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On $ij-\wedge_\delta^s$ Sets in Bitopological Spaces

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Abstract

In this present paper, by utilizing $ij-\delta$ semi open sets, we introduce $ij-\wedge_\delta^s$, $ij-\vee_\delta^s$, $g-ij-\wedge_\delta^s$, $g-ij-\vee_\delta^s$ sets and investigate some of their properties in bitopological spaces. Also, we present and study the notions of $ij-T^{\vee_\delta^s}$ space, $ij-\wedge_\delta^s$ continuous, $ij-\wedge_\delta^s$ irresolute, $ij-\wedge_\delta^s$ open and $ij-\wedge_\delta^s$ homeomorphism functions.

Keywords: *$ij-\delta$ semi open, $ij-\wedge_\delta^s$ sets, $ij-\vee_\delta^s$ sets, $g-ij-\wedge_\delta^s$ sets, $g-ij-\vee_\delta^s$ sets, $ij-T^{\vee_\delta^s}$ space, $ij-\wedge_\delta^s$ continuous, $ij-\wedge_\delta^s$ irresolute, $ij-\wedge_\delta^s$ open, $ij-\wedge_\delta^s$ homeomorphism.*

1 Introduction

The class of generalized $\wedge$ - sets studied by H. Maki in [17]. Caldas et al. ([3],[6]) introduced the concept $\wedge_\delta^s$ - sets (resp. $\vee_\delta^s$ - sets) in topological spaces, which is the intersection of δ - semiopen (resp. union of δ - semiclosed) sets. F. H. Khedr and H. S. Al-saadi[15] introduced and studied the concept of $ij-s\wedge$-semi-θ-Closed and pairwise θ-generalized $s\wedge$-set in bitopological spaces, which is an extension of the class of generalized $\wedge$-sets. The aim of this paper is to introduce the notions of $ij-\wedge_\delta^s$ and $g-ij-\wedge_\delta^s$ sets and study some of their fundamental properties. Also, we study the new notions of $ij-T^{\vee_\delta^s}$ space, $ij-\wedge_\delta^s$ continuous, $ij-\wedge_\delta^s$ irresolute, $ij-\wedge_\delta^s$ open and $ij-\wedge_\delta^s$ homeomorphism functions and its properties.

2 Preliminaries

Throughout the present paper, (X,τ_1,τ_2)(or briefly X) always mean a bitopological space. Also $i,j=1,2$ and $i\neq j$. Let A be a subset of (X,τ_1,τ_2). By $i-int(A)$ and $i-cl(A)$, we mean respectively the interior and the closure of A in the topological space (X,τ_i) for $i=1,2$. A subset A of X is called ij - regular open [13] if $A=i-int[j-cl(A)])$. A point x of X is called an $ij-\delta$-cluster point of A if $i-int(j-cl(U))\cap A\neq\phi$ for every τ_i - open set U containing x. The set of all $ij-\delta$-cluster points of A is called the $ij-\delta$-closure of A and is denoted by $ij-\delta cl(A)$.

Definition 2.1 *A subset A is said to be $ij-\delta$ closed if $ij-\delta cl(A)=A$. The complement of an $ij-\delta$ closed set is said to be $ij-\delta$ open. The set of all $ij-\delta$ open (resp. $ij-\delta$ closed) sets of X will be denoted by $ij-\delta O(X)$(resp. $ij-\delta C(X)$).*

3 $ij-\wedge_\delta^s$ Sets and Generalized $ij-\wedge_\delta^s$ Sets

Definition 3.1 *A subset A of a bitopological space $(X,\tau_1,\tau_2)$is called $ij-\delta$ semi open if there exists an $ij-\delta$ open set U such that $U\subseteq A\subseteq j-cl(U)$.*

Definition 3.2 *For a subset A of a bitopological space (X,τ_1,τ_2), we define $A^{\delta s\wedge_{ij}}$ and $A^{\delta s\vee_{ij}}$ as follows, $A^{\delta s\wedge_{ij}}=\bigcap\{U:A\subseteq U, U\in ij-\delta SO(X)\}$ and $A^{\delta s\vee_{ij}}=\bigcup\{U:U\subseteq A, U^C\in ij-\delta SO(X)\}$.*

Definition 3.3 *A subset A of a bitopological space $(X,\tau_1,\tau_2)$is called,*

(a) $ij-\wedge_\delta^s$ set if $A=A^{\delta s\wedge_{ij}}$.

