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Reverse Degree Distance of Composite Graphs

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Abstract

The reverse degree distance of a connected graph G is defined as ${}^{r}D'(G) = 2(|V(G)|-1)\,|E(G)|\,d(G) - \frac{1}{2}\sum_{u,v\in V(G)}(d_G(u)+d_G(v))d_G(u,v)$, where $d(G)$ is the diameter of G. In this paper, we present the exact formulae for the reverse degree distance of some graph operations, such as tensor product, strong product and join of two connected graphs.

Keywords: *Reverse degree distance, Composite graph.*

1 Introduction

Let G be a simple connected graph with vertex set $V(G)$ and edge set $E(G)$. For $u \in V(G)$, let $d_G(u)$ be the degree of u in G and $d_G(u,v)$ is the distance between the vertices u and v in G. For a vertex v of G, the eccentricity $e(v)$ is the distance between v and a vertex farthest from v. The maximum eccentricity is its diameter, denoted by $d(G)$. The *degree distance* of G is defined as [6, 8, 9] $DD(G) = \frac{1}{2}\sum_{u,v\in V(G)}(d_G(u) + d_G(v))d_G(u,v)$.

It is a useful molecular descriptor [16]. Earlier as noted in [13, 15], this graph invariant appeared to be part of the molecular topological index (or Schultz index) [14], which may be expressed as $DD(G) + \sum_{u \in V(G)} d_G(u)^2$, see [9, 12], where the latter part $\sum_{u \in V(G)} d_G(u)^2$ is known as the first Zagreb index [10]. Thus the degree distance is also called the true Schultz index in chemical literature [4]. Tomescu [18] showed that the star is the unique graph with minimum degree distance in the class of connected graphs with n vertices. Further work on the minimum degree distance (especially for unicyclic and bicyclic graphs) may be found in A.I. Tomescu [17], Tomescu [19] and Bucicovschi and Cioaba [2]. Dankelmann et al. [3] gave asymptotically sharp upper bounds for the degree distance.

The Wiener index of G is, denoted by $W(G)$, defined as $W(G) = \frac{1}{2} \sum_{u,v \in V(G)} d_G(u, v)$. Gutman [9] showed that if G is a tree with n vertices, then $DD(G) = 4W(G) - n(n-1)$. Thus there is no need to study the degree distance for trees because this is equivalent to the study of the Wiener index, see [5]. Balaban et al. [1] introduced the concept of *reverse Wiener index,* which is defined to be $\Lambda(G) = \frac{|V(G)|(|V(G)|-1)d(G)}{2} - W(G)$. Let $\Lambda'(G) = \frac{(|V(G)|-1)^2 d(G)}{2} - W(G)$, which is a revised version of the reverse Wiener index of G.

The reverse degree distance of a connected graph G is defined in discrete mathematical chemistry as ${}^rD'(G) = 2(|V(G)| - 1)\,|E(G)|\,d(G) - \frac{1}{2} \sum_{u,v \in V(G)} (d_G(u) + d_G(v))d_G(u, v)$.

Some basic properties of the reverse degree distance have been established by Zhou and Trinajstic [21], and in particular, it was shown that the reverse degree distance satisfies the basic requirement to be a branching index usable in chemistry. In continuation to the study of the reverse degree distance, a natural starting point is the reverse degree distances of unicyclic graphs. In [24] the graphs with maximum reverse degrees distance in the class of unicyclic graphs with given girth, number of pendant vertices and maximum degree are determined. The degree distance and reverse degree distance of one tetragonal carbon nanocones are determined by Momen and Alaeiyan in [23]. In this paper, we present the exact formulae for the reverse degree distance of some graph operations, such as tensor product, strong product and join of two connected graphs.

2 Main Results

The first Zagreb index and second Zagerb index are defined as $M_1(G) = \sum_{u\in V(G)} d_G(u)^2$ and $M_2(G) = \sum_{uv\in E(G)} d_G(u)d_G(v)$. In fact, one can rewrite the first Zagreb index as $M_1(G) = \sum_{uv\in E(G)} (d_G(u) + d_G(v))$. Similarly, the first Zagreb coindex and second Zagerb coindex are defined as $\overline{M}_1(G) = \sum_{uv\notin E(G)} (d_G(u) + d_G(v))$ and $\overline{M}_2(G) = \sum_{uv\notin E(G)} d_G(u)d_G(v)$. The Zagreb indices are found to have appilications in QSPR and QSAR studies as well.

2.1 Tensor Product

For two simple graphs G and H their tensor product, denoted by $G \times H$, has vertex set $V(G) \times V(H)$ in which (g_1, h_1) and (g_2, h_2) are adjacent whenever g_1g_2 is an edge in G and h_1h_2 is an edge in H. Note that if G and H are connected graphs, then $G \times H$ is connected only if at least one of them is nonbipartite.

The proof of the following lemma follows easily from the properties and structure of $G \times K_r$. The lemma is used in the proof of the main theorem of this section.

Lemma 2.1. *Let G be a connected graph on $n \geq 2$ vertices. For any pair of vertices x_{ij}, $x_{kp} \in V(G \times K_r)$, $r \geq 3$*
(i) If $v_iv_k \in E(G)$, then

$$d_{G\times K_r}(x_{ij}, x_{kp}) = \begin{cases} 1, & \text{if } j \neq p, \\ 2, & \text{if } j = p \text{ and } v_iv_k \text{ is on a triangle of } G, \\ 3, & \text{if } j = p \text{ and } v_iv_k \text{ is not on a triangle of } G. \end{cases}$$

(ii) If $v_iv_k \notin E(G)$, then $d_{G\times K_r}(x_{ij}, x_{kp}) = d_G(v_i, v_k)$.
(iii) $d_{G\times K_r}(x_{ij}, x_{ip}) = 2$.

