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Effects of Thermal Dissipation, Heat Generation/Absorption on MHD Mixed Convection Boundary Layer Flow over a Permeable Vertical Flat Plate Embedded in an Anisotropic Porous Medium

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Abstract

In this paper, we investigated the effects of thermal dissipation, heat generation or absorption on MHD mixed convection boundary layer flow over a permeable vertical flat plate embedded in an anisotropic porous medium saturated with viscous Newtonian fluid. The non-linear partial differential equations describing the problem were reduced to coupled ordinary differential equations by means of dimensional analysis and appropriate transformations. The resulting boundary layer equations were solved numerically using a shooting quadrature implemented on an efficient and reliable software package. The influences of the embedded flow parameters sequel to the flow and heat transfer were attested on pertinent thermo fluid characteristics, such as the skin friction coefficient, the Nusselt number in table and plots while attributes due to the dimensionless velocity and temperature were demonstrated graphically. Considerable effects of all the governing flow parameters were observed *quantitatively on all important physical quantities. It is conjectured that the results obtained will not only provide useful information for applications but also serve as a complement to the previous studies related to anisotropic porous media. There has not been documented archive on this particular study of ours yet. The present work can thusly be adduced original.*

Keywords: *Boundary layer, Porous medium anisotropy, Thermal dissipation, Heat generation or absorption.*

1 Introduction

The problems of mixed (or combined free and forced) convection flows through saturated porous media due to heated or cooled plates exhibit important basic scenarios in the classical theory of the boundary layer flow and heat transfer. Many an author in the resent past had demonstrated rekindled interest into the studies concerning the boundary layer flow and heat transfer problems due to heated solid surfaces encompassed in a stream of electrically conducting gases or liquid metals exposed to prescribed magnetic fields in the presence of isotropic porous medium. The rewarding applications are found in many engineering and geophysical applications such as geothermal reservoirs, thermal insulation, enhanced oil recovery, packed-bed catalytic reactors, cooling of nuclear reactors most especially during an emergency shutdown. The closed form analysis of forced convection boundary layer flow past an impermeable surface embedded in a non-porous medium due to a variable velocity of an ambient steady hydrodynamic fluid was initiated by Hiemenz [1]. Among the early pioneering studies of importance in heat transfer and boundary layer flow about a vertical heated plate was that due to Ostrach [2] who reported the effects of Archemedian (buoyancy) force induced convective flow and forced convection induced due to pressure drop or an agitator about a fixed surface in a quiescent fluid. Influence of transpiration and magnetic field on Hiemenz flow had been considered by Yih [3], Chamkha and Khaled [4] extended it with inclusion of Darcian porous medium and Lorentz force alongside the mass flux, and much later by Kechil and Hashim [5] who respectively carried out their investigations via numerical and approximate analytical methods. Crane [6] initiated for the first time, the boundary layer flow and heat transfer caused by a stretching surface in an otherwise still hydrodynamic fluid encompassed in a porous–free medium, he obtained exact solutions for the velocity and temperature fields. The effects of surface mass flux (suction or injection) on a related problem was tackled exactly by Chen and Char [7], Ayeni and Okoya [8]. The latter who revisited the study of Hossain and Shayo [9], pointed out that the skin-friction function might not necessarily vanish for some pertinent thermofluid parameters. Nonetheless, Bejan and Poulikakos [10] employed Forchheimer's equation (Forchheimer, [11]) to account for the effect of a quadratic drag on free convection in a highly porous medium. Excellent theoretical support of existence and validity of Darcy's law (Darcy, [12]) permeability tensor which largely depends on the geometry of the porous medium had been demonstrated by Whitaker [13] alongside the needed

Forchheimer correction in the momentum equation. In the recent past and very recently, many a researcher (Chamkha et al., [14]; Makinde, [15]; Abel et al., [16], Kaya, [17]; Adeniyan and Ogwuegbu, [18]) amongst others; had investigated other various aspects of boundary layer flow and mass/heat transfer characteristics over an impermeable and/or permeable surface embedded wholly in fully or partially saturated porous medium of homogeneous isotropic permeability.

Anisotropy in the porous medium arises from the geometry of the grains or agglomeration of solid sediments which allow the passages of percolating velocity of the fluid. In the generalized Darcy's law the permeability property is considered as a general tensor ascribable to direction dependency (see Tyvand and Storhaug [19]); monograph). Worthy of note and observation from all studies mentioned above accounting for the permeability effects, the porous media were assumed to be homogeneous and isotropic whereas, in several real life situations and applications the solid porous matrices are anisotropic. In spite of this fact, natural and/or forced convection in such anisotropic media has received sparse attention. Several scientific and engineering devices involving flow and heat transfer through porous media of anisotropic nature with a variety of pressing applications, such as in geothermal operations, petroleum industries, agriculture, steel plates and rods manufacturing industries, filtration and adsorption processes, nuclear waste repositories, permafrost degradation control, soil lofting due to frost heave and so forth provided further impetus for in depth scholarly investigations by researchers. A cursory survey of literature revealed that Castinel and Combarnous [20] initiated a novel study of convective flow in a saturated horizontal porous medium layer with anisotropy subjected to heat from below. Others whose studies were in tow with the latter include Straughan and Walker [21], Degan et al. [22], Capone et al. [23] and the very recent paper of Tyvand and Storesletten [24]. A comprehensive review of documented reports on the convective flow in porous media can be retrieved from Storesletten [25]. The studies on heat and momentum transfer for a vertical channel or cavity filled with anisotropic porous media were presented by Degan et al. [26], who analyzed numerically and in closed form both hydrodynamically and thermally isotropic medium natural convection flow in a vertical cavity, whereas Safi and Benissaad [27] reported a numerical study of heat and mass transfer also in a vertical rectangular impermeable wall enclosure abounded in porous medium with anisotropy both hydrodynamically and thermally. Degan and Vasseur [28] who carried out their study analytically and numerically on natural convection heat transfer in a hydrodynamically anisotropic porous layer within rectangular cavity whose vertical walls are subjected to constant heat flux. Thasekhar et al. [29] numerically investigated free convection in a vertical cylindrical annulus filled with anisotropic porous medium. However, Muasavi and Shahnazari [30] undertook study of double-diffusive natural convection orthotropic porous medium vertical channel flow in a vertical enclosed channel and obtained numerical results. Another novel and classical problem in hydrodynamics is the mixed (free-forced) convective boundary-layer flow, of an incompressible and viscous hydromagnetic fluid over an infinite or semi-infinite hot or cold vertical plate permeated by a transverse magnetic field with anisotropy in porous permeability. This type of problem features very often in engineering devices and technological processes but has not yet been examined research-wise, to the fullest. A few of the scanty works taking into account of some of these aspects on buoyancy-induced convective flows over a vertical or horizontal flat plate encompassed in the porous medium of anisotropic permeability would include those of Hene [31], Vasseur and Degan [32] and Bachok et al. [33]. Lately, Sanya et al.[34] adopted Brinkman-Darcy flow model to investigate liquid film condensation past a vertical surface embedded with large anisotropic permeability.