(b) $ij-\vee_\delta^s$ set if $A=A^{\delta s\vee_{ij}}$.

The family of all $ij-\wedge_\delta^s$ sets (resp. $ij-\vee_\delta^s$) is denoted by $ij-\wedge_\delta^s(X,\tau_1,\tau_2)$ (resp. $ij-\vee_\delta^s(X,\tau_1,\tau_2)$).

Theorem 3.4 *Let A be a subset of a bitopological space (X,τ_1,τ_2). Then $A^{\delta s\wedge_{ij}}=(A^{\delta s\wedge_{ij}})^{\delta s\wedge_{ij}}$*

Proof: We have $(A^{\delta s\wedge_{ij}})^{\delta s\wedge_{ij}} = \bigcap\{U : U \in ij-\delta SO(X,\tau_1,\tau_2), A^{\delta s\wedge_{ij}} \subseteq U\} = \bigcap\{U : U \in ij-\delta SO(X,\tau_1,\tau_2), (\bigcap\{V : V \in ij-\delta SO(X,\tau_1,\tau_2), A \subseteq V\}) \subseteq U\} \subseteq \bigcap\{U : U \in ij-\delta SO(X,\tau_1,\tau_2), A \subseteq U\} = A^{\delta s\wedge_{ij}}$. This means $(A^{\delta s\wedge_{ij}})^{\delta s\wedge_{ij}} \subseteq A^{\delta s\wedge_{ij}}$. On the other hand, $A \subseteq A^{\delta s\wedge_{ij}}$ for each subset A. Then $A^{\delta s\wedge_{ij}} \subseteq (A^{\delta s\wedge_{ij}})^{\delta s\wedge_{ij}}$. Therefore $A^{\delta s\wedge_{ij}} = (A^{\delta s\wedge_{ij}})^{\delta s\wedge_{ij}}$.

Theorem 3.5 *For any subsets A and B of a bitopological space (X,τ_1,τ_2), the following are hold:*

(a) $A \subseteq A^{\delta s\wedge_{ij}}$.

(b) If $A \subseteq B$, then $A^{\delta s\wedge_{ij}} \subseteq B^{\delta s\wedge_{ij}}$.

(c) If $A \in ij-\delta SO(X,\tau_1,\tau_2)$, then $A = A^{\delta s\wedge_{ij}}$.

(d) $(A^C)^{\delta s\wedge_{ij}} = (A^{\delta s\wedge_{ij}})^C$.

(e) $A^{\delta s\vee_{ij}} \subseteq A$.

(f) If $A \in ij-\delta SC(X,\tau_1,\tau_2)$, then $A = A^{\delta s\vee_{ij}}$.

Proof: (a) Obviously. By the definition of $A^{\delta s\wedge_{ij}}$, $A \subseteq A^{\delta s\wedge_{ij}}$.

(b) Suppose that $x \notin B^{\delta s\wedge_{ij}}$. Then there exists a subset $U \in ij-\delta SO(X,\tau_1,\tau_2)$ such that $U \supseteq B$ with $x \notin U$. Since $B \supseteq A$, then $x \notin A^{\delta s\wedge_{ij}}$ and thus $A^{\delta s\wedge_{ij}} \subseteq B^{\delta s\wedge_{ij}}$.

(c) By the definition of $A^{\delta s\wedge_{ij}}$ and $A \in ij-\delta SO(X,\tau_1,\tau_2)$, we have $A^{\delta s\wedge_{ij}} \subseteq A$. By (a), we have $A = A^{\delta s\wedge_{ij}}$.

(d) By the definition, $(A^{\delta s\vee_{ij}})^C = \bigcap\{U^C : U^C \supseteq A^C, U^C \in ij-\delta SO(X,\tau_1,\tau_2) = (A^C)^{\delta s\wedge_{ij}}$.

(e) Obviously. Clear by the definition.

(f) If $A \in ij-\delta SC(X,\tau_1,\tau_2)$, then $A^C \in ij-\delta SO(X,\tau_1,\tau_2)$. By (d) and (e), we have $A^C = (A^C)^{\delta s\wedge_{ij}} = (A^{\delta s\wedge_{ij}})^C$. Therefore $A = A^{\delta s\vee_{ij}}$.

