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Study of Time Dependent Cosmological Term Λ in a Six Dimensional Cosmological Model

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Abstract

This paper deals the variation of cosmological term Λ with time in a six dimensional cosmological model. We have also shown that this model verify the energy condition given by Ellis and also satisfies the dominant energy condition given by Hawking and Ellis.

Keywords: *Cosmology, Six-dimensional metric equation, Einstein field equation, Cosmological constant, Hubble parameter.*

1 Introduction

The formation of our universe have been described through a paper published by Alebert Einstein in 1917. In this paper, Albert Einstein studied the formation of universe by a mathematical model. Einstein gave a new idea of "Static Universe" or "Einstein Universe". According to him this type of space is neither expanding nor contracting but slightly dynamically stable. Albert Einstein introduced a new type of term and called it cosmological term Λ in the equations of general relativity for the studied of dynamical effects of gravity. The cosmological constant problem and the study of cosmology with time varying cosmological constant have been studied by A. D. Dolgov and J. Silk [2], Tsagas and Maartens [5], Sahni and Starobinsky [15], Peebles [11], Padmanabhan [14], Vishwakarma [12] and M. S. Berman and M. M. Som [8]. Some Physicists and Mathematicians studied different cosmological model with the dependency of comological constant ($\Lambda \propto t^{-2}$), e.g. Endo and Fku [7], Lau [16] and Berman [9, 10]. In 1990, Chen and Wu [17] considered Λ which is inversely proportional to square of scale factor R. Carvalho, Lima and I. Waga [6] generalised it by taking $\Lambda = \alpha R^{-2} + \beta H^{-2}$, where R is the scale factor of Robertson Walker metric, H is the Hubble parameter and α, β are adjustable dimensionless parameters on the basis of quantum field estimations in the curved expanding background. Bulk viscouse Bianchi type-I cosmological model with time dependent cosmological term Λ has been studied by Raj Bali and J.P. Singh [13] in 2008. In 1974, Linde [1] published a paper in which he verify that the cosmological term Λ is function of temperature and also related to spantaneous symmetric breaking process. LRS Bianchi-I cosmological universe models with varying cosmological term have been studied by Anurudh Pradhan and Ambarish Kumar [3] in 2001. Recently B.B. Chaturvedi and B.K. Gupta [4] published some investigations on a cosmological model in 2016. In the consequencess of these studies we motivated to study the six dimensional cosmological model in which cosmological term varies with time.

2 The Metric and Field Equations

We have considered the following six-dimensional metric

$$ds^2 = dt^2 - A^2 dx^2 - B^2(dy^2 + dz^2 + d\mu^2 + d\nu^2), \tag{1}$$

where A and B are functions of t only.
The energy momentum tensor T_i^j for the perfect fluid is defined by

$$T_i^j = (\rho + p)u_i u^j - p\, g_i^j - \Lambda(t) g_i^j, \tag{2}$$

where p, ρ and $\Lambda(t)$ are anistropic pressure, energy density and cosmological term, respectively. We are taking u^i as the six-velocity of the particles such that $u^i = (0, 0, 0, 0, 0, 1)$, to have $u^i u_i = 1$.
In 1916, Albert Einstein gave the equation

$$R_{ij} - \frac{1}{2} R g_{ij} = -8\pi T_{ij}, \tag{3}$$

called the Einstein field equation, where R, R_{ij}, k and T are respectively the scalar curvature, Ricci tensor, gravitational constant and energy momentum tensor.
Taking into consideration our model, the coefficeints of the metric (1) form

the matrix

$$K = \begin{pmatrix} -A^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & -B^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -B^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -B^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -B^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