The motivation of our present communication is thusly pivoted on the ongoing review of literature, the necessity to extend Bachok et al. [33] by incorporating the combined effect of the surface mass flux (suction or injection), Archemedes (buoyancy) and Lorentz forces on laminar viscous and incompressible flow of an electrically and thermally conducting fluid past a stationary vertical permeable flat plate encompassed in a porous medium with homogeneous anisotropic permeability.

Without a controversy, it has been shown by many a researcher such as Ridha [35]; that dual solutions not only exist in the case for which the mixed buoyancy parameter is negative but also when it is positive in values. Our research is centered on cooling problems in engineering and industrial processes, where the positive mixed convection parameter is of relevance (see Makinde [36]; Olanrewaju and Hayat, [37]) consequent upon the choice of the latter. As pointed out by Ridha [35], dual solutions might be merely academic exercise due to lack of bearings to real life situations in sequel to which we settle for the latter in the present study.

2 Problem Formulation

We consider steady two-dimensional, laminar boundary layer flow of an incompressible and electrically conducting fluid past a permeable vertical plate of infinite extent awash with a uniform transverse magnetic field B_0, in which the ambient fluid is streaming with velocity $u_e(\bar{x})$. It is assumed that the plate with prescribed wall temperature $\bar{T}_w(\bar{x})$ is embedded in a porous medium of homogeneous anisotropic permeability, the fluid is thermally conducting, in addition, both the fluid and the solid porous matrix are in thermal equilibrium. The induced magnetic field is negligible in comparison with the applied magnetic field in consequence the effects of Hall currents and the iron-slips are inconsequential. All physical compositions of the model are as depicted in Figure 1.

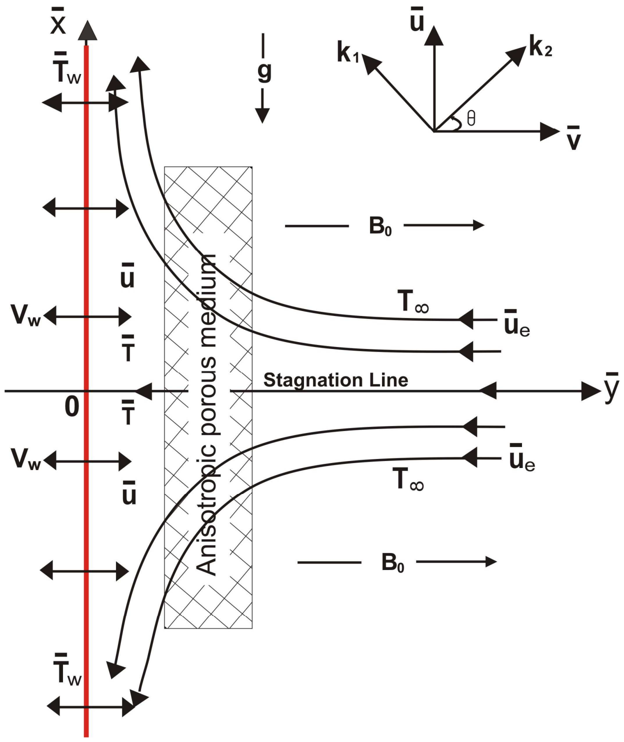

Figure 1: Physical model and coordinate system

The $\bar{x}$ and $\bar{y}$ axes are respectively along and perpendicular to the plate, velocity components along these axes are $\bar{u}$ and $\bar{v}$ while K_1, K_2 denote the permeability components along the two principal axes of the porous medium wherein θ is the orientation angle between $\bar{y}$ and K_2 directions. V_w is the normal velocity component at the wall of the stationary plate such that $V_w < o$ indicates fluid extraction (or suction), $V_w > 0$ indicates fluid blowing (or injection) and $V_w = 0$ characterizes that the plate is impermeable. On the basis of Brinkman-extended Darcy (BED) model (Nield and Bejan, [38], Ingham and Pop, [39]), Boussinesq approximation and all the afore-stated assumptions, the governing equations for mass conservation, momentum balance and energy balances are posited as (Bera and Khalili, [40]; Mobedi et al., [41]; Bachok et al., [33]):