Theorem 3.6 *Let A and $\{A_\alpha, \alpha \in J\}$ be the subsets of a bitopological space (X,τ_1,τ_2). Then the following are valid:*

(a) $[\bigcup_{\alpha\in J} A_\alpha]^{\delta s\wedge_{ij}} = \bigcup_{\alpha\in J}(A_\alpha)^{\delta s\wedge_{ij}}$.

(b) $[\bigcap_{\alpha\in J} A_\alpha]^{\delta s\wedge_{ij}} \subseteq \bigcap_{\alpha\in J}(A_\alpha)^{\delta s\wedge_{ij}}$.

(c) $[\bigcup_{\alpha\in J} A_\alpha]^{\delta s\vee_{ij}} \supseteq \bigcup_{\alpha\in J}(A_\alpha)^{\delta s\vee_{ij}}$.

Proof: (a) Suppose there exists a point x such that $x \notin [\bigcup_{\alpha\in J} A_\alpha]^{\delta s\wedge_{ij}}$. Then there exists a subset $U \in ij-\delta SO(X,\tau_1,\tau_2)$, such that $\bigcup_{\alpha\in J} A_\alpha \subseteq U$ and $x \notin U$. Thus for each $\alpha \in J$, we have $x \notin (A_\alpha)^{\delta s\wedge_{ij}}$. This implies that $x \notin \bigcup_{\alpha\in J}(A_\alpha)^{\delta s\wedge_{ij}}$.

Conversely, Suppose there exists a point x such that $x \notin \bigcup_{\alpha\in J}(A_\alpha)^{\delta s\wedge_{ij}}$. Then by the definition, there exist subsets $U_\alpha \in ij-\delta SO(X,\tau_1,\tau_2)$, for each $\alpha \in J$ such that $x \notin U_\alpha$ and $A_\alpha \subseteq U_\alpha$. Let $U = \bigcup_{\alpha\in J} U_\alpha$. Then $x \notin \bigcup_{\alpha\in J} U_\alpha, \bigcup_{\alpha\in J} A_\alpha \subseteq U$ and $U \in ij-\delta SO(X,\tau_1,\tau_2)$. Thus $x \notin [\bigcup_{\alpha\in J} A_\alpha]^{\delta s\wedge_{ij}}$.

(b) Suppose there exists a point x such that $x \notin \bigcap_{\alpha\in J}(A_\alpha)^{\delta s\wedge_{ij}}$, then there exists $\alpha \in J$ such that $x \notin (A_\alpha)^{\delta s\wedge_{ij}}$. Hence there exists $U \in ij-\delta SO(X,\tau_1,\tau_2)$ such that $U \supseteq A_\alpha$ and $x \notin U$. Thus $x \notin [\bigcap_{\alpha\in J} A_\alpha]^{\delta s\wedge_{ij}}$.

(c) $[\bigcup_{\alpha\in J} A_\alpha]^{\delta s\vee_{ij}} = [[(\bigcup_{\alpha\in J} A_\alpha)^C]^{\delta s\wedge_{ij}}]^C = [[\bigcap_{\alpha\in J} A_\alpha^C]^{\delta s\wedge_{ij}}]^C$.
$\supseteq [\bigcap_{\alpha\in J}(A_\alpha^C)^{\delta s\wedge_{ij}}]^C = [\bigcap_{\alpha\in J}(A_\alpha^{\delta s\vee_{ij}})^C]^C$, by theorem 3.5(d). By (b), we have $[\bigcup_{\alpha\in J} A_\alpha]^{\delta s\vee_{ij}} \supseteq \bigcup_{\alpha\in J} A_\alpha^{\delta s\vee_{ij}}$.

Definition 3.7 *A subset A of a bitopological space $(X,\tau_1,\tau_2)$is called,*

(a) generalized $ij-\wedge_\delta^s$ set ($g-ij-\wedge_\delta^s$ set) if $A^{\delta s\wedge_{ij}} \subseteq U$ whenever $A \subseteq U$ and $U \in ji-\delta SC(X,\tau_1,\tau_2)$.

