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Some Remarks on Fuzzy P-Spaces

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Abstract

In this paper we discuss several characterizations of fuzzy P-spaces and the conditions under which fuzzy topological spaces become fuzzy P-spaces, are investigated.

Keywords: *Fuzzy G_δ-set, Fuzzy F_σ-set, Fuzzy dense set, Fuzzy nowhere dense set, Fuzzy sub maximal space, Fuzzy hyper connected space, Fuzzy Baire space, Fuzzy Volterra space.*

1 Introduction

The concept of fuzzy sets and fuzzy set operations were first introduced by L.A. Zadeh in his classical paper [19] in the year 1965. There after the paper of C.L. Chang [5] in 1968 paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts. Since then much attention has been paid to generalize the basic concepts of general topology in fuzzy setting and thus a modern theory of fuzzy topology has been developed.

A.K. Mishra [8] introduced the concepts of P-spaces as a generalization of$\boldsymbol{\omega_\alpha}$– additive spaces of Sikorski [9]and L.W. Cohen and C. Goffman [6]. The concept of P-spaces in fuzzy setting was introduced by G. Balasubramanianin [10]. Almost P-spaces in classical topology was introduced by A.I. Veksler [18] and was also studied further by R. Levy [7]. The concept of almost P-spaces in fuzzy setting was introduced by the authors in [17]. In this paper, in section 3, we discuss several characterizations of fuzzy P-spaces and in section 4, the conditions under which a fuzzy sub maximal space becomes a fuzzy P-space, are investigated. In section 5, fuzzy Baire spaces, fuzzy D-Baire spaces, fuzzy hyper connected spaces, fuzzy second category spaces and fuzzy Volterra spaces are studied along with fuzzy P-spaces.

2 Preliminaries

Now we introduce some basic notions and results used in the sequel. In this work by (X, T) or simply by X, we will denote a fuzzy topological space due to Chang (1968).

Definition 2.1: *Let λ and μ be any two fuzzy sets in a fuzzy topological space (X, T). Then we define $\lambda \vee \mu : X \to [0,1]$ as follows:*

$(\lambda \vee \mu)(x) = \text{Max}\{\lambda(x), \mu(x)\}$. Also we define $\lambda \wedge \mu : X \to [0,1]$ as follows:

$(\lambda \wedge \mu)(x) = \text{Min}\{\lambda(x), \mu(x)\}$.

For a family $\{\lambda_i / i \in I\}$ of fuzzy sets in (X, T), the union$\psi = \vee_i (\lambda_i)$ and the intersection $\delta = \wedge_i (\lambda_i)$ are defined respectively as $\psi(x) = sup_i \{\lambda_i(x), x \in X\}$ and $\delta(x) = inf_i \{\lambda_i(x), x \in X\}$.

Definition 2.2: *Let (X, T) be a fuzzy topological space and λ be any fuzzy set in (X, T). We define the interior and the closure of λ respectively as follows:*

(i) $\text{int}(\lambda) = \vee\{\mu / \mu \leq \lambda, \mu \in T\}$,
(ii) $\text{cl}(\lambda) = \wedge\{\mu / \lambda \leq \mu, 1-\mu \in T\}$.

Lemma 2.1 [1]: *For a fuzzy set λ of a fuzzy topological space X,*

(i) $1- \text{int}(\lambda) = \text{cl}(1-\lambda)$,
(ii) $1- \text{cl}(\lambda) = \text{int}(1-\lambda)$

Definition 2.3 [11]: *A fuzzy set λ in a fuzzy topological space (X, T) is called fuzzy dense if there exists no fuzzy closed set μ in (X, T) such that$\lambda < \mu < 1$. That is, $cl(\lambda) = 1$.*

Definition 2.4 [12]: *A fuzzy set λ in a fuzzy topological space (X, T) is called fuzzy nowhere dense if there exists no non-zero fuzzy open set μ in (X, T) such that $\mu < cl(\lambda)$. That is, int cl (λ) = 0.*