$$\nabla \bullet \mathbf{v} = 0 \tag{1}$$

$$\mathbf{v} = -\frac{\overline{\overline{K}}}{\mu}\left(\nabla \bar{p} - \tilde{\mu}\nabla^2 \mathbf{v} + \rho \mathbf{g}\beta(\bar{T} - T_\infty) + \sigma(\mathbf{v} \wedge \mathbf{B}_0\right) \tag{2}$$

$$\nabla \bullet (\mathbf{v}\bar{T}) = \alpha \nabla^2 \bar{T} + \frac{\tilde{\mu}}{\rho c_p}\left(\frac{\partial \bar{u}}{\partial y}\right)^2 + \frac{Q_0}{\rho c_p}\left(\bar{T} - T_\infty\right) \tag{3}$$

where $\rho,\ \mu,\ \tilde{\mu},\ \alpha,\ \sigma, \bar{T},\ T_\infty,\ Q_0,\ \bar{p},\ c_p$ are respectively the fluid density, dynamic viscosity, effective or apparent dynamic viscosity, thermal diffusivity, electrical conductivity, fluid temperature, ambient fluid temperature, volumetric heat generation or absorption, fluid pressure and specific heat of the fluid at constant pressure. Here, the fluid velocity **v**, gravitational acceleration **g**, and uniform transverse magnetic field $\mathbf{B}_0$, are prescribed as

$$\mathbf{v} = (\bar{u}, \bar{v}), \quad \mathbf{g} = (-g, 0), \quad \mathbf{B}_0 = (0, B_0) \tag{4}$$

while the second-order symmetrical permeability tensor is defined as Degan and Vasseur [28]:

$$\overline{\overline{K}} = \begin{pmatrix} K_1\cos^2\theta + K_2\sin^2\theta & (K_1 - K_2)\sin\theta\cos\theta \\ (K_1 - K_2)\sin\theta\cos\theta & K_2\cos^2\theta + K_1\sin^2\theta \end{pmatrix} \tag{5}$$

Invoking (4) and (5) into eqs. (1)$-$(3), we write

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0, \tag{6}$$

$$\left.\begin{aligned} a\bar{u} - b\bar{v} &= \frac{\tilde{\mu}}{\mu}K_1\bar{\nabla}^2\bar{u} - \frac{K_1}{\mu}\frac{\partial \bar{p}}{\partial \bar{x}} + \frac{\rho g \beta K_1}{\mu}\left(\bar{T} - T_\infty\right) - \frac{K_1\sigma B_0^2\bar{u}}{\mu} \\ c\bar{v} &= \frac{\bar{\mu}}{\mu}K_1\bar{\nabla}^2\bar{v} - \frac{K_1}{\mu}\frac{\partial \bar{p}}{\partial \bar{y}} \end{aligned}\right\} \tag{7}$$

$$\bar{u}\frac{\partial \bar{T}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{T}}{\partial \bar{y}} = \alpha\bar{\nabla}^2\bar{T} + \frac{Q_o}{\rho C_p}\left(\bar{T} - T_\infty\right) + \frac{\tilde{\mu}}{\rho C_p}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2 \tag{8}$$

With the imposed boundary conditions (Bachok et al., [33]):

$$\left.\begin{aligned} &\bar{v} = \bar{V}_w, \quad \bar{u} = 0, \quad \bar{T} = \bar{T}_w(x) \quad at \quad \bar{y} = 0, \\ &\bar{u} \to \bar{u}_e(x), \quad \bar{T} \to T_\infty, \quad as \quad \bar{y} \to \infty \end{aligned}\right\} \tag{9}$$

Where

$$\left.\begin{aligned} a &= \cos^2\theta + K^*\sin^2\theta \\ b &= \left(K^* - 1\right)\sin 2\theta \\ c &= \sin^2\theta + K^*\cos^2\theta \end{aligned}\right\} \tag{10}$$

and $K^* = K_1 / K_2$ is the anisotropic permeability ratio.

By introducing the following non-dimensional variables:

$$\left.\begin{aligned} & x=\frac{\bar{x}}{\ell},\ \ y=\frac{\bar{y}}{\ell}\sqrt{Pe},\quad u=\frac{\bar{u}}{U},\ \ v=\frac{\bar{v}}{U}\sqrt{Pe}, \\ & T=\frac{\bar{T}-T_\infty}{\Delta\bar{T}},\quad u_e(x)=\frac{\bar{u}_e(\bar{x})}{U},\quad V_w=\frac{\bar{V}_w}{U}\sqrt{Pe}, \end{aligned}\right\} \tag{11}$$

into the eqs. $(6)-(8)$, wherein $Pe = U\ell/\alpha$ is the Peclet number, and in addition, ℓ, U and $\Delta\bar{T}$ are respectively the reference length, the reference velocity and the reference temperature difference, they reduce to the following form:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{12}$$

$$a\frac{\partial u}{\partial y}=\varepsilon Da\frac{\partial^3 u}{\partial y^3}+\lambda\frac{\partial T}{\partial y}-m\frac{\partial u}{\partial y} \tag{13}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\partial^2 T}{\partial y^2}+QT+\Pr Ec\left(\frac{\partial u}{\partial y}\right)^2 \tag{14}$$

Along with the dimensionless boundary conditions:

$$\left.\begin{aligned} & v=V_w,\qquad u=0,\qquad T=T_w(x)\qquad at\ \ y=0, \\ & u\to u_e(x),\qquad T\to 0\qquad as\quad y\to\infty, \end{aligned}\right\} \tag{15}$$