(b) generalized $ij-\vee_\delta^s$ set ($g-ij-\vee_\delta^s$ set) if A^C is a $g\wedge_\delta^s$ - set.

The family of all $g-ij-\wedge_\delta^s$ sets and $g-ij-\vee_\delta^s$ sets are denoted by $ij-D^{\wedge_\delta^s}$ and $ij-D^{\vee_\delta^s}$.

Theorem 3.8 *(a) The ϕ and X are $ij-\wedge_\delta^s$ sets and $ij-\vee_\delta^s$ sets.*

(b) Every union of $ij-\wedge_\delta^s$ sets (resp. $ij-\vee_\delta^s$) is a $ij-\wedge_\delta^s$ set (resp. $ij-\vee_\delta^s$).

(c) Every intersection of $ij-\wedge_\delta^s$ sets (resp. $ij-\vee_\delta^s$ sets) is a $ij-\wedge_\delta^s$ set ($ij-\vee_\delta^s$ set).

(d) A subset A of $(X,\tau_1,\tau_2)$is a $ij-\wedge_\delta^s$ set if and only if A^C is a $ij-\vee_\delta^s$ set.

Proof: (a) Obvious.

(b) Let $\{A_\alpha, \alpha \in J\}$ be a family of $ij - \wedge_\delta^s$ sets in a bitopological space (X, τ_1, τ_2). Then by theorem 3.5(a), we have $\bigcup_{\alpha\in J} A_\alpha = \bigcup_{\alpha\in J}(A_\alpha)^{\delta s\wedge_{ij}} = [\bigcup_{\alpha\in J} A_\alpha]^{\delta s\vee_{ij}}$.

(c) Let $\{A_\alpha, \alpha \in J\}$ be a family of $ij - \wedge_\delta^s$ sets in a bitopological space (X, τ_1, τ_2). By theorem 3.5(b), we have $[\bigcap_{\alpha\in J} A_\alpha]^{\delta s\wedge_{ij}} \subseteq \bigcap_{\alpha\in J}(A_\alpha)^{\delta s\wedge_{ij}} = \bigcap_{\alpha\in J} A_\alpha$. Hence by theorem 3.4(b), we have $\bigcap_{\alpha\in J} A_\alpha = [\bigcap_{\alpha\in J} A_\alpha]^{\delta s\wedge_{ij}}$.

(d) Obvious.

Remark 3.9 *(a) In general* $[A_1 \cap A_2]^{\delta s\wedge_{ij}} \neq A_1^{\delta s\wedge_{ij}} \cap A_2^{\delta s\wedge_{ij}}$.

(b) The family of all $ij - \wedge_\delta^s$ sets and $ij - \vee_\delta^s$ sets are the topologies on X containing all $ij - \delta$ semi open and $ij - \delta$ semi closed sets respectively. Let $\tau_{ij}^{\wedge_\delta^s} = ij - \wedge_\delta^s(X, \tau_1, \tau_2)$ and $\tau_{ij}^{\vee_\delta^s} = ij - \vee_\delta^s(X, \tau_1, \tau_2)$. Clearly $(X, \tau_{ij}^{\wedge_\delta^s})$ and $(X, \tau_{ij}^{\vee_\delta^s})$ are Alexandroff spaces, i.e. arbitrary intersection of open sets is open.

Theorem 3.10 *Let (X, τ_1, τ_2)be a bitopological space. Then*

(a) Every $ij - \wedge_\delta^s$ set is a $g - ij - \wedge_\delta^s$ set.

(b) Every $ij - \vee_\delta^s$ set is a $g - ij - \vee_\delta^s$ set.

(c) If $A_\alpha \in ij - D^{\wedge_\delta^s}$ for all $\alpha \in J$, then $\bigcup_{\alpha\in J} A_\alpha \in ij - D^{\wedge_\delta^s}$.

(d) If $A_\alpha \in ij - D^{\wedge_\delta^s}$ for all $\alpha \in J$, then $\bigcap_{\alpha\in J} A_\alpha \in ij - D^{\wedge_\delta^s}$.

Proof: (a) Obvious. Follows from definition.

