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Coefficient Estimates for Bi-Mocanu-Convex Functions of Complex Order

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Abstract

In this paper, we propose to investigate the coefficient estimates for certain subclasses bi-Mocanu-convex functions in the open unit disk $\mathbb{U}$. *The results presented in this paper would generalize and improve some recent works.*

Keywords: *Analytic functions, Univalent functions, Bi-univalent functions, Bi-starlike and Bi-convex functions, Bi-Mocanu-convex functions.*

1 Introduction

Let $\mathcal{A}$ denote the class of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1}$$

which are analytic in the open unit disc $\mathbb{U} = \{z : z \in \mathbb{C} \text{ and } |z| < 1\}$. Further, by $\mathcal{S}$ we shall denote the class of all functions in $\mathcal{A}$ which are univalent in $\mathbb{U}$.

For two functions f and g, analytic in $\mathbb{U}$, we say that the function $f(z)$ is subordinate to $g(z)$ in $\mathbb{U}$, and write

$$f(z) \prec g(z) \qquad (z \in \mathbb{U})$$

if there exists a Schwarz function $w(z)$, analytic in $\mathbb{U}$, with

$$w(0) = 0 \quad \text{and} \quad |w(z)| < 1 \qquad (z \in \mathbb{U})$$

such that

$$f(z) = g(w(z)) \qquad (z \in \mathbb{U}).$$

In particular, if the function g is univalent in $\mathbb{U}$, the above subordination is equivalent to

$$f(0) = g(0) \quad \text{and} \quad f(\mathbb{U}) \subset g(\mathbb{U}).$$

Some of the important and well-investigated subclasses of the univalent function class $\mathcal{S}$ includes (for example) the class $\mathcal{S}^*(\beta)$ of starlike functions of order β $(0 \leqq \beta < 1)$ in $\mathbb{U}$ and the class $\mathcal{SS}^*(\alpha)$ of strongly starlike functions of order α $(0 < \alpha \leqq 1)$ in $\mathbb{U}$. For every $f \in \mathcal{S}$ there exists an inverse function f^{-1} which is defined in some neighborhood of the origin. According to the Koebe one-quarter theorem f^{-1} is defined in some disk containing the disk $|w| < 1/4$. In some cases this inverse function can be extended to whole $\mathbb{U}$. Clearly, f^{-1} is also univalent.

A function $f \in \mathcal{A}$ is said to be bi-univalent in $\mathbb{U}$ if both f and f^{-1} are univalent in $\mathbb{U}$. We denote by Σ the class of all bi-univalent functions in $\mathbb{U}$. We observe that for $f \in \Sigma$ of the form (1) the inverse function f^{-1} has the Taylor-Maclaurin series expansion

$$f^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2 a_3 + a_4)w^4 + \dots . \qquad (2)$$

Analogous to the function class $\mathcal{S}$, the bi-univalent function class Σ includes (for example) the class $\mathcal{S}^*_\Sigma(\beta)$ of bi-starlike functions of order β $(0 \leqq \beta < 1)$ in $\mathbb{U}$ and the class $\mathcal{SS}^*_\Sigma(\alpha)$ of bi-strongly starlike functions of order α $(0 < \alpha \leqq 1)$ in $\mathbb{U}$. For a brief history, interesting examples and other fascinating subclasses of the bi-univalent function class Σ see [1, 6, 12] and the related references therein.

In fact, the study of the coefficient problems involving bi-univalent functions was revived recently by Srivastava et al. [12]. Various subclasses of the bi-univalent function class Σ were introduced and non-sharp estimates on the first two Taylor-Maclaurin coefficients $|a_2|$ and $|a_3|$ of functions in these subclasses were found in several recent investigations (see, for example, [1, 2], [4] - [9] and [11] - [13]). The aforecited all these papers on the subject were motivated by the pioneering work of Srivastava et al. [12]. But the coefficient problem for each of the following Taylor-Maclaurin coefficients $|a_n|$ $(n \in \mathbb{N} \setminus \{1, 2\};$ $\mathbb{N} := \{1, 2, 3, \cdots\})$ is still an open problem.

Motivated by the aforecited works (especially [4, 13]), we introduce the following subclass $\mathcal{M}_{\Sigma}^{\varphi,\psi}(\gamma;\lambda)$ of the analytic function class $\mathcal{A}$.