The determinant K is given by

$$|K| = -A^2\, B^8. \tag{4}$$

The non-vanishing components of christoffel symbols corresponding to the metric are

$$\begin{aligned} &\Gamma^6_{1\,1} = A_6 A, \Gamma^6_{2\,2} = \Gamma^6_{3\,3} = \Gamma^6_{4\,4} = \Gamma^6_{5\,5} = B_6 B, \\ &\Gamma^1_{6\,1} = \frac{A_6}{A}, \Gamma^2_{6\,2} = \Gamma^3_{6\,3} = \Gamma^4_{6\,4} = \Gamma^5_{6\,5} = \frac{B_6}{B}, \end{aligned} \tag{5}$$

where the index 6 denotes the derivative with respect to the time i.e. $A_6 = \frac{\partial A}{\partial t}$. we know that the Ricci tensor of type (0,2) is defined as

$$R_{i\,j} = \frac{\partial \Gamma^k_{k\,i}}{\partial x^j} - \frac{\partial \Gamma^k_{i\,j}}{\partial x^k} + \Gamma^q_{k\,i}\Gamma^k_{q\,j} - \Gamma^q_{i\,j}\Gamma^k_{q\,k}. \tag{6}$$

By using (5) in (6), we get all the non-zero components of the Ricci tensor

$$\begin{aligned} &R_{1\,1} = -A\,A_{6\,6} - 4\frac{A\,A_6\,B_6}{B}, \\ &R_{2\,2} = R_{3\,3} = R_{4\,4} = R_{5\,5} = -B\,B_{6\,6} - \frac{B\,A_6\,B_6}{A} - 3B_6^2, \\ &R_{6\,6} = \frac{A_{6\,6}}{A} + 4\frac{B_{6\,6}}{B}, \end{aligned} \tag{7}$$

then the scalar curvature tensor is given by

$$R = R_{1\,1}\, g^{1\,1} + R_{2\,2}\, g^{2\,2} + R_{3\,3}\, g^{3\,3} + R_{4\,4}\, g^{4\,4} + R_{5\,5}\, g^{5\,5} + R_{6\,6}\, g^{6\,6}, \tag{8}$$

and further, by using (7) in (8), we have

$$R = 8\frac{A_6\,B_6}{A\,B} + 12\frac{B_6^2}{B^2} + 2\frac{A_{6\,6}}{A} + 8\frac{B_{6\,6}}{B}. \tag{9}$$

This is the required scalar curvature in terms of scale factor.
By the help of (3), (7) and (9), we get

$$\begin{aligned} &(a)\, 4\frac{B_{6\,6}}{B} + 6\frac{B_6^2}{B^2} = -8\pi(p+\Lambda), \\ &(b)\, \frac{A_{6\,6}}{A} + 3[\frac{B_{6\,6}}{B} + \frac{B_6\,A_6}{B\,A} + \frac{B_6^2}{B^2}] = -8\pi(p+\Lambda), \\ &(c)\, 2\frac{A_6\,B_6}{A\,B} + 3\frac{B_6^2}{B^2} = 4\pi(\rho-\Lambda). \end{aligned} \tag{10}$$

Thus we conclude:

Theorem 2.1 *In a six dimensional cosmological model defined by (1) the Einstein field equation (3) have the form (10).*

3 Solving the Field Equations

Subtracting (10(a)) from (10(b)), we have

$$\frac{A_{66}}{A} - \frac{B_{66}}{B} + 3\frac{A_6\, B_6}{A\, B} - 3(\frac{B_6}{B})^2 = 0, \tag{11}$$

Integrating (11) with respect to t, we get

$$B^4\, A_6 - A\, B^3\, B_6 = k, \tag{12}$$

where k is an integration constant.
It is clear that equation (12) represent linear differential equation in A(t), where B(t) is an arbitrary function and the solution is given by

$$A = k_1 B + kB \int \frac{dt}{B^5}, \tag{13}$$

where k_1 is an integration constant.
Similarly the equation (12) represent linear differential equation in B(t), where A(t) is an arbitrary function and the solution is given by

$$B^4 = k_2\, A^4 - kA^4 \int \frac{dt}{4\, A^4}, \tag{14}$$

where k_2 is an integration constant.
We know that the energy conservation equation is given by

$$T^{i\,j}_{,i} = 0, \tag{15}$$

therefore, from equations (2), (5) and (15), we obtain

$$\rho_6 - \Lambda_6 + (\rho + p)(\frac{A_6}{A} + 4\frac{B_6}{B}) = 0. \tag{16}$$