(b) Let A be a $ij - \vee_\delta^s$ subset of (X, τ_1, τ_2). Then $A = A^{\delta s\vee_{ij}}$. By theorem 3.4(d), $[A^C]^{\delta s\wedge_{ij}} = [A^{\delta s\vee_{ij}}]^C = A^C$. Therefore by (a), A is a $g - ij - \vee_\delta^s$ set.

(c) Let $A_\alpha \in ij - D^{\wedge_\delta^s}$ for all $\alpha \in J$. Then by theorem 3.5(a), $[\bigcup_{\alpha\in J} A_\alpha]^{\delta s\wedge_{ij}} = \bigcup_{\alpha\in J}(A_\alpha)^{\delta s\wedge_{ij}}$. Hence by hypothesis and definition, $\bigcup_{\alpha\in J}(A_\alpha) \in ij - D^{\wedge_\delta^s}$.

(d) Obvious. Follows from (c) and definition 3.2.

Theorem 3.11 *A subset A of a bitopological space $(X, \tau_1, \tau_2)$is a $g - ij - \vee_\delta^s$ set if and only if $U \subseteq A^{\delta s\vee_{ij}}$, whenever $U \subseteq A$ and $U \in ij - \delta SO(X, \tau_1, \tau_2)$.*

Proof: Let U be a $ij - \delta$ semi open subset of (X, τ_1, τ_2)such that $U \subseteq A$. Then since U^C is $ij - \delta$ semi closed and $U^C \supseteq A^C$, we have $U^C \subseteq V_\delta^s(A^C)$, by definition 3.2. Hence by theorem 3.8(d), $U^C \supseteq [V_\delta^s(A)]^C$. Thus $U \subseteq V_\delta^s(A)$. Conversely, let U be a $ij-\delta$ semi closed subset of (X, τ_1, τ_2)such that $A^C \subseteq U$. Since U^C is $ij-\delta$ semi open and $U^C \subseteq A$, by assumption we have $U^C \subseteq V_\delta^s(A)$. Then $U \supseteq [V_\delta^s(A)]^C = \wedge_\delta^s(A^C)$ by theorem 3.8(d). Therefore A^C is a $g-ij-\wedge_\delta^s$ set. Thus A is a $g-ij-\vee_\delta^s$.

A bitopological space $(X, \tau_1, \tau_2)$is called $ij-\delta s-T_1$ if for each distinct points $x, y \in X$, there exist two $ij-\delta$ semi open sets U and V such that $x \in U \backslash V$ and $y \in V \backslash U$. If X is $12-\delta s-T_1$ and $21-\delta s-T_1$, then it is called pairwise $\delta s-T_1$ $(P-\delta s-T_1)$. Also let $(X, \tau_1, \tau_2)$is $ij-\delta s-T_1$ if and only if every singleton $\{x\}$ is $ij-\delta$ semi closed.

Theorem 3.12 *Let $ij-\delta SC(X, \tau_1, \tau_2)$ be closed by unions. Then for a bitopological space (X, τ_1, τ_2), the following are equivalent,*

(a) $(X, \tau_1, \tau_2)$is $ij-\delta s-T_1$.

(b) Every subset of $(X, \tau_1, \tau_2)$is a $ij-\wedge_\delta^s$ set.

(c) Every subset of $(X, \tau_1, \tau_2)$is a $ij-\vee_\delta^s$ set.

Proof: (a) $\Longrightarrow$ (c) Suppose that $(X, \tau_1, \tau_2)$is $ij-\delta s-T_1$. Let A be any subset of X. Since every singleton $\{x\}$ is $ij-\delta$ semi closed and $A = \bigcup\{\{x\}, x \in A\}$, then A is the union of $ij-\delta$ semi closed sets. Hence A is a $ij-\vee_\delta^s$ set.
(c) $\Longrightarrow$ (a) Since by (c), we have that every singleton is the union of $ij-\delta$ semi closed sets, i.e., it is $ij-\delta$ semi closed, then $(X, \tau_1, \tau_2)$is a $ij-\delta s-T_1$ space.
(b) $\Longrightarrow$ (c) Obvious.

Theorem 3.13 *Let (X, τ_1, τ_2)be a bitopological space. Then*

(a) $(X, \tau_{ij}^{\wedge_\delta^s})$ and $(X, \tau_{ij}^{\vee_\delta^s})$ are always $\delta-T_{1/2}$ spaces.