Definition 1.1 *Let $f \in \mathcal{A}$ and the functions $\varphi, \psi : \mathbb{U} \to \mathbb{C}$ be convex univalent functions such that*

$$\min\{\Re(\varphi(z)), \Re(\psi(z))\} > 0 \qquad (z \in \mathbb{U}) \quad \text{and} \quad \varphi(0) = \psi(0) = 1.$$

Assume that $\gamma \in \mathbb{C}\backslash\{0\}$, $0 \leqq \lambda \leqq 1$. We say that $f \in \Sigma$ is in the class $\mathcal{M}_{\Sigma}^{\varphi,\psi}(\gamma;\lambda)$ if the following conditions are satisfied:

$$1 + \frac{1}{\gamma}\left((1-\lambda)\frac{zf'(z)}{f(z)} + \lambda\left(1 + \frac{zf''(z)}{f'(z)}\right) - 1\right) \in \varphi(\mathbb{U}) \;\; \text{for all} \;\; z \in \mathbb{U} \qquad (3)$$

and for $g = f^{-1}$ we have

$$1 + \frac{1}{\gamma}\left((1-\lambda)\frac{wg'(w)}{g(w)} + \lambda\left(1 + \frac{wg''(w)}{g'(w)}\right) - 1\right) \in \psi(\mathbb{U}) \;\; \text{for all} \;\; w \in \mathbb{U}. \qquad (4)$$

We note that, for the different choices of the functions φ and ψ, we get interesting known or new subclasses of the analytic function class Σ. For example, if we set

$$\varphi(z) = \left(\frac{1+z}{1-z}\right)^{\alpha} \qquad \text{and} \qquad \psi(z) = \left(\frac{1-z}{1+z}\right)^{\alpha} \qquad (0 < \alpha \leqq 1;\; z \in \mathbb{U}),$$

then the class $\mathcal{M}_{\Sigma}^{\varphi,\psi}(\gamma;\lambda)$ becomes the class $\mathcal{SS}_{\Sigma}^{*}(\alpha,\gamma;\lambda)$ of bi-strongly Mocanu-convex functions of complex order γ ($\gamma \in \mathbb{C}\backslash\{0\}$). Also, $f \in \mathcal{SS}_{\Sigma}^{*}(\alpha,\gamma;\lambda)$ if the following conditions are satisfied :

$$f \in \Sigma, \qquad \left|\arg\left(1 + \frac{1}{\gamma}\left((1-\lambda)\frac{zf'(z)}{f(z)} + \lambda\left(1 + \frac{zf''(z)}{f'(z)}\right) - 1\right)\right)\right| < \frac{\alpha\pi}{2}$$

$$(0 < \alpha \leqq 1;\; 0 \leqq \lambda \leqq 1; \gamma \in \mathbb{C}\backslash\{0\};\; z \in \mathbb{U})$$

and for $g = f^{-1}$ we have

$$\left|\arg\left(1 + \frac{1}{\gamma}\left((1-\lambda)\frac{wg'(w)}{g(w)} + \lambda\left(1 + \frac{wg''(w)}{g'(w)}\right) - 1\right)\right)\right| < \frac{\alpha\pi}{2}$$

$$(0 < \alpha \leqq 1;\; 0 \leqq \lambda \leqq 1; \gamma \in \mathbb{C}\backslash\{0\};\; w \in \mathbb{U}).$$

Similarly, if we let

$$\varphi(z) = \frac{1 + (1-2\beta)z}{1-z} \qquad \text{and} \qquad \psi(z) = \frac{1 - (1-2\beta)z}{1+z} \qquad (0 \leqq \beta < 1;\; z \in \mathbb{U}),$$

in the class $\mathcal{M}_{\Sigma}^{\varphi,\psi}(\gamma;\lambda)$ then we get $\mathcal{M}_{\Sigma}(\beta,\gamma;\lambda)$ (which are now referred to as bi-Mocanu-convex functions of complex order γ ($\gamma \in \mathbb{C}\backslash\{0\}$)). Further, we say that $f \in \mathcal{M}_{\Sigma}(\beta,\gamma;\lambda)$ if the following conditions are satisfied :

$$f \in \Sigma, \qquad \Re\left(1+\frac{1}{\gamma}\left((1-\lambda)\frac{zf'(z)}{f(z)}+\lambda\left(1+\frac{zf''(z)}{f'(z)}\right)-1\right)\right) > \beta$$

$$(0 \leqq \beta < 1;\ 0 \leqq \lambda \leqq 1; \gamma \in \mathbb{C}\backslash\{0\};\ z \in \mathbb{U})$$

and

$$\Re\left(1+\frac{1}{\gamma}\left((1-\lambda)\frac{wg'(w)}{g(w)}+\lambda\left(1+\frac{wg''(w)}{g'(w)}\right)-1\right)\right) > \beta$$

$$(0 \leqq \beta < 1;\ 0 \leqq \lambda \leqq 1; \gamma \in \mathbb{C}\backslash\{0\};\ w \in \mathbb{U}),$$

where g is the extension of f^{-1} to $\mathbb{U}$.