Definition 2.5 [2]: *A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy F_σ-set in (X, T) if $\lambda = \bigvee_{i=1}^{\infty} (\lambda_i)$ where $1 - \lambda_i \in T$ for $i \in I$.*

Definition 2.6 [2]: *A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy G_δ-set in (X, T) if $\lambda = \bigwedge_{i=1}^{\infty} (\lambda_i)$ where $\lambda_i \in T$ for $i \in I$.*

Definition 2.7 [1]: *A fuzzy set λ in a fuzzy topological space (X, T) is called*

(i) A fuzzy regular open set in (*X*, *T*) if Int cl (λ) = λ and
(ii) A fuzzy regular closed set in (*X*, *T*) if cl int (λ) = λ.

Lemma 2.2 [1]: *For a family $\mathcal{A}$ of $\{\lambda_\alpha\}$ of fuzzy sets of a fuzzy topological space (X, T), $\vee cl(\lambda_\alpha) \leq cl(\vee\lambda_\alpha)$. In case $\mathcal{A}$ is a finite set, $\vee cl(\lambda_\alpha) = cl(\vee(\lambda_\alpha))$. Also $\vee int(\lambda_\alpha) \leq int(\vee\lambda_\alpha)$ in (X, T).*

Definition 2.8 [12]: *A fuzzy topological space (X, T) is called a fuzzy Baire space if $int(\bigvee_{i=1}^{\infty}(\lambda_i)) = 0$, where $(\lambda_i)'s$ are fuzzy nowhere dense sets in (X, T).*

Definition 2.9 [12]: *A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy first category set if $\lambda = \bigvee_{i=1}^{\infty}(\lambda_i)$, where $(\lambda_i)'s$ are fuzzy nowhere dense sets in (X, T). Any other fuzzy set in (X, T) is said to be of fuzzy second category.*

Definition 2.10 [11]: *A fuzzy topological space (X, T) is called fuzzy first category if $\bigvee_{i=1}^{\infty}(\lambda_i) = 1_X$, where $(\lambda_i)'s$ are fuzzy nowhere dense sets in (X, T). A topological space which is not of fuzzy first category is said to be of fuzzy second category.*

Definition 2.11 [2]: *A fuzzy topological space (X, T) is called a fuzzy submaximal space if for each fuzzy set λ in (X, T) such that $cl(\lambda) = 1$, then $\lambda \in T$ in (X, T).*

3 Fuzzy P-Spaces

Definition 3.1 [10]: *A fuzzy topological space (X, T) is called a fuzzy P-space if countable intersection of fuzzy open sets in (X, T) is fuzzy open. That is, every non-zero fuzzy G_δ-set in (X, T), is fuzzy open in (X, T).*

Proposition 3.1: *If the fuzzy topological space (X, T) is a fuzzy P-space, then $int(\bigwedge_{i=1}^{\infty}(\mu_i)) = \bigwedge_{i=1}^{\infty} int(\mu_i)$, where $(\mu_i)'s$ are non-zero fuzzy open sets in (X, T).*

Proof: Let $(\mu_i)'s$ be non-zero fuzzy open sets in a fuzzy P-space (*X*, *T*). Then $\mu = \wedge_{i=1}^{\infty} (\mu_i)$, is a fuzzy G_δ-set in (*X*, *T*). Since (*X*, *T*) is a fuzzy P-space, the fuzzy G_δ-set μ is fuzzy open in (*X*, *T*). Hence, we have int $(\mu) = \mu$. This implies that int $(\wedge_{i=1}^{\infty} (\mu_i) = \wedge_{i=1}^{\infty} (\mu_i) = \wedge_{i=1}^{\infty}$ int (μ_i), (since $\mu_i \in T$, int$(\mu_i) = \mu_i$) and hence int $(\wedge_{i=1}^{\infty} (\mu_i)) = \wedge_{i=1}^{\infty}$ (int (μ_i)), where $(\mu_i)'s$ are non-zero fuzzy open sets in (*X*, *T*).