(b) If $(X, \tau_1, \tau_2)$is $ij-\delta s-T_1$, then both $(X, \tau_{ij}^{\wedge_\delta^s})$ and $(X, \tau_{ij}^{\vee_\delta^s})$ are discrete spaces.

(c) The identity function $id : (X, \tau_{ij}^{\wedge_\delta^s}) \longrightarrow (X, \tau_i)$ is δ - continuous.

(d) The identity function $id : (X, \tau_{ij}^{\vee_\delta^s}) \longrightarrow (X, \tau_i)$ is δ - contra continuous.

Proof: (a) Let $x \in X$. Then $\{x\}$ is $ij-\delta$ open or $ij-\delta$ semi closed in X. If $\{x\}$ is $ij-\delta$ open, then it is $ij-\delta$ semi open. Therefore $\{x\} \in \tau_{ij}^{\wedge_\delta^s}$. If $\{x\}$

is $ij - \delta$ semi closed then $X - \{x\}$ is $ij - \delta$ semi open and so $X - \{x\} \in \tau_{ij}^{\wedge_\delta^s}$, i.e., $\{x\}$ is $\tau_{ij}^{\wedge_\delta^s}$ - closed. Hence $(X, \tau_{ij}^{\wedge_\delta^s})$ is $\delta - T_{1/2}$. In similar manner, we can prove $(X, \tau_{ij}^{\vee_\delta^s})$ is $\delta - T_{1/2}$.

(b) Obvious. The proof follows from theorem 3.8.

(c) If A is $ij - \delta$ open, then A is $ij - \delta$ semi open. Hence $A \in \tau_{ij}^{\wedge_\delta^s}$

(d) If A is $ij - \delta$ open, then A is $ij - \delta$ semi open. This implies that $X - A$ is $ij - \delta$ semi closed and hence $X - A$ is $\tau_{ij}^{\vee_\delta^s}$ - open or A is $\tau_{ij}^{\vee_\delta^s}$ - closed.

4 Applications

Definition 4.1 *A bitopological space $(X, \tau_1, \tau_2)$is called an $ij - \delta s - R_0$ space if for every $ij - \delta$ semi open set U, $x \in U$ implies $ij - \delta scl(\{x\}) \subseteq U$.*

Definition 4.2 *A bitopological space $(X, \tau_1, \tau_2)$is called a $ij - T^{\vee_\delta^s}$ space if $\tau_{ij}^{\wedge_\delta^s} = \tau_{ij}^{\vee_\delta^s}$.*

Theorem 4.3 *For a bitopological space $(X, \tau_1, \tau_2)$the following are equivalent,*

(a) $(X, \tau_1, \tau_2)$is $ij - \delta s - R_0$.

(b) $(X, \tau_{ij}^{\wedge_\delta^s})$ is discrete space.

(c) $(X, \tau_{ij}^{\vee_\delta^s})$ is discrete space.

(d) For each $x \in X$, $\{x\}$ is a $ij - \wedge_\delta^s$ set of (X, τ_1, τ_2).

(e) For each $ij - \delta$ semi open set U of X, $U = U^{\delta s \vee_{ij}}$.

(f) $(X, \tau_1, \tau_2)$is $ij - T^{\vee_\delta^s}$ space.

(g) $(X, \tau_{ij}^{\wedge_\delta^s})$ is R_0 - space.

Proof: (a) $\Longrightarrow$ (b) For any $x \in X$ we have $\{x\}^{\delta s \wedge_{ij}} = \bigcap\{U : \{x\} \subseteq U, U is ij - \delta semiopen\}$. Since X is $ij - \delta s - R_0$ space, then each $ij - \delta$ semi open set U containing x contains $ij - \delta scl(\{x\})$. Hence $ij - \delta scl(\{x\}) \subseteq \{x\}^{\delta s \wedge_{ij}}$. Then by theorem 3.13, $(X, \tau_{ij}^{\wedge_\delta^s})$ is discrete space.

(b) $\Longrightarrow$ (c) Suppose that $(X, \tau_{ij}^{\wedge_\delta^s})$ is discrete space. By the definition of $A^{\delta s \vee_{ij}}$, $A^{\delta s \wedge_{ij}} = [(X - A)^{\delta s \vee_{ij}}]^C$. Therefore if X is $ij - \wedge_\delta^s$ set, then $X - A$ is $ij - \vee_\delta^s$ set. Then $(X, \tau_{ij}^{\vee_\delta^s})$ is discrete space.