In addition, we observe that,

$\mathcal{M}_{\Sigma}^{\varphi,\psi}(1;0) =: \mathcal{B}_{\Sigma}^{\varphi,\psi}$, (see Bulut [4]),
and

$\mathcal{SS}_{\Sigma}^{*}(\alpha,1;\lambda) =: \mathcal{SS}_{\Sigma}^{*}(\alpha;\lambda)$ and $\mathcal{M}_{\Sigma}(\beta,1;\lambda) =: \mathcal{M}_{\Sigma}(\beta;\lambda)$, (see Li and Wang [8]).

In order to derive our main result, we have to recall here the following lemma.

Lemma 1.2 *[10] Let the function $\varphi(z)$ given by*

$$\varphi(z) = \sum_{n=1}^{\infty} \varphi_n z^n \qquad (z \in \mathbb{U})$$

be convex univalent in $\mathbb{U}$. Suppose also that the function $h(z)$ given by

$$h(z) = \sum_{n=1}^{\infty} h_n z^n \qquad (z \in \mathbb{U})$$

is holomorphic in $\mathbb{U}$. If

$$h(z) \prec \varphi(z) \qquad (z \in \mathbb{U}),$$

then

$$|h_n| \leqq |\varphi_1| \qquad (n \in \mathbb{N}).$$

In our investigation of the estimates for the Taylor-Maclaurin coefficients $|a_2|$ and $|a_3|$ for functions in the above-defined general bi-univalent function class $\mathcal{M}_{\Sigma}^{\varphi,\psi}(\gamma;\lambda)$, which indeed provides a bridge between the classes of bi-convex functions in $\mathbb{U}$ and bi-starlike functions in $\mathbb{U}$. Several related classes are also considered, and connection to earlier known results are made.

2 Main Result

In this section we state and prove our general results involving the bi-univalent function class $\mathcal{M}_{\Sigma}^{\varphi,\psi}(\gamma;\lambda)$ given by Definition 1.1.

Theorem 2.1 *Let $f(z)$ be of the form (1). If $f \in \mathcal{M}_{\Sigma}^{\varphi,\psi}(\gamma;\lambda)$, then*

$$|a_2| \leqq \min\left\{\frac{|\gamma|}{1+\lambda}\sqrt{\frac{|\varphi'(0)|^2+|\psi'(0)|^2}{2}},\ \sqrt{\frac{|\gamma|[|\varphi'(0)|+|\psi'(0)|]}{2(1+\lambda)}}\right\} \tag{5}$$

and

$$|a_3| \leqq \min\left\{\frac{|\gamma|^2[|\varphi'(0)|^2+|\psi'(0)|^2]}{2(1+\lambda)^2}+\frac{|\gamma|[|\varphi'(0)|+|\psi'(0)|]}{4(1+2\lambda)},\right.$$
$$\left.\frac{|\gamma|[(3+5\lambda)|\varphi'(0)|+(1+3\lambda)|\psi'(0)|]}{4(1+2\lambda)(1+\lambda)}\right\}. \tag{6}$$

Proof: From Definition 1.1, we thus have

$$1+\frac{1}{\gamma}\left((1-\lambda)\frac{zf'(z)}{f(z)}+\lambda\left(1+\frac{zf''(z)}{f'(z)}\right)-1\right)\in\varphi(\mathbb{U})\quad\text{for all}\quad z\in\mathbb{U}$$

and for $g=f^{-1}$ we have

$$1+\frac{1}{\gamma}\left((1-\lambda)\frac{wg'(w)}{g(w)}+\lambda\left(1+\frac{wg''(w)}{g'(w)}\right)-1\right)\in\psi(\mathbb{U})\quad\text{for all}\quad w\in\mathbb{U}.$$

Setting

$$p(z)=1+\frac{1}{\gamma}\left((1-\lambda)\frac{zf'(z)}{f(z)}+\lambda\left(1+\frac{zf''(z)}{f'(z)}\right)-1\right) \tag{7}$$

and

$$q(w)=1+\frac{1}{\gamma}\left((1-\lambda)\frac{wg'(w)}{g(w)}+\lambda\left(1+\frac{wg''(w)}{g'(w)}\right)-1\right). \tag{8}$$