Where

$$\left.\begin{aligned} & T_w(x)=\frac{\bar{T}(\bar{x})-T_\infty}{\Delta\bar{T}},\ \ m=\frac{K_1\sigma B_o^2}{\mu},\ \ Q=\frac{\ell Q_o}{U\rho C_p},\quad Ra=\frac{gK_1\beta\Delta\bar{T}\ell}{\upsilon\alpha}, \\ & \Pr=\frac{\tilde{\mu}}{U\rho\ell},\ \ Ec=\frac{U^2}{C_p\Delta T},\ \ Da=\frac{K_1}{\ell^2},\ \ Pe=\frac{U\ell}{\alpha},\ \ \lambda=\frac{Ra}{Pe},\ \ \varepsilon=\frac{\tilde{\mu}}{\mu}Pe, \end{aligned}\right\} \tag{16}$$

are the dimensionless wall temperature, the magnetic parameter, the heat generation or absorption parameter, Rayleigh number, Prandtl number, Eckert number, Darcy-Brinkman number, Peclet number, mixed convection parameter and modified Peclet number respectively.

It is noteworthy here to mention some specific cases for which the temperature difference characterizing the thermal boundary conditions prescribed at the plate surface may be taken as $\Delta\bar{T}=\bar{T}_w-T_\infty$. If $\Delta\bar{T}>0$, the case for which the plate is heated or $\Delta\bar{T}<0$ when the plate is cooled or the steady equilibrium thermal case for which $\Delta\overline{T}=0$ in which $\Delta\overline{T}=\nu^2/(\ell^2 c_p)$ or $\mu U^2/k$, (ν is kinematic viscosity, k is thermal conductivity) may be employed as succor to overcome a breakdown of the transformation process Chamkha [42], Baletta [43], while it may thusly be replaced by putting $\Delta\overline{T}=q_w\ell/k$ when the plate is subjected to uniform heat flux q_w. In this present study, our interest is centered on the case of a heated and not a cooled plate.

The dimensionless velocity components can now be defined in terms of the Stokes stream function $\psi(x,y)$ as

$$u=\frac{\partial\psi}{\partial y},\; v=-\frac{\partial\psi}{\partial x} \tag{17}$$

such that eq. (12) is satisfied identically. One can now integrate eq. (13) partially with respect to y, after the substitution of (17) using the second row of boundary condition (15) to obtain

$$a\left(\frac{\partial\psi}{\partial y}-u_e(x)\right)=\varepsilon Da\frac{\partial^3\psi}{\partial y^3}+\lambda T-m\left(\frac{\partial\psi}{\partial y}-u_e(x)\right). \tag{18}$$

Invoking (17) into the dimensionless energy eq. (14) and the boundary conditions (15) yields

$$\frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y}=\frac{\partial^2\psi}{\partial y^2}+QT+\Pr Ec\left(\frac{\partial^2\psi}{\partial y^2}\right)^2 \tag{19}$$

$$\left.\begin{aligned}&-\frac{\partial\psi}{\partial x}=V_w \quad \frac{\partial\psi}{\partial y}=0, \quad T=T_w(x) \quad at \quad y=0\\ &\frac{\partial\psi}{\partial y}\to u_e(x), \quad T\to 0 \quad as \quad y\to\infty\end{aligned}\right\} \tag{20}$$

The engineering quantities of importance are the skin-friction coefficient and the Nusselt number. These quantities may be defined respectively in terms of the wall sheer stress and the surface heat flux as

$$C_f=\frac{\tau_w}{\rho U^2}, \quad Nu=\frac{\ell q_w}{k\Delta\overline{T}} \tag{21}$$

Where

$$\tau_w=\mu\left.\frac{\partial\overline{u}}{\partial\overline{y}}\right|_{\overline{y}=0}, \; q_w=-k\left.\frac{\partial\overline{T}}{\partial\overline{y}}\right|_{\overline{y}=0} \tag{22}$$

They are transformed by (11) to yield their respective nondimensional forms written as

$$\frac{\sqrt{Pe}}{\Pr} C_f = x \left.\frac{\partial u}{\partial y}\right|_{y=0}, \quad \frac{1}{\sqrt{Pe}} Nu = -x \left.\frac{\partial T}{\partial y}\right|_{y=0} \tag{23}$$

referred to as the reduced local skin-friction coefficient and reduced local Nusselt number respectively.

The system of nonlinear coupled partial differential equations (PDEs) $(18)-(19)$ along with the boundary conditions (20) can now be reduced to a set of nonlinear ordinary differential equations (ODEs) by means of the appropriate similarity transformations Ref. [33]:

$$\psi(x, y) = xf(y), \quad T(x, y) = x\theta(y), \tag{24}$$

such that the ambient dimensionless fluid velocity and the plate wall temperature are

$$u_e(x) = x,, \quad T_w(x) = x. \tag{25}$$

Application of similarity variables of eqs. (24) and then (25) in eqs. $(18)-(20)$ produce

$$f''' - A(f' - 1) + \Lambda\theta - M(f' - 1) = 0 \tag{26}$$

$$\theta'' + f\theta' - f'\theta + Q\theta + \Pr.Ec\ f'' = 0, \tag{27}$$

$$f(0) = f_{o,} \quad f'(0) = 0, \theta(0) = 1, \quad f'(\infty) = 1, \theta(\infty) = 0 \tag{28}$$

Where

$$f_0 = -V_w, A = \frac{a}{\varepsilon Da}, M = \frac{m}{\varepsilon Da}, \Lambda = \frac{\lambda}{\varepsilon Da} \tag{29}$$

are wall mass flux parameter ($f_0 > o$: indicates suction, $f_0 < 0$: indicates injection, $f_0 = 0$: adduces impermeable plate), anisotropic parameter, modified magnetic parameter and modified mixed convection parameter respectively, additionally, for well-posed model, $\varepsilon\text{Da} \neq 0$.