Now we propose:

Theorem 3.1 *In six dimensional cosmological model the energy density ρ is directly proportional to cosmological term Λ if only if $AB^4 = constant$.*

Proof: If we take $\rho_6 = \Lambda_6$, then $\rho = c\Lambda$, where c is integration constant. Replacing this in (16), we get

$$(\rho + p)(\frac{A_6}{A} + 4\frac{B_6}{B}) = 0. \tag{17}$$

Since $\rho \neq -p$, then

$$\frac{A_6}{A} + 4\frac{B_6}{B} = 0. \tag{18}$$

this infers that $AB^4 = constant$. Conversely, let $AB^4 = constant$, then

$$log\, A + 4log\, B = log\,(constant). \tag{19}$$

Differentiating this with respect to t, we get

$$\frac{A_6}{A} + 4\frac{B_6}{B} = 0, \tag{20}$$

and using this in (16), we get $\rho_6 = \Lambda_6$. This implies $\rho = c\Lambda$, where c is integration constant.

We know that perfect-gas equation of state for the complete determination of a system is given by

$$p = \gamma\, \rho. \quad 0 \leq \gamma \leq 1 \tag{21}$$

Therefore, we choose any arbitrary function B(t) from equation (13), one can obtain A(t). Hence from equations (10), (16) and (21), p, ρ and Λ can be calculated, i.e. for any given function B(t), the field equations are solvable. Similarly by using equation (14) the field equations (10) , (16) and (21) can be solved for any given function A(t).

In 1994, A. Mazumdar considered

$$B = t^{\frac{1-n}{2}}, \tag{22}$$

where n is real number and $n \neq \frac{1}{3}$.

Using equation (22) in (13), we have

$$A = k_1 t^{\frac{1-n}{2}} + \frac{2}{5n-3} k t^{(2n-1)}. \tag{23}$$

Now putting (22) and (23) in (1), we get

$$ds^2 = dt^2 - (k_1 t^{\frac{1-n}{2}} + \frac{2}{5n-3} k\, t^{(2n-1)})^2 dx^2 - t^{1-n}(dy^2 + dz^2 + d\mu^2 + d\nu^2). \tag{24}$$

Subtracting (10(c)) from (10(a)), we get

$$\frac{B_{6\,6}}{B} - \frac{A_6\, B_6}{A\, B} = -2\pi(p + \rho). \tag{25}$$

Now using (21), (22) and (23) in (25), we get

$$\rho = \frac{1}{4\pi(1+\gamma)} \frac{\left[(1-n)k_1 t^{\frac{(-5n+3)}{2}} + k\frac{(5n-1)(1-n)}{5n-3}\right]}{t^2[k_1\, t^{\frac{-5n+3}{2}} + \frac{2k}{5n-3}\,]}. \tag{26}$$

Now using (22) and (23) in (10(c)), we have

$$\begin{aligned}\Lambda =& \frac{(4(1-n)-5(1+\gamma)(1-n)^2)\,k_1\,t^{\frac{(-5n+3)}{2}}}{16\pi\,(1+\gamma)\,t^2[k_1\,t^{\frac{-5n+3}{2}}+\frac{2k}{5n-3}\,]} \\ &+\frac{\frac{k}{5n-3}\,[4(5n-1)(1-n)-(1+\gamma)(\frac{6(1-n)^2}{2}+4(1-n)(2n-1))]}{16\pi\,(1+\gamma)\,t^2[k_1\,t^{\frac{-5n+3}{2}}+\frac{2k}{5n-3}\,]}.\end{aligned} \tag{27}$$

Using (26) in (21), we get

$$p=\frac{\gamma}{4\pi(1+\gamma)}\frac{\left[(1-n)k_1t^{\frac{(-5n+3)}{2}}+k\frac{(5n-1)(1-n)}{5n-3}\right]}{t^2[k_1\,t^{\frac{-5n+3}{2}}+\frac{2k}{5n-3}\,]}. \tag{28}$$

Equations (26), (27) and (28) represent the energy density, cosmological term and pressure in term of time.