Theorem 3.1 [17]: *If the fuzzy topological space (X, T) is a fuzzy P-space and if λ is a fuzzy first category set in (X, T), then λ is not a fuzzy dense set in (X, T).*

Proposition 3.2: *If λ is a fuzzy residual set in a fuzzy P-space (X, T), then $int\ (\lambda) \neq 0$.*

Proof: Let λ be a fuzzy residual set in a fuzzy P-space (*X*, *T*). Then, $(1-\lambda)$ is a fuzzy first category set in (*X*, *T*) and hence by theorem 3.1, $(1-\lambda)$ is not a fuzzy dense set in (*X*, *T*). That is, cl$(1-\lambda) \neq 1$. This implies that $1-$ int $(\lambda) \neq 1$ and hence we have int $(\lambda) \neq 0$.

Theorem 3.2 [17]: *If $(\lambda_i)'s$ are fuzzy regular closed sets in a fuzzy P-space (X, T), then cl $(\vee_{i=1}^{\infty} (\lambda_i)) = \vee_{i=1}^{\infty} (\lambda_i)$.*

Theorem 3.3 [15]: *If λ is a fuzzy dense and fuzzy G_δ-set in a fuzzy topological space (X, T), then $1-\lambda$ is a fuzzy first category set in (X, T).*

Proposition 3.3: *If λ is a fuzzy dense and fuzzy G_δ-set in a fuzzy P-space (X, T), then $int\ (\lambda) \neq 0$.*

Proof: Let λ be a fuzzy dense and fuzzy G_δ-set in a fuzzy P-space (*X*, *T*). By theorem 3.3, $1-\lambda$ is a fuzzy first category set in (*X*, *T*). Since (*X*, *T*) is a fuzzy P-space, by theorem 3.1, then $1-\lambda$ is not a fuzzy dense set in (*X*, *T*) and hence cl$(1-\lambda) \neq 1$. This implies that $1-$ int $(\lambda) \neq 1$and hence we have int $(\lambda) \neq 0$.

Proposition 3.4: *If $(\mu_i)'s$ are fuzzy regular open sets in a fuzzy P-space (X, T), then int $(\wedge_{i=1}^{\infty} (\mu_i)) = \wedge_{i=1}^{\infty} (\mu_i)$, where $(\mu_i)'s$ are non-zero fuzzy regular open sets in (X, T).*

Proof: Let $(\mu_i)'s$ be fuzzy regular open sets in a fuzzy P-space (*X*, *T*). Then $(1-\mu_i)'s$ are non-zero fuzzy regular closed sets in (*X*, *T*). Then, by theorem 3.2, cl $(\vee_{i=1}^{\infty}(1-\mu_i))) = \vee_{i=1}^{\infty}(1-\mu_i)$. This implies that cl $(1-\wedge_{i=1}^{\infty}(\mu_i)) = 1 - [\wedge_{i=1}^{\infty} (\mu_i)]$ and hence $1-$int$(\wedge_{i=1}^{\infty}(\mu_i)) = 1-[\wedge_{i=1}^{\infty} (\mu_i)]$. Therefore int $(\wedge_{i=1}^{\infty}(\mu_i)) = \wedge_{i=1}^{\infty}(\mu_i)$, where$(\mu_i)'s$ are non-zero fuzzy regular open sets in (*X*, *T*).

4 Fuzzy P-Spaces and Fuzzy Submaximal Spaces

The class of submaximal spaces was introduced by N. Bourbaki in Topologie G´en´erale [3]. This concept in fuzzy setting was introduced by G. Balasubramanian in [11].

