{V},\sigma_{\mathcal{R}}(H))$, by theorem (3.3) part (i), then $\mathcal{K}$ is a Ngαg-CS in $(\mathcal{V},\sigma_{\mathcal{R}}(H))$. Since h is a Ngαg-irresolute, then $h^{-1}(\mathcal{K})$ is a Ngαg-CS in $(\mathcal{U},\tau_{\mathcal{R}}(G))$. Thus, h is a Ngαg-continuous.

Definition 4.10: *A nts $(\mathcal{U},\tau_{\mathcal{R}}(G))$ is said to be a nano $T_{\frac{1}{2}}$-space (in short $NT_{\frac{1}{2}}$-space) if every Ng-CS in it is a N-CS.*

Definition 4.11: *A nts $(\mathcal{U},\tau_{\mathcal{R}}(G))$ is said to be a nano $T_{g\alpha g}$-space (in short $NT_{g\alpha g}$-space) if every $Ng\alpha g$-CS in it is a N-CS.*

Proposition 4.12: *Every $NT_{\frac{1}{2}}$-space is a $NT_{g\alpha g}$-space.*

Proof: Let $(\mathcal{U}, \tau_{\mathcal{R}}(G))$ be a N$T_{\frac{1}{2}}$-space and let $\mathcal{C}$ be a Ngαg-CS in $\mathcal{U}$. Then $\mathcal{C}$ is a Ng-CS, by theorem (3.3) part (ii). Since $(\mathcal{U}, \tau_{\mathcal{R}}(G))$ is a N$T_{\frac{1}{2}}$-space, then $\mathcal{C}$ is a N-CS in $\mathcal{U}$. Hence, $(\mathcal{U}, \tau_{\mathcal{R}}(G))$ is a N$T_{g\alpha g}$-space.

The following example demonstrates that the contrary of the above proposition not be true.

Example 4.13: *Let $\mathcal{U} = \{p, q, r\}$ with $\mathcal{U}/\mathcal{R} = \{\{p\}, \{q, r\}\}$ and $G = \{p, r\}$. Let $\tau_{\mathcal{R}}(G) = \{\phi, \{p\}, \{q, r\}, \mathcal{U}\}$ be a nts. Then $(\mathcal{U}, \tau_{\mathcal{R}}(G))$ is a $NT_{g\alpha g}$-space, but not $NT_{\frac{1}{2}}$-space.*

Theorem 4.14: *If $h_1: (\mathcal{U}, \tau_{\mathcal{R}}(G)) \longrightarrow (\mathcal{V}, \sigma_{\mathcal{R}}(H))$ is a $Ng\alpha g$-continuous map and $h_2: (\mathcal{V}, \sigma_{\mathcal{R}}(H)) \longrightarrow (\mathcal{W}, \rho_{\mathcal{R}}(I))$ is a Ng-continuous map and $(\mathcal{V}, \sigma_{\mathcal{R}}(H))$ is a $NT_{\frac{1}{2}}$-space. Then $h_2 \circ h_1: (\mathcal{U}, \tau_{\mathcal{R}}(G)) \longrightarrow (\mathcal{W}, \rho_{\mathcal{R}}(I))$ is a $Ng\alpha g$-continuous map.*

Proof: Let $\mathcal{K}$ be a N-CS in $\mathcal{W}$. Since h_2 is a Ng-continuous map and $(\mathcal{V}, \sigma_{\mathcal{R}}(H))$ is a N$T_{\frac{1}{2}}$-space, ${h_2}^{-1}(\mathcal{K})$ is a N-CS in $\mathcal{V}$. Since h_1 is a Ngαg-continuous map, ${h_1}^{-1}({h_2}^{-1}(\mathcal{K}))$ is a Ngαg-CS in $\mathcal{U}$. Thus, $h_2 \circ h_1$ is a Ngαg-continuous.

Theorem 4.15: *Let $(\mathcal{U}, \tau_{\mathcal{R}}(G))$ and $(\mathcal{V}, \sigma_{\mathcal{R}}(H))$ be nts, and $h: (\mathcal{U}, \tau_{\mathcal{R}}(G)) \longrightarrow (\mathcal{V}, \sigma_{\mathcal{R}}(H))$ be a map:*

(i) If $(\mathcal{U}, \tau_{\mathcal{R}}(G))$ is a $NT_{\frac{1}{2}}$-space then h is a Ng-continuous iff it is a $Ng\alpha g$-continuous.