4 Discussion for Dust Universe ($\gamma = 0$) and Perfect Fluid ($\gamma = 1$)

Putting $\gamma = 0$ and $\gamma = 1$ in equation (26), we have

$$\rho=\frac{1}{4\pi}\frac{\left[(1-n)k_1t^{\frac{(-5n+3)}{2}}+k\frac{(5n-1)(1-n)}{5n-3}\right]}{t^2[k_1\,t^{\frac{-5n+3}{2}}+\frac{2k}{5n-3}\,]},\quad (\gamma=0) \tag{29}$$

and

$$\rho=\frac{1}{8\pi}\frac{\left[(1-n)k_1t^{\frac{(-5n+3)}{2}}+k\frac{(5n-1)(1-n)}{5n-3}\right]}{t^2[k_1\,t^{\frac{-5n+3}{2}}+\frac{2k}{5n-3}\,]}.\quad (\gamma=1) \tag{30}$$

From equation (26) and (28) it is clear that the energy condition given by Ellis i.e

$$\begin{aligned}&(i)\rho+p>0\\&(ii)\rho+3p>0,\\&(iii)\rho>0,\end{aligned}$$

are satisfies provided $k > 0$, $k_1 > 0$ and $n < 1$.
Also the dominant energy conditions given by Hawking and Ellis i.e.

$$\begin{aligned}&(i)\rho-p\geq,\\&(ii)\rho+p\geq 0,\end{aligned}$$

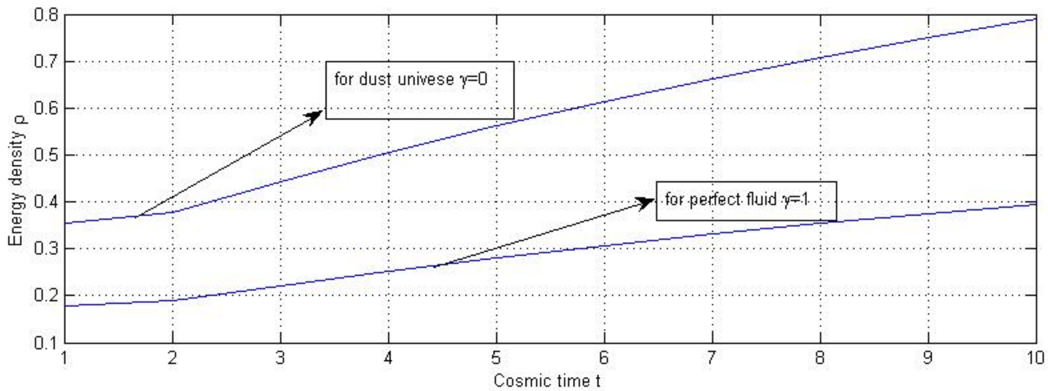

Figure 1: Energy density vs cosmic time t, where $n = k_1 = k = 0.5$ and $\pi = 3.14$

are satisfies provided $k > 0$, $k_1 > 0$ and $n < 1$.
Putting $\gamma = 0$ and $\gamma = 1$ in equation (27), we have

$$\Lambda = \frac{[(5n-1)(1-n)\,k_1\,t^{\frac{(-5n+3)}{2}} + k(\frac{(15n-3)(1-n)}{5n-3}\,]}{16\pi\,t^2[k_1\,t^{\frac{-5n+3}{2}} + \frac{2k}{5n-3}\,]}, \quad (\gamma = 0) \tag{31}$$

and

$$\Lambda = \frac{[2(5n-3)(1-n))\,k_1\,t^{\frac{(-5n+3)}{2}} + \frac{k(1-n)(10n-2)}{(5n-3)}]}{32\pi\;t^2[k_1\,t^{\frac{-5n+3}{2}} + \frac{2k}{5n-3}\,]}. \quad (\gamma = 1) \tag{32}$$