(c) $\Longrightarrow$ (d) For each $x \in X$, $\{x\}$ is $\tau_i j^{\wedge_\delta^s}$ - open and $\{x\}$ is a $ij - \wedge_\delta^s$ set of (X, τ_1, τ_2).

(d) $\Longrightarrow$ (e) Let U be a $ij - \delta$ semi open set. Let $x \in U^C$. By assumption $\{x\} = x^{\delta s \wedge_{ij}}$ and therefore $x^{\delta s \wedge_{ij}} \subseteq U^C$. Hence $U^C \supseteq \bigcup\{\{x\}^{\delta s \wedge_{ij}} : x \in U^C\} = [\bigcup\{x : x \in U^C\}]^{\delta s \wedge_{ij}} = [U^C]^{\delta s \wedge_{ij}}$. This shows that $U^C = [U^C]^{\delta s \wedge_{ij}}$ and By the definition of $A^{\delta s \vee_{ij}}$, $A^{\delta s \wedge_{ij}} = [(X - A)^{\delta s \vee_{ij}}]^C$, we have $U = U^{\delta s \vee_{ij}}$.

(e) $\Longrightarrow$ (f) By (e) $ij - \delta SO(X, \tau_1, \tau_2) \subseteq \tau_{ij}^{\vee_\delta^s}$. First we show that $\tau_{ij}^{\wedge_\delta^s} \subseteq \tau_{ij}^{\vee_\delta^s}$. Let A be any $ij - \wedge_\delta^s$ of (X, τ_1, τ_2). Then $A = A^{\delta s \wedge_{ij}} = \bigcap\{U : U \subseteq A, U \in ij - \delta SO(X)\}$. Since $ij - \delta SO(X, \tau_1, \tau_2) \subseteq \tau_{ij}^{\vee_\delta^s}$, by theorem 3.10 we have $A \in \tau_{ij}^{\vee_\delta^s}$ and $\tau_{ij}^{\wedge_\delta^s} \subseteq \tau_{ij}^{\vee_\delta^s}$. Next, let $A \in \tau_{ij}^{\vee_\delta^s}$. Then $X - A \in \tau_{ij}^{\wedge_\delta^s} \subseteq \tau_{ij}^{\vee_\delta^s}$. Therefore $A \in \tau_{ij}^{\vee_\delta^s}$ and $\tau_{ij}^{\wedge_\delta^s} \subseteq \tau_{ij}^{\vee_\delta^s}$. Hence $(X, \tau_1, \tau_2)$is a $ij - T^{\vee_\delta^s}$ space.

(f) $\Longrightarrow$ (g) Let $U \in \tau_{ij}^{\wedge_\delta^s}$ and $x \in U$. Since $(X, \tau_1, \tau_2)$is a $ij - T^{\vee_\delta^s}$ space, $U \in \tau_{ij}^{\vee_\delta^s}$ and $U^C \in \tau_{ij}^{\wedge_\delta^s}$. Since $\{x\} \cap U^C = \phi$, $\tau_{ij}^{\wedge_\delta^s} - cl(\{x\}) \cap U^C = \phi$ and $\tau_{ij}^{\wedge_\delta^s} - cl(\{x\}) \subseteq U$. Hence $(X, \tau_{ij}^{\wedge_\delta^s} - cl(\{x\}))$ is R_0 - space.

(g) $\Longrightarrow$ (a) Let $U \in ij - \delta SO(X, \tau_1, \tau_2)$ and $x \in U$. Since $ij - \delta SO(X, \tau_1, \tau_2) \subseteq \tau_{ij}^{\wedge_\delta^s}$, by (g), $\tau_{ij}^{\wedge_\delta^s} - cl(\{x\}) \subseteq U$. Since $\tau_{ij}^{\wedge_\delta^s} - cl(\{x\}) \in \tau_{ij}^{\vee_\delta^s} - cl(\{x\})$, $\tau_{ij}^{\vee_\delta^s} - cl(\{x\}) = \bigcup\{F : F \in \tau_{ij}^{\wedge_\delta^s} - cl(\{x\}) and F \in ij - \delta SC(X, \tau_1, \tau_2)\}$ and $x \in \tau_{ij}^{\wedge_\delta^s} - cl(\{x\})$. Therefore for some $F \in ij - \delta SC(X, \tau_1, \tau_2)$, $x \in F$ and hence $ij - \delta scl(\{x\}) \subseteq F \subseteq \tau_{ij}^{\wedge_\delta^s} - cl(\{x\}) \subseteq U$. This shows that $(X, \tau_1, \tau_2)$is $ij - \delta s - R_0$.