We deduce so that

$$p(0)=\varphi(0)=1,\qquad p(z)\in\varphi(\mathbb{U})\qquad(z\in\mathbb{U})$$

and

$$q(0) = \psi(0) = 1, \qquad q(w) \in \psi(\mathbb{U}) \qquad (w \in \mathbb{U}).$$

Therefore, from Definition 1.1, we have

$$p(z) \prec \varphi(z) \qquad (z \in \mathbb{U})$$

and

$$q(w) \prec \psi(z) \qquad (w \in \mathbb{U}).$$

According to Lemma 1.2, we obtain

$$|p_m| = \left|\frac{p^{(m)}(0)}{m!}\right| \leqq |\varphi'(0)| \qquad (m \in \mathbb{N})$$

and

$$|q_m| = \left|\frac{q^{(m)}(0)}{m!}\right| \leqq |\psi'(0)| \qquad (m \in \mathbb{N}).$$

On the other hand, we find from (7) and (8) that

$$(1-\lambda)\frac{zf'(z)}{f(z)} + \lambda\left(1 + \frac{zf''(z)}{f'(z)}\right) = 1 + \gamma\,(p(z) - 1) \qquad (z \in \mathbb{U})$$

and

$$(1-\lambda)\frac{wg'(w)}{g(w)} + \lambda\left(1 + \frac{wg''(w)}{g'(w)}\right) = 1 + \gamma\,(q(w) - 1) \qquad (w \in \mathbb{U}),$$

respectively.

Next, we suppose that

$$p(z) = 1 + p_1 z + p_2 z^2 + \ldots$$

and

$$q(w) = 1 + q_1 w + q_2 w^2 + \ldots .$$

Now, upon equating the coefficients of $(1-\lambda)zf'(z)/f(z)+\lambda\,(1 + zf''(z)/f'(z))$ with those of $1 + \gamma\,(p(z) - 1)$ and the coefficients of $(1 - \lambda)wg'(w)/g(w) + \lambda\,(1 + wg''(w)/g'(w))$ with those of $1 + \gamma\,(q(w) - 1)$, we get

$$\frac{1}{\gamma}(\lambda + 1)a_2 = p_1, \tag{9}$$

$$\frac{1}{\gamma}[(2 + 4\lambda)a_3 - (1 + 3\lambda)a_2^2] = p_2, \tag{10}$$

$$-\frac{1}{\gamma}(\lambda + 1)a_2 = q_1 \tag{11}$$

and

$$\frac{1}{\gamma}[(3+5\lambda)a_2^2-(2+4\lambda)a_3]=q_2. \tag{12}$$

From (9) and (11), we get

$$p_1=-q_1 \tag{13}$$

and

$$\frac{2(1+\lambda)^2}{\gamma^2}a_2^2=p_1^2+q_1^2. \tag{14}$$

From (10) and (12), we obtain

$$\frac{2(1+\lambda)}{\gamma}a_2^2=p_2+q_2. \tag{15}$$

Therefore, we find from (14) and (15) that

$$a_2^2=\frac{\gamma^2(p_1^2+q_1^2)}{2(1+\lambda)^2} \tag{16}$$

and

$$a_2^2=\frac{\gamma(p_2+q_2)}{2(1+\lambda)}. \tag{17}$$

From (16) and (17) we have

$$|a_2|^2 \leqq \frac{|\gamma|^2[|\varphi'(0)|^2+|\psi'(0)|^2]}{2(1+\lambda)^2}$$

and

$$|a_2|^2 \leqq \frac{|\gamma|[|\varphi'(0)|+|\psi'(0)|]}{2(1+\lambda)}$$

respectively. So we get the desired estimate on $|a_2|$ as asserted in (5).

Next, in order to find the bound on $|a_3|$, by subtracting (12) from (10), we get

$$\frac{1}{\gamma}(4+8\lambda)a_3-\frac{1}{\gamma}(4+8\lambda)a_2^2=p_2-q_2. \tag{18}$$

Upon substituting the values of a_2^2 from (16) and (17) into (18), we have

$$a_3=\frac{\gamma^2(p_1^2+q_1^2)}{2(1+\lambda)^2}+\frac{\gamma(p_2-q_2)}{4(1+2\lambda)}$$

and

$$a_3=\frac{\gamma[(3+5\lambda)p_2+(1+3\lambda)q_2]}{(4+8\lambda)(1+\lambda)}$$

respectively. We thus find that

$$|a_3| \leqq \frac{|\gamma|^2[|\varphi'(0)|^2 + |\psi'(0)|^2]}{2(1+\lambda)^2} + \frac{|\gamma|[|\varphi'(0)| + |\psi'(0)|]}{4(1+2\lambda)},$$

and

$$|a_3| \leqq \frac{|\gamma|[(3+5\lambda)|\varphi'(0)| + (1+3\lambda)|\psi'(0)|]}{4(1+2\lambda)(1+\lambda)}.$$

This completes the proof of Theorem 2.1.