The kinematical parameters are found to have the following expressions

$$\theta = \frac{\left[5k_1\frac{(-n+1)}{2}\,t^{-\frac{(n+1)}{2}} + \frac{2k}{5n-3}t^{2(n-1)}\right]}{k_1 t^{\frac{1-n}{2}} + \frac{2}{5n-3}kt^{(2n-1)}}, \tag{33}$$

$$H = \frac{\left[5k_1\frac{(-n+1)}{2}\,t^{-\frac{(n+1)}{2}} + \frac{2k}{5n-3}t^{2(n-1)}\right]}{5(k_1 t^{\frac{1-n}{2}} + \frac{2}{5n-3}kt^{(2n-1)})}, \tag{34}$$

$$\begin{aligned}\sigma^2 =& \frac{1}{5}[[\frac{\frac{(1-n)}{2}\,k_1 t^{\frac{-(n+1)}{2}} + \frac{2k(2n-1)}{(5n-3)}\,t^{2n-2}}{(k_1 t^{\frac{1-n}{2}} + \frac{2}{5n-3}kt^{(2n-1)})}]^2 \\ &- (\frac{(1-n)^2}{4}\,t^{-2}) - \frac{(1-n)^2\,t^{\frac{-(n+3)}{2}} + 4\frac{k(2n-1)(1-n)}{5n-3}\,t^{(2n-3)}}{k_1 t^{\frac{1-n}{2}} + \frac{2}{5n-3}kt^{(2n-1)}}],\end{aligned} \tag{35}$$

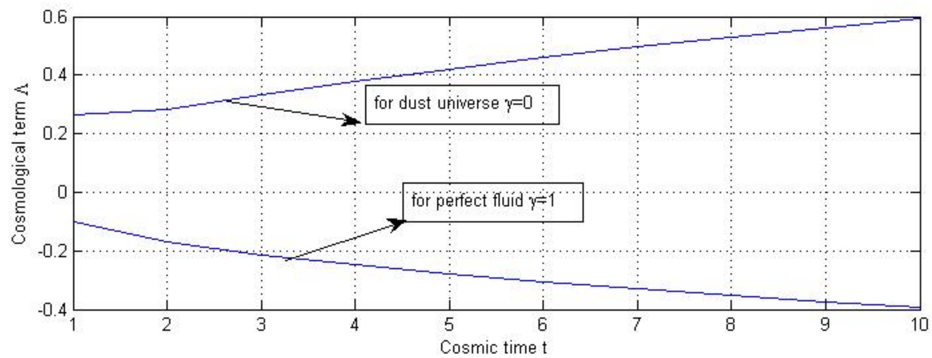

Figure 2: Cosmological term γ vs cosmic time t, where $n = k_1 = k = 0.5$ and $\pi = 3.14$

equations (33), (34) and (35) represent expasion scalar, Hubble parameter and shear scalar respectively. If we take $k_1 = 0$ then from equation (27) we observed that the cosmological constant Λ is inversely proportional to t^2 i.e $\Lambda \propto t^{-2}$. There are so many Physicists and Mathematicians shown the dependency of Λ with time. Berman and Som also obtained that $\Lambda \propto t^{-2}$ and said that this play a very important role in cosmology. This verified the statment of Berman and Som and we can say that for a six dimensional cosmological model in the cosmological term Λ will be time dependent and inversely proportional to t^2, we also observed that $\Lambda > 0$ providing $k > 0$, $k_1 > 0$ and $n < 1$. For dust universe ($\gamma = 0$) we have $\Lambda > 0$. Therefore we obtained a possitive cosmologial Λ-term under the same conditions which satisfies the dominant energy condition given by Ellis and Hawking

5 Conclusion

We have drawn the graphs given in fig.1 and fig.2 which shown the variation of energy density with time and cosmological term with time. A number of author have discussed that the cosmological term ($\Lambda \propto t^{-2}$) play a very important role in cosmology. We have seen that with present metric the cosmological term is $\Lambda \propto t^{-2}$. Also we have seen the energy condition given by Ellis and dominant energy condition given by Hawking and Ellis verify, therefore we can say that the present model is very useful for the study of universe.

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