Proposition 4.1: *If each fuzzy G_δ-set is a fuzzy dense set in a fuzzy submaximal space (X, T), then (X, T) is a fuzzy P-space.*

Proof: Let λ be a fuzzy G_δ-set in a fuzzy submaximal space (X, T). Then, by hypothesis, λ is a fuzzy dense set in (X, T). Since (X, T), is a fuzzy submaximal space, the fuzzy dense set λ in (X, T), is a fuzzy open set in (X, T). That is, every fuzzy G_δ-set set in (X, T) is a fuzzy open set in (X, T). Therefore (X, T) is a fuzzy P-space.

Proposition 4.2: *If int $(\lambda)= 0$, where λ is a fuzzy F_σ-set in a fuzzy submaximal space (X, T), then (X, T) is a fuzzy P-space.*

Proof: Let λ be a fuzzy G_δ-set in a fuzzy submaximal space (X, T). Then, $(1-\lambda)$ is a fuzzy F_σ-set in (X, T). Then, by hypothesis, int $(1-\lambda) = 0$, for the fuzzy F_σ-set λ in (X, T). This implies that $\mathrm{cl}(\lambda) = 1$. Then λ is a fuzzy dense set in (X, T). Since (X, T) is a fuzzy submaximal space, the fuzzy dense set λ in (X, T), is a fuzzy open set in (X, T). That is, every fuzzy G_δ-set in (X, T), is a fuzzy open set in (X, T). Therefore (X, T) is a fuzzy P-space.

Proposition 4.3: *If each fuzzy F_σ-set is a fuzzy nowhere dense set in a fuzzy submaximal space (X, T), then (X, T) is a fuzzy P-space.*

Proof: Let λ be a fuzzy F_σ-set in a fuzzy submaximal space (X, T) such that int $\mathrm{cl}(\lambda) = 0$. Then, int $(\lambda) \leq$ int cl (λ), implies that int $(\lambda) = 0$. Now $\mathrm{int}(\lambda) = 0$ for a fuzzy F_σ-set in a fuzzy submaximal space (X, T). Then, by proposition 4.2, (X, T) is a fuzzy P-space.

Proposition 4.4: *If cl int$(\lambda) = 1$, for each fuzzy G_δ-set λ in a fuzzy submaximal space (X, T), then (X, T) is a fuzzy P-space.*

Proof: Let λ be a fuzzy F_σ-set in a fuzzy submaximal space (X, T). Then $(1-\lambda)$ is a fuzzy G_δ-set in (X, T). By hypothesis, cl $\mathrm{int}(1-\lambda) = 1$. Then $1-$cl $\mathrm{int}(1-\lambda)=0$. This implies that $1- [1-$ int $\mathrm{cl}(\lambda)] = 0$. That is, int cl $(\lambda) = 0$ and hence λ is a fuzzy nowhere dense set in (X, T). Thus the fuzzy F_σ-set λ is a fuzzy nowhere dense set in a fuzzy submaximal space (X, T). Hence, by proposition 4.3, (X, T) is a fuzzy P–space.

Proposition 4.5: *If λ is a fuzzy residual set in a fuzzy submaximal space (X, T), then λ is a fuzzy G_δ-set in (X, T).*

Proof: Let λ be a fuzzy residual set in a fuzzy submaximal space (*X*, *T*). Then $1-\lambda$ is a fuzzy first category set in (*X*, *T*) and hence $1-\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $(\lambda_i)'$sare fuzzy nowhere dense sets in (*X*, *T*). Since $(\lambda_i)'$s are fuzzy nowhere dense (*X*, *T*), int cl $(\lambda_i) = 0$. Then, int $(\lambda_i) \leq$ intcl (λ_i), implies that int $(\lambda_i) = 0$. This implies that $1-$int $(\lambda_i) = 1$ and hence cl $(1-\lambda_i) = 1$. Since (*X*, *T*) is a fuzzy submaximal space, the fuzzy dense sets $(1-\lambda_i)$'s are fuzzy open sets in (*X*, *T*). Then $(\lambda_i)'$s are fuzzy closed sets in (*X*, *T*). Hence $1-\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $(\lambda_i)'$s are fuzzy closed sets in (*X*, *T*), implies that $1-\lambda$ is a fuzzy F_σ-set in (*X*, *T*). Therefore λ is a fuzzy G_δ-set in (*X*, *T*).