Remark 2.2 *For $\gamma = 1$ and $\lambda = 0$ Theorem 2.1 becomes the results obtained in [4, Theorem 2.1].*

If we choose

$$\varphi(z) = \left(\frac{1+z}{1-z}\right)^\alpha \qquad \text{and} \qquad \psi(z) = \left(\frac{1-z}{1+z}\right)^\alpha \qquad (0 < \alpha \leqq 1,\ z \in \mathbb{U})$$

in Theorem 2.1, we have the following corollary.

Corollary 2.3 *Let $f(z)$ be of the form (1) and in the class $\mathcal{SS}^*_\Sigma(\alpha, \gamma; \lambda)$, $\gamma \in \mathbb{C}\backslash\{0\}$, $0 < \alpha \leqq 1$ and $0 \leqq \lambda \leqq 1$. Then*

$$|a_2| \leqq \sqrt{\frac{2\alpha|\gamma|}{1+\lambda}} \qquad \text{and} \qquad |a_3| \leqq \frac{2\alpha|\gamma|}{1+\lambda}.$$

Taking $\gamma = 1$ in Corollary 2.3, we get the following corollary for the class $\mathcal{SS}^*_\Sigma(\alpha, 1; \lambda) =: \mathcal{SS}^*_\Sigma(\alpha; \lambda)$ of bi-strongly Mocanu-convex functions.

Corollary 2.4 *Let $f(z)$ be of the form (1) and in the class $\mathcal{SS}^*_\Sigma(\alpha; \lambda)$, $0 < \alpha \leqq 1$ and $0 \leqq \lambda \leqq 1$. Then*

$$|a_2| \leqq \sqrt{\frac{2\alpha}{1+\lambda}} \qquad \text{and} \qquad |a_3| \leqq \frac{2\alpha}{1+\lambda}.$$

Remark 2.5 *Corollary 2.4 is an improvement of [8, Theorem 2.2]. Further, for $\lambda = 0$ (bi-strongly starlike function) Corollary 2.4, would obviously yields an improvement of [3, Theorem 2.1].*

If we set

$$\varphi(z) = \frac{1+(1-2\beta)z}{1-z} \qquad \text{and} \qquad \psi(z) = \frac{1-(1-2\beta)z}{1+z} \qquad (0 \leqq \beta < 1,\ z \in \mathbb{U})$$

in Theorem 2.1, we readily have the following corollary.

Corollary 2.6 *Let $f(z)$ be of the form* (1) *and in the class $\mathcal{M}_\Sigma(\beta,\gamma;\lambda)$, $0 \leqq \beta < 1$, $\gamma \in \mathbb{C}\backslash\{0\}$ and $0 \leqq \lambda \leqq 1$. Then*

$$|a_2| \leqq \sqrt{\frac{2|\gamma|(1-\beta)}{1+\lambda}} \qquad \text{and} \qquad |a_3| \leqq \frac{2|\gamma|(1-\beta)}{1+\lambda}.$$

Taking $\gamma = 1$ in Corollary 2.6, we get the following corollary for the class $\mathcal{M}_\Sigma(\beta, 1; \lambda) =: \mathcal{M}_\Sigma(\beta;\lambda)$ of bi-Mocanu-convex functions.

Corollary 2.7 *Let $f(z)$ be of the form* (1) *and in the class $\mathcal{M}_\Sigma(\beta;\lambda)$, $0 \leqq \beta < 1$ and $0 \leqq \lambda \leqq 1$. Then*

$$|a_2| \leqq \sqrt{\frac{2(1-\beta)}{1+\lambda}} \qquad \text{and} \qquad |a_3| \leqq \frac{2(1-\beta)}{1+\lambda}.$$

Remark 2.8 *Corollary 2.7 is an improvement of [8, Theorem 3.2]. Further, for $\lambda = 0$ (bi-starlike function) Corollary 2.7, would obviously yields an improvement of [3, Theorem 4.1]. Similarly, various other interesting corollaries and consequences of our main result can be derived by choosing different φ and ψ. The details involved may be left to the reader.*

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