It is worthy to convert the parameters in (23) to those that will be independent of the dimensionless position x in the flow regime as to account for similarity solutions. Of course, this can be achieved by assuming that the dynamic viscosity (μ) and the thermal conductivity (k) are each proportional to x^{-1} (see Makinde, [36]; Olanrewaju and Hayat, [37]. Consequently, (23) then gives

$$\left(C_f, Nu\right) \propto \left(f''(0), -\theta'(0)\right) \tag{30}$$

3 Solution Methods

3.1 Some Important Cases

3.1.1 Negligible Magnetic Field

The situation where the effect of magnetic field is inconsequential is frequently associated with purely hydrodynamic boundary layer flow. In which case, $M = 0$ in eq. (26) lends itself to the nonlinear momentum equation presented and discussed numerically in Ref. [33] for some special cases of the present study.

3.1.2 Pure Forced Convection $(\Lambda = 0)$

In the case of purely forced convection, when the free or natural convection effect is inconsequential, then $\Lambda = 0$ so that eq. (26) reduces to uncoupled ODE

$$f''' - R(f' - 1) = 0, \tag{31}$$

Where $R = M + A$ is the combined effect of magnetic field and anisotropic permeability. **Isotropic mixed convection** $(A = 1)$

If the porous medium is isotropic $(A = 1)$, then eq. (26) transforms to

$$f''' - (1 + M) f' + \Lambda\theta = 0 \tag{32}$$

which has been numerically solved and discussed for particular cases in the foregoing section.

3.1.3 Negligible Inertial Effect $(\varepsilon Da = 0)$

Under this assumption, $(\varepsilon Da = 0)$ and eq. (26) is replaced by

$$f' + \lambda^{*}\theta - 1 = 0 \tag{33}$$

wherein $\lambda^{*} = -\lambda / (a + m)$ and $(a + m)$ is the combined constant due to the influence of anisotropy and magnetic field. This particular case without wall normal mass velocity had been derived and mentioned in Merkin [44] and Ref. [33].

3.1.4 Negligible Dissipative and Generative Heating Effects $(Ec = 0, Q = 0)$

In the absence of viscous heat dissipation and thermal heat generation/absorption, $(Ec = 0, Q = 0)$ the energy balance eq. (27) is customized as those of Ref. [33] for purely hydrodynamic boundary layer flow $(M = 0)$.

3.2 Numerical Simulations

Eqs (26) and (27) subject to the boundary conditions (28) were solved computationally using symbolic algebraic software Maple-17 Heck [45]. As the BVP is a two-point type with unspecified initial conditions $f''(0)$ and $\theta'(0)$, use were made of shoot Lib of the package in our codes. Its accuracy and robustness had been repeatedly confirmed in previous publications (Makinde and Olanrewaju, [46], Olanrewaju and Adeniyan, [47]).

4 Results and Discussion

Table1 has been prepared to illustrate the effects of Prandtl number, Eckert number, modified mixed convective parameter, anisotropic parameter, modified magnetic effect parameter, suction/injection parameter on Skin friction coefficient and Nusselt number. From the table, we observed that as the anisotropic parameter increases, the skin friction coefficient increases while the Nusselt number decreases. It is also observed that modified magnetic effect parameter and modified mixed convective parameter follow the same trend. The skin friction coefficient and Nusselt number also increase as Prandtl number increases. Heat generation/absorption and Eckert number follow the same pattern. Moreover, it is observed that the suction parameter (f_o > 0) tends to decrease the local heat transfer rate, while the opposite trend is observed for the injection parameter (f_o < 0). This is because as the mass injection is applied at the surface, the momentum or mass transport is reduced near the surface thereby causing a reduction in the Nusselt number.

4.1 Effects of Parameter Variation on Velocity Profiles

The numerical results for the velocity profiles are displayed in Figures 2-8 and follow the pattern of characteristic of natural convection boundary layer flow over a vertical plate. In Figure 2, the velocity increases from a zero value at the plate to a peak value within the boundary layer and maintain a specified free stream velocity value of $f'(\infty) = 1$ as anisotropic parameter increases and satisfy the boundary conditions. M, Pr, Q, Λ, Ec and f_o > 0 for suction, f_o < 0 for injection also have similar trend on velocity profiles as shown in Figures 3-8. It is noteworthy that all the velocity profiles increase with strengthening the magnetic parameter M or the anisotropy parameter A, unlike the impeding influence that frequently features for isotropic porous medium. Both parameters as observed, demonstrate accelerating influence on the fluid flow. But the reverse is the case for their influences on temperature profiles.

4.2 Effects of Parameter Variation on Temperature Profiles

The numerical results for the temperature profiles are shown in Figures 9-15. It is seen from Figure 9 that the fluid temperature attains its maximum value at the plate surface and decreases significantly to the free stream zero value away from the plate satisfying the boundary layer conditions as anisotropic parameter increases and the temperature profiles moves closer to the wall. As observed in Figures 10-12, the thermal boundary layer thickness increases with an increase in the intensity of the magnetic parameter M, surface mass flux f_o as well as the mixed convection parameter Λ. The same responses unveil for Pr, Q and Ec as depicted in Figures 13-15. The responses demonstrated by the thermal profiles consequent upon increment in M as well as A clearly reveal the opposite features to the usual occurrence in the case of isotropic porous medium. In Figure 14 the dimensionless temperature reduces as the heat absorption parameter increases while the temperature profiles advance with increasing heat generation parameter.