Proposition 4.6: *If λ is a fuzzy residual set in a fuzzy submaximal and fuzzy P-space (X, T), then λ is a fuzzy open set in (X, T).*

Proof: Let λ be a fuzzy residual set in a fuzzy submaximal and fuzzy P-space (*X*, *T*). Since λ is a fuzzy residual set in a fuzzy submaximal space (*X*, *T*), by proposition 4.5, λ is a fuzzy G_δ-set in (*X*, *T*). Since (*X*, *T*) is a fuzzy P-space, the fuzzy G_δ-set in (*X*, *T*) is a fuzzy open set in (*X*, *T*). Hence a fuzzy residual set λ in a fuzzy submaximal and fuzzy P- space (*X*, *T*) is a fuzzy open set in (*X*,*T*).

Remarks: *In view of the proposition we have the following result: every fuzzy first category set in a fuzzy submaximal and fuzzy P-space (X, T) is a fuzzy closed set in (X, T).*

Proposition 4.7: *If λ is a fuzzy nowhere dense set in a fuzzy submaximal space (X, T), then λ is a fuzzy closed set in (X, T).*

Proof: Let λ be a fuzzy nowhere dense set in a fuzzy submaximal space (*X*, *T*). Then we have int cl $(\lambda) = 0$ and int $(\lambda) \leq$ int cl (λ), implies that int $(\lambda) = 0$. Then $1-$int $(\lambda) = 1$ implies that cl $(1-\lambda) = 1$ and hence $1-\lambda$ is a fuzzy dense set in (*X*, *T*). Since (*X*, *T*) is a fuzzy submaximal space, $1-\lambda$ is a fuzzy open set in (*X*, *T*). Therefore the fuzzy nowhere dense set λ is a fuzzy closed set in (*X*, *T*).

Proposition 4.8: *If $(\lambda_i)'s$ (i = 1 to ∞) are fuzzy nowhere dense sets in a fuzzy submaximal and fuzzy P-space (X, T), then, cl$(\vee_{i=1}^{\infty}(\lambda_i)) = \vee_{i=1}^{\infty}(\lambda_i)$.*

Proof: Let $(\lambda_i)'$s (i= 1 to ∞) be fuzzy nowhere dense sets in a fuzzy submaximal and fuzzy P-space (*X*, *T*). Since $(\lambda_i)'$s are fuzzy nowhere dense set in a fuzzy submaximal space (*X*, *T*), by proposition 4.8, $(\lambda_i)'$s are fuzzy closed sets in (*X*, *T*) and hence $(1-\lambda_i)'$ s are fuzzy open sets in (*X*, *T*). Now $\mu = \wedge_{i=1}^{\infty}(1-\lambda_i)$ is a non-zero fuzzy G_δ-set in (*X*, *T*). Since (*X*, *T*) is a fuzzy P- space, μ is a fuzzy open set in (*X*, *T*) and hence we have int$(\mu) = \mu$. This implies that int$(\wedge_{i=1}^{\infty}(1-\lambda_i)) = \wedge_{i=1}^{\infty}(1-\lambda_i)$. Then, int $(1-\vee_{i=1}^{\infty}(\lambda_i)) = 1-\vee_{i=1}^{\infty}(\lambda_i)$ and hence $1-$cl $(\vee_{i=1}^{\infty}(\lambda_i)) = 1-\vee_{i=1}^{\infty}(\lambda_i)$. Therefore we have cl$(\vee_{i=1}^{\infty}(\lambda_i)) = \vee_{i=1}^{\infty}(\lambda_i)$.

5 Fuzzy P-Spaces and Other Fuzzy Topological Spaces

Definition 5.1 [4]: *A fuzzy topological space X is said to be fuzzy hyper connected if every non-null fuzzy open subset of X is fuzzy dense in X. That is, a fuzzy topological space(X, T) is fuzzy hyper connected if cl* (μ_i) *=1, for all*$\mu_i \in T$.