Definition 4.4 *A function $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ is called,*

(a) $ij - \wedge_\delta^s$ continuous, if $f^{-1}(V)$ is a $ij - \wedge_\delta^s$ set in $(X, \tau_1, \tau_2)$for each σ_i - open set V of (Y, σ_1, σ_2).

(b) $ij - \wedge_\delta^s$ irresolute, if $f^{-1}(V)$ is a $ij - \wedge_\delta^s$ set in $(X, \tau_1, \tau_2)$for each $ij - \wedge_\delta^s$ set V of (Y, σ_1, σ_2).

(c) $ij - \wedge_\delta^s$ open if $f(U)$ is a $ij - \wedge_\delta^s$ set in (Y, σ_1, σ_2) for each $ij - \wedge_\delta^s$ set U of (X, τ_1, τ_2).

Definition 4.5 *A function $f : (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ is called a $ij - \wedge_\delta^s$*

homeomorphism if f is a $ij-\wedge_\delta^s$ irresolute, $ij-\wedge_\delta^s$ open and bijective.

Theorem 4.6 *Let $f:(X,\tau_1,\tau_2)\longrightarrow(Y,\sigma_1,\sigma_2)$ be a function.*

(a) If f is $ij-\wedge_\delta^s$ irresolute, injection and (Y,σ_1,σ_2) is a $ij-T^{\vee_\delta^s}$ space, then $(X,\tau_1,\tau_2)$is a $ij-T^{\vee_\delta^s}$ space.

(b) If f is $ij-\wedge_\delta^s$ open surjection and $(X,\tau_1,\tau_2)$is a $ij-T^{\vee_\delta^s}$ space, then (Y,σ_1,σ_2) is a $ij-T^{\vee_\delta^s}$ space.

(c) If f is $ij-\wedge_\delta^s$ homeomorphism, then $(X,\tau_1,\tau_2)$is a $ij-T^{\vee_\delta^s}$ space if and only if (Y,σ_1,σ_2) is a $ij-T^{\vee_\delta^s}$ space.

Proof: (a) Since (Y,σ_1,σ_2) is a $ij-T^{\vee_\delta^s}$ space, (Y,σ_1,σ_2) is discrete by theorem 4.3. Then $\{f(x)\}\in\sigma_{ij}^{\wedge_\delta^s}$ for every $x\in X$. Since f is $ij-\wedge_\delta^s$ irresolute, $f^{-1}(f(x))\in\tau_{ij}^{\wedge_\delta^s}$ for every $x\in X$. This implies $\{x\}\in\tau_{ij}^{\wedge_\delta^s}$ for every $x\in X$, since f is injective. Therefore $(X,\tau_{ij}^{\wedge_\delta^s})$ is discrete and by Theorem 4.3, $(X,\tau_1,\tau_2)$is a $ij-T^{\vee_\delta^s}$ space.

(b) Let $y\in Y$. $\{f^{-1}(y)\}\neq\phi$, since f is surjective. Since $(X,\tau_{ij}^{\wedge_\delta^s})$ is discrete, $\{f^{-1}(y)\}\in\tau_{ij}^{\wedge_\delta^s}$ for every $y\in Y$. Since f is $ij-\wedge_\delta^s$ open, $f(\{f^{-1}(y)\})\in\sigma_{ij}^{\wedge_\delta^s}$ for every $y\in Y$. This implies $\{y\}\in\sigma_{ij}^{\wedge_\delta^s}$ for every $y\in Y$ or $(Y,\sigma_{ij}^{\wedge_\delta^s})$ is discrete. Hence (Y,σ_1,σ_2) is a $ij-T^{\vee_\delta^s}$ space.

(c) Follows from (a) and (b).

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