4.3 Effects of Parameter Variation on Flow Characteristics

Figure 16 displays the variation of skin friction coefficient with M in the presence of various values of A, as it can be seen the value of skin friction coefficient increases as M increases with raise in A. Modified mixed convective parameter also has a similar trend as shown in figure 17. Also, Figure 18 depicts the variation of Nusselt number with M for selected values of A, and the numerical value of Nusselt number reduces as M increases with a raise in A. Modified mixed convective parameter also has a similar trend on variation of Nusselt number with M as shown in Figure 19.

4.4 Tables and Plots

Table 1: Computations showing values of $f''(0), and\ -\theta'(0)$ for various values of the basic flow parameters

A	M	Pr	Λ	Q	Ec	f_0	f''(0)	-θ'(0)
0	1	1	1	1	1	0.2	1.661767	0.384933209
1	1	1	1	1	1	0.2	1.9454821	0.354581293
2	1	1	1	1	1	0.2	2.1909401	0.332708388
3	1	1	1	1	1	0.2	2.4106759	0.315771129
1	1	1	1	1	1	0.2	1.9454821	0.354581293
1	2	1	1	1	1	0.2	2.1909401	0.332708388
1	3	1	1	1	1	0.2	2.4106759	0.315771129
1	1	0.72	1	1	1	0.2	1.921923	0.126159774

1	1	1	1	1	1	0.2	1.9454821	0.354581293
1	1	3	1	1	1	0.2	2.1043803	2.044795811
1	1	1	1	1	1	0.2	1.9454821	0.354581293
1	1	1	2	1	1	0.2	2.4149282	0.303916776
1	1	1	3	1	1	0.2	2.8349876	0.258248189
1	1	1	1	0	1	0.2	1.8599512	-0.30813804
1	1	1	1	1	1	0.2	1.9454821	0.354581293
1	1	1	1	2	1	0.2	2.1038735	1.429523983
1	1	1	1	1	1	0.2	1.9454821	0.354581293
1	1	1	1	1	2	0.2	2.0269437	1.187879019
1	1	1	1	1	3	0.2	2.1043803	2.044795811
1	1	1	1	1	1	-0.2	1.9901061	0.539657939
1	1	1	1	1	1	-0.1	1.9790916	0.498647404
1	1	1	1	1	1	0	1.9679558	0.4540968
1	1	1	1	1	1	0.1	1.9567393	0.406051851
1	1	1	1	1	1	0.2	1.9454821	0.354581293

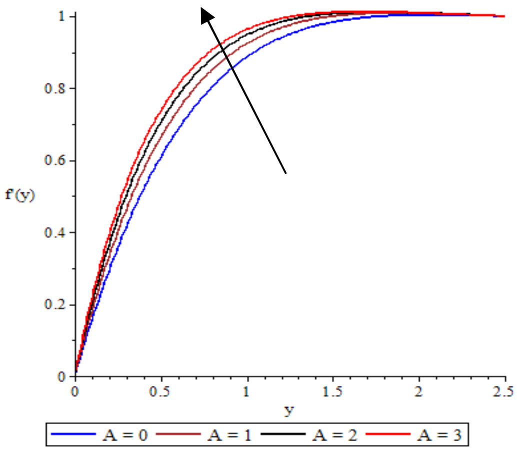

Figure 2: Velocity profiles for various values of A when M= Λ =Q =Pr=Ec=1 and $f_o = 0.2$

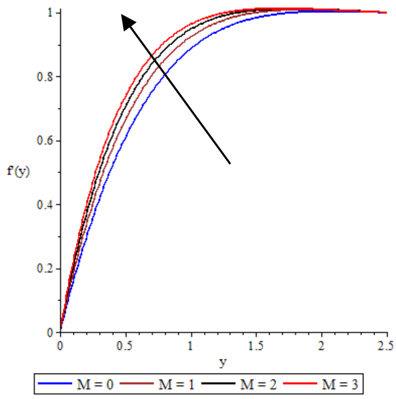

Figure 3: Variation of velocity profiles for various values of M when A=Pr= Λ =Q=Ec=1 and f_o =0.2

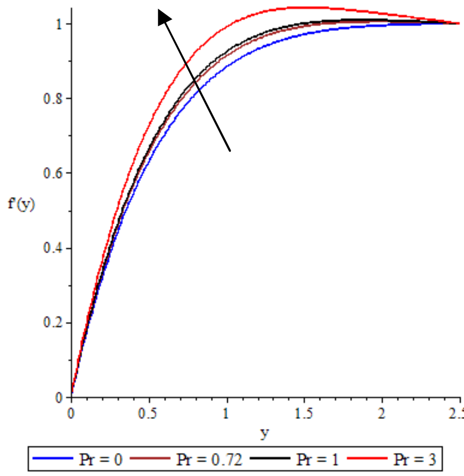

Figure 4: Variation of velocity profile for various values of Pr when A=M= Λ =Q=Ec=1 and f_o =0.2

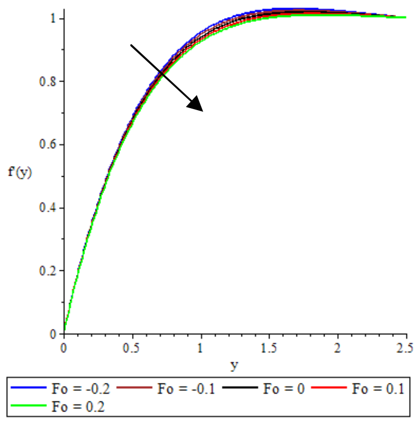

Figure 5: Variation of velocity profiles for various values of f_0 when A=M= Pr= Λ =Q=Ec=1