Proposition 5.1: *If a fuzzy P-space (X, T) is a fuzzy hyper connected space, then (X, T) is a fuzzy Baire space.*

Proof: Let λ be a fuzzy G_δ-set in a fuzzy P-space (*X*, *T*). Since(*X*, *T*) is a fuzzy P-space, is a fuzzy open set in (*X*, *T*). Since the fuzzy space (*X*, *T*) is a fuzzy hyper connected space, the fuzzy open set λ in (*X*, *T*) is a fuzzy dense set in (*X*, *T*). That is, cl $(\lambda) = 1$. Hence λ is a fuzzy G_δ-set and a fuzzy dense set in (*X*, *T*). Then, by proposition 3.3, $(1-\lambda)$ is a fuzzy first category set in (*X*, *T*). Therefore $(1-\lambda) = \bigvee_{i=1}^{\infty}(\lambda_i)$, where $(\lambda_i)'s$ are fuzzy nowhere dense sets in (*X*, *T*). Then, $\text{int}[\bigvee_{i=1}^{\infty}(\lambda_i)] = \text{int}[1-\lambda] = 1 - \text{cl}(\lambda) = 1 - 1 = 0$. Hence we have $\text{int}[\bigvee_{i=1}^{\infty}(\lambda_i)] = 0$, where $(\lambda_i)'s$ are fuzzy nowhere dense sets in (*X*, *T*). Therefore (*X*, *T*) is a fuzzy Baire space.

Proposition 5.2: *If a fuzzy P-space (X, T) is a fuzzy hyper connected space, then (X, T) is a fuzzy second category space.*

Proof: Let the fuzzy P-space (*X*, *T*) be a fuzzy hyper connected space. Then, by proposition 5.1, (*X*, *T*) is a fuzzy Baire space and hence int $[\bigvee_{i=1}^{\infty}(\lambda_i)] = 0$, where $(\lambda_i)'s$ are fuzzy nowhere dense sets in (*X*, *T*). We claim that $\bigvee_{i=1}^{\infty}(\lambda_i) \neq 1$. Suppose that $\bigvee_{i=1}^{\infty}(\lambda_i) = 1$. This would imply that $\text{int}[\bigvee_{i=1}^{\infty}(\lambda_i)] = int(1) = 1 \neq 0$, a contradiction. Hence we must have $\bigvee_{i=1}^{\infty}(\lambda_i) \neq 1$, where $(\lambda_i)'s$ are fuzzy nowhere dense sets in (*X*, *T*). Therefore (*X*, *T*) is a fuzzy second category space.

Definition 5.2 [16]: *A fuzzy topological space (X, T) is called a fuzzy Volterra space if cl* $(\bigwedge_{i=1}^{N}(\lambda_i)) = 1$, *where* $(\lambda_i)'s$ *are fuzzy dense and fuzzy* G_δ *sets in (X, T).*

Proposition 5.3: *If there are N fuzzy* G_δ*-sets* $(\lambda_k)'s$ *(k = 1 to N) in a fuzzy hyper connected and fuzzy P-space (X, T), then (X, T) is a fuzzy Volterra space.*