(ii) If $(\mathcal{U}, \tau_{\mathcal{R}}(G))$ is a $NT_{g\alpha g}$-space then h is a N-continuous iff it is a $Ng\alpha g$-continuous.

Proof:

(i) Let $\mathcal{K}$ be any N-CS in $(\mathcal{V}, \sigma_{\mathcal{R}}(H))$. Since h is a Ng-continuous, $h^{-1}(\mathcal{K})$ is a Ng-CS in $(\mathcal{U}, \tau_{\mathcal{R}}(G))$. By $(\mathcal{U}, \tau_{\mathcal{R}}(G))$ is a N$T_{\frac{1}{2}}$-space, which implies, $h^{-1}(\mathcal{K})$ is a N-CS. By theorem (3.3) part (i); $h^{-1}(\mathcal{K})$ is a Ngαg-CS in $(\mathcal{U}, \tau_{\mathcal{R}}(G))$. Hence, h is a Ngαg-continuous.

Conversely, suppose that h is a Ngαg-continuous. Let $\mathcal{K}$ be any N-CS in $(\mathcal{V}, \sigma_{\mathcal{R}}(H))$. Then $h^{-1}(\mathcal{K})$ is a Ngαg-CS in $(\mathcal{U}, \tau_{\mathcal{R}}(G))$. By theorem (3.3) part (ii); $h^{-1}(\mathcal{K})$ is a Ng-CS in $(\mathcal{U}, \tau_{\mathcal{R}}(G))$. Hence, h is a Ng-continuous.

(ii) Let $\mathcal{K}$ be any N-CS in $(\mathcal{V}, \sigma_{\mathcal{R}}(H))$. Since h is a N-continuous, $h^{-1}(\mathcal{K})$ is a N-CS in $(\mathcal{U}, \tau_{\mathcal{R}}(G))$. By theorem (3.3) part (i); $h^{-1}(\mathcal{K})$ is a Ngαg-CS in $(\mathcal{U}, \tau_{\mathcal{R}}(G))$. Hence, h is a Ngαg-continuous.

Conversely, suppose that h is a Ngαg-continuous. Let $\mathcal{K}$ be any N-CS in $(\mathcal{V}, \sigma_{\mathcal{R}}(H))$. Then $h^{-1}(\mathcal{K})$ is a Ngαg-CS in $(\mathcal{U}, \tau_{\mathcal{R}}(G))$. By $(\mathcal{U}, \tau_{\mathcal{R}}(G))$ is a N$T_{g\alpha g}$-space, which implies $h^{-1}(\mathcal{K})$ is a N-CS in $(\mathcal{U}, \tau_{\mathcal{R}}(G))$. Hence, h is a N-continuous.

Remark 4.16: *The following diagram shows the relation between the different types of N-continuous maps:*

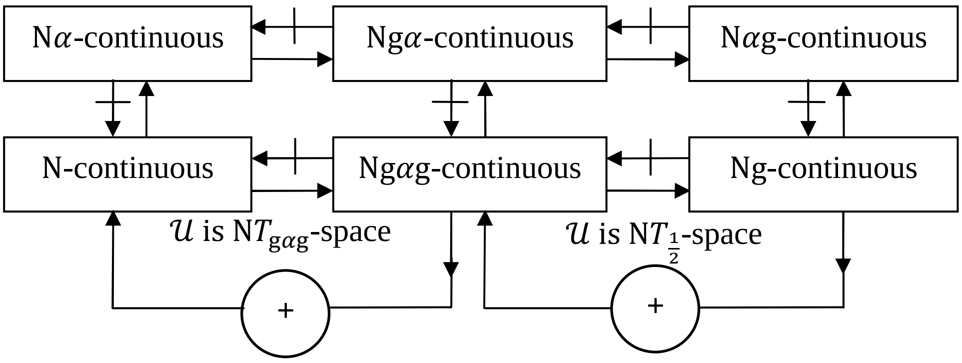

Diagram (4.1)

5 Conclusion

The class of Ngαg-CS characterized utilizing Nαg-CS forms a nano topology and lies between the class of N-CS and the class of Ng-CS. We likewise present Ngαg-continuous maps by utilizing Ngαg-CS. The Ngαg-CS can be utilized to determine another nano separation axiom.

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