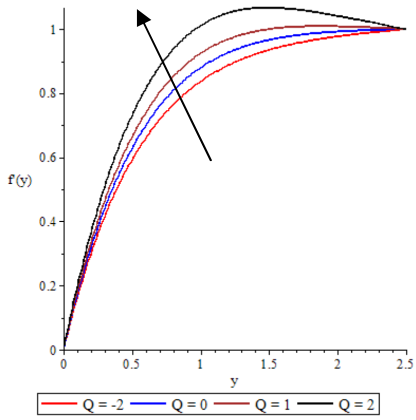

Figure 6: Variation of velocity profiles for various values of Q when A=M=Pr= Λ =Ec=1 and f_o=0.2

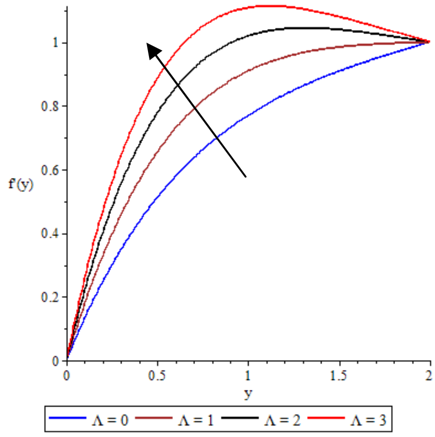

Figure 7: Variation of velocity profile for various values of Λ when A=M=Pr=Q=Ec=1 and f_o=0.2

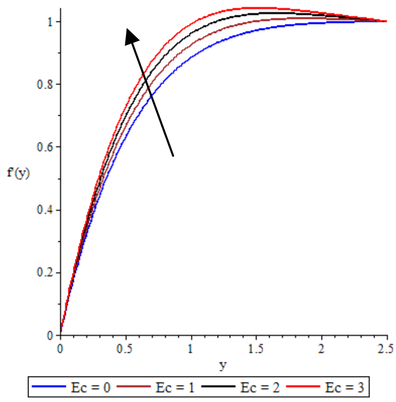

Figure 8: Variation of velocity profile for various values of Ec when A=M= Λ =Pr=Q=1 and f_o=0.2

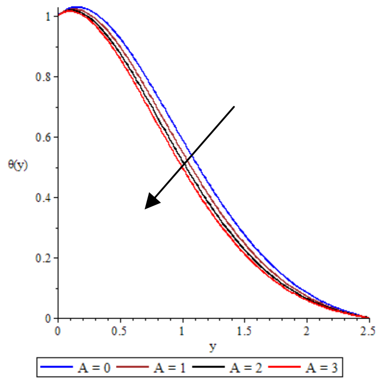

Figure 9: Variation of Temperature profiles for various values of A when M= Λ =Q =Pr=Ec=1 and f_o =0.2

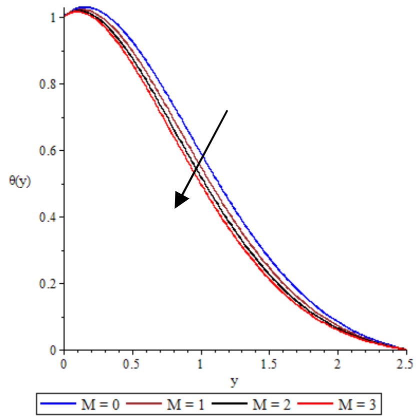

Figure 10: Variation of Temperature profiles for various values of M when A= Λ =Q =Pr=Ec=1 and f_o =0.2

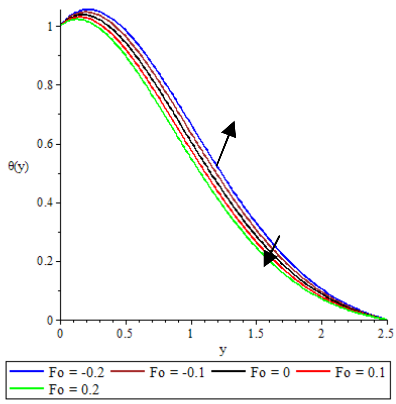

Figure 11: Variation of Temperature profiles for various values of Fo(= f_0) when A= M= Λ =Q =Pr=Ec=1

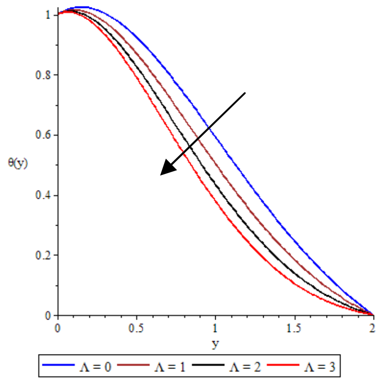

Figure 12: Variation of Temperature profiles for various values of Λ when M= Λ =Q =Pr=Ec=1 and f_o

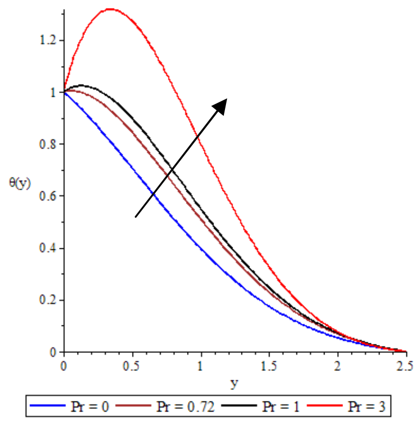

Figure 13: Variation of Temperature profiles for various values of **Pr** when M= Λ =Q =Pr=Ec=1 and f_o =0.2