Proof: Let $(\lambda_k)'s$ (*k*= 1 to *N*) are fuzzy G_δ-sets in a fuzzy hyper connected and fuzzy P-space (*X*, *T*). Then $\lambda = \bigvee_{k=1}^{N}(\lambda_k)$, is also a fuzzy G_δ-set in (*X*,*T*). Since (*X*, *T*) is a fuzzy P-space, λ is a fuzzy open set in (*X*, *T*). Again, since (*X*, *T*) is a fuzzy hyper connected space, the fuzzy open set λ is fuzzy dense in (*X*, *T*). Then we have cl $(\lambda) = 1$. This implies that cl $(\bigwedge_{i=1}^{N}(\lambda_k)) = 1$. Now cl $(\bigwedge_{i=1}^{N}(\lambda_k)) \leq \bigwedge_{i=1}^{N}\text{cl}((\lambda_k))$, implies that $1 \leq \bigwedge_{i=1}^{N}\text{cl}(\lambda_k)$. That is, $\bigwedge_{i=1}^{N}\text{cl}(\lambda_k) = 1$. Then we have cl $(\lambda_k) = 1$, for *k*= 1 to *N*. Hence $(\lambda_k)'s$ are fuzzy dense sets in (*X*, *T*). Thus, we have $cl(\wedge_{i=1}^{N}(\lambda_k)) = 1$, where (λ_k)'s are fuzzy dense and fuzzy G_δ-sets in (*X*, *T*). Therefore (*X*, *T*) is a fuzzy Volterra space.

Definition 5.3 [14]: *A fuzzy topological space (X, T) is called a fuzzy D-Baire space in (X, T) if every fuzzy first category set in (X, T) is a fuzzy nowhere dense set in (X, T). That is, (X, T) is a fuzzy D-Baire space if int cl (λ)= 0, for each fuzzy first category set λ in (X, T).*

Theorem 5.1 [17]: *If the fuzzy topological space (X,T) is a fuzzy P-space and if λ is a fuzzy first category set in (X, T), then λ is not a fuzzy nowhere dense set in (X, T).*

Proposition 5.4: *If (X, T) is a fuzzy P-space, then (X, T) is not a fuzzy D-Baire space.*

Proof: Let λ be a fuzzy category set in a fuzzy P-space (*X*, *T*). By theorem 5.1, the fuzzy first category set λ in (*X*, *T*), is not a fuzzy nowhere dense set in (*X*, *T*). Thus the fuzzy first category set in (*X*, *T*) is not a fuzzy nowhere dense set in (*X*,*T*). Therefore (*X*, *T*) is not a fuzzy D-Baire space.

Proposition 5.5: *If a fuzzy P-space (X, T) is a fuzzy submaximal and fuzzy Baire space, then (X, T) is a fuzzy D-Baire space.*

Proof: Let the fuzzy P-space (*X*, *T*) be a fuzzy submaximal and fuzzy Baire space. Let λ be a fuzzy first category set in (*X*, *T*). Since (*X*, *T*) is a fuzzy Baire space, int $(\lambda) = 0$. Then, $1 -$int $(\lambda) = 1$. This implies that cl $(1-\lambda) = 1$ and hence $(1-\lambda)$ is a fuzzy dense set in (*X*, *T*). Since (*X*, *T*) is a fuzzy submaximal space, $(1-\lambda)$ is a fuzzy open set in (*X*, *T*). Then, λ is a fuzzy closed set in (*X*, *T*) and hence cl $(\lambda) = \lambda$. Now int cl $(\lambda) = int(\lambda)$, implies that int cl $(\lambda) = 0$. Then λ is a fuzzy nowhere dense set in (*X*, *T*). Hence, each fuzzy first category set in (*X*, *T*) is a fuzzy nowhere dense set in (*X*, *T*). Therefore (*X*, *T*) is a fuzzy D-Baire space.

Theorem 5.2 [13]: *If the fuzzy topological space (X, T) is a fuzzy Baire space, then no non-zero open set is a fuzzy first category set in (X, T).*

Proposition 5.6: *If λ is a fuzzy G_δ-set in a fuzzy Baire and fuzzy P-space (X, T), then (X, T) is a fuzzy second category set in (X, T).*

Proof: Letλ be a fuzzy G_δ-set in a fuzzy Baire and fuzzy P-space (*X*, *T*). Since (*X*, *T*) is a fuzzy P-space, the fuzzy set G_δ-setλ is a fuzzy open in (*X*, *T*). Again since (*X*, *T*) is a fuzzy Baire space, by theorem5.2, the open set λ is not a first category set in (*X*, *T*). Hence λ is a fuzzy second category set in (*X*, *T*).

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