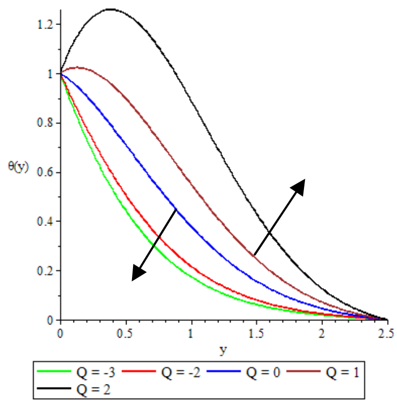

Figure 14: Variation of Temperature profiles for various values of Q when A= M= Λ =Pr=Ec=1 and f_o =0.2

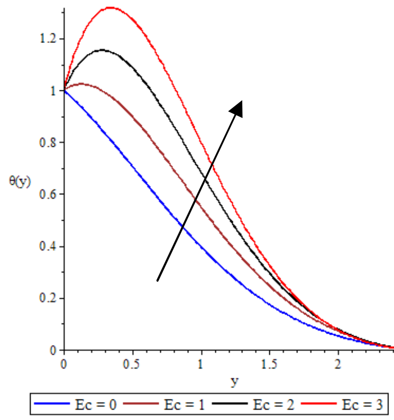

Figure 15: Variation of Temperature profiles for various values of Ec when A=M= Λ =Q =Pr=1 and f_o =0.2

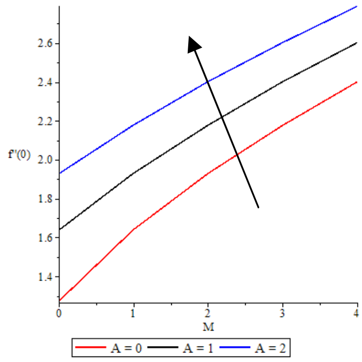

Figure 16: Variation of skin friction coefficient with M in the present of various values of A when Λ =Q =Pr=Ec=1 and f_o =0.2

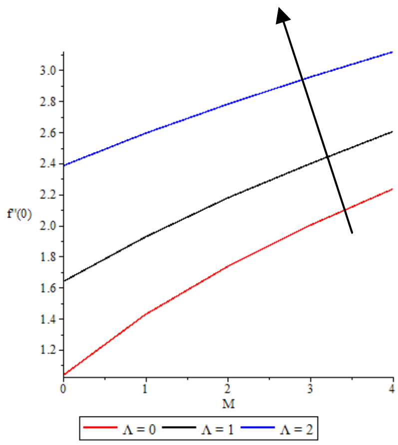

Figure 17: Variation of skin friction coefficient with M in the present of various values of Λ when Λ =Q =Pr=Ec=1 and f_o =0.2

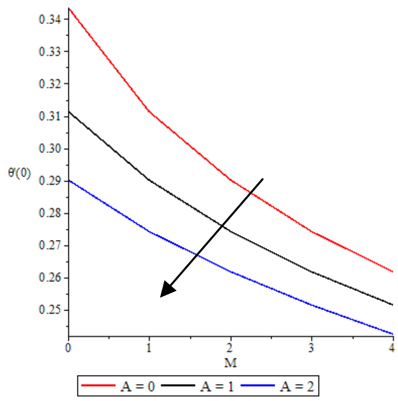

Figure 18: Variation of Nusselt number with M in the present of various values of A when Λ =Q =Pr=Ec=1 and f_o =0.2

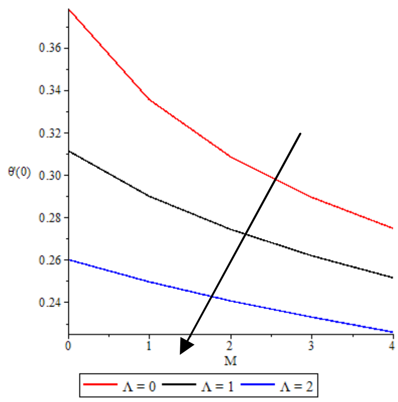

Figure 19: Variation of Nusselt number with M in the present of various values of Λ of when A=Q =Pr=Ec=1 and f_o =0.2

5 Conclusions

The effects of thermal dissipation, heat generation or absorption on MHD mixed convection boundary layer flow of an electrically conducting fluid over a permeable vertical flat plate embedded in an anisotropic porous medium with non-ignorable influence of Lorentz force and suction/injection have been carried out for various heat and momentum transfer characteristics**.** Our findings are summarized as follows:

- The anisotropic parameter, modified magnetic parameter and modified mixed convective parameter can increase the skin friction coefficient and decrease the Nusselt number. Both the skin friction coefficient and Nusselt number can be increased due to parameter strengthening of the Prandtl number, heat generation and Eckert number while the suction parameter

(f_o > 0) tends to decrease the local heat transfer rate only, with reverse case for the injection parameter (f_o < 0).

- Both the magnetic and anisotropic parameters accelerate the flow velocity which may consequently erode the boundary-layer conceptual control giving rise to an early separation and eventual appearance of turbulence. On the other hand, both parameters heat up the tender boundary layer due to parameter intensification.
- The presence of porous medium anisotropy leads to a violation of the usual motion impeding influence of the magnetic field and porous medium permeability parameters on the flow field.
- The usual characteristics of the temperature profiles in the boundary-layer regime still feature as temperature rises significantly with increase in M, f_o, Λ, Pr, Q and Ec while thermal boundary layer thickens across the wall plate.
- Improving suction strength decreases both the momentum and boundary-layer thicknesses and in consequence delays the boundary layer separation while injection acts the other way round.

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