Theorem 2.2. *Let G be a connected graph with $n \geq 2$ vertices and m edges. Then ${}^rD'(G \times K_r) = 2(nr-1)mr(r-1)d(G) - r(r-1)\Big[rDD(G) + 4m(r-1) + M_1(G) + \sum_{u_iu_k\in E_2} d_G(u_i)d_G(u_k)\Big]$, where $r \geq 3$.*

Proof: Set $V(G) = \{u_1, u_2, \ldots, u_n\}$ and $V(K_r) = \{v_1, v_2, \ldots, v_r\}$. Let x_{ij} denote the vertex (u_i, v_j) of $G \times K_r$. The degree of the vertex x_{ij} in $G \times K_r$ is

$d_G(u_i)d_{K_r}(v_j)$, that is $d_{G\times K_r}(x_{ij}) = (r-1)d_G(u_i)$. By the definition of reverse degree distance

$$
\begin{aligned}
& {}^rD'(G\times K_r) = 2(|V(G\times K_r)|-1)\,|E(G\times K_r)|\,d(G\times K_r) \\
& -\frac{1}{2}\sum_{x_{ij},\,x_{kp}\,\in V(G\times K_r)} (d_{G\times K_r}(x_{ij}) + d_{G\times K_r}(x_{kp}))d_{G\times K_r}(x_{ij},x_{kp}) \\
= \;& 2(nr-1)mr(r-1)d(G) - \frac{1}{2}\sum_{i=0}^{n-1}\sum_{\substack{j,p=0\\ j\neq p}}^{r-1} (d_{G\times K_r}(x_{ij}) + d_{G\times K_r}(x_{ip}))d_{G\times K_r}(x_{ij},x_{ip}) \\
& -\frac{1}{2}\sum_{\substack{i,k=0\\ i\neq k}}^{n-1}\sum_{j=0}^{r-1}(d_{G\times K_r}(x_{ij}) + d_{G\times K_r}(x_{kj}))d_{G\times K_r}(x_{ij},x_{kj}) \\
& -\frac{1}{2}\sum_{\substack{i,k=0\\ i\neq k}}^{n-1}\sum_{\substack{j,p=0\\ j\neq p}}^{r-1} (d_{G\times K_r}(x_{ij}) + d_{G\times K_r}(x_{kp}))d_{G\times K_r}(x_{ij},x_{kp}). \qquad (1)
\end{aligned}
$$

We shall calculate the above sums are separately.

First we compute S_1. By Lemma 2.1, we have

$$
\begin{aligned}
S_1 &= \sum_{i=0}^{n-1}\sum_{\substack{j,p=0\\ j\neq p}}^{r-1} (d_{G\times K_r}(x_{ij}) + d_{G\times K_r}(x_{ip}))d_{G\times K_r}(x_{ij},x_{ip}) \\
&= \sum_{i=0}^{n-1}\sum_{\substack{j,p=0\\ j\neq p}}^{r-1} 2((r-1)d_G(u_i) + (r-1)d_G(u_i)) \\
&= 8r(r-1)^2 m.
\end{aligned}
$$

Let $E_1 = \{uv \in E(G)\,|\,uv \text{ is on a } C_3 \text{ in } G\}$ and $E_2 = E(G) - E_1$.

$$
\begin{aligned}
S_2' &= \sum_{\substack{i,k=0\\ i\neq k}}^{n-1} (d_{G\times K_r}(x_{ij}) + d_{G\times K_r}(x_{kj}))d_{G\times K_r}(x_{ij},x_{kj}) \\
&= \Bigg(\sum_{\substack{i,k=0\\ i\neq k\\ u_iu_k\notin E(G)}}^{n-1} + \sum_{\substack{i,k=0\\ i\neq k\\ u_iu_k\in E_1}}^{n-1} + \sum_{\substack{i,k=0\\ i\neq k\\ u_iu_k\in E_2}}^{n-1}\Bigg)(d_{G\times K_r}(x_{ij}) + d_{G\times K_r}(x_{kj}))d_{G\times K_r}(x_{ij},x_{kj})
\end{aligned}
$$

By Lemma 2.1, we obtain:

$$
\begin{aligned}
S_2' = \Bigg(& \sum_{\substack{i,k=0 \\ i\neq k \\ u_iu_k\notin E(G)}}^{n-1} (r-1)(d_G(u_i)+d_G(u_k))d_G(u_i,u_k) \\
& + \sum_{\substack{i,k=0 \\ i\neq k \\ u_iu_k\in E_1}}^{n-1} 2(r-1)(d_G(u_i)+d_G(u_k)) + \sum_{\substack{i,k=0 \\ i\neq k \\ u_iu_k\in E_2}}^{n-1} 3(r-1)(d_G(u_i)+d_G(u_k))\Bigg) \\
= \; & (r-1)\Big\{\Big(\sum_{\substack{i,k=0 \\ i\neq k \\ u_iu_k\notin E(G)}}^{n-1} (d_G(u_i)+d_G(u_k))d_G(u_i,u_k) + \sum_{\substack{i,k=0 \\ i\neq k \\ u_iu_k\in E_1}}^{n-1} (d_G(u_i)+d_G(u_k))d_G(u_i,u_k) \\
& + \sum_{\substack{i,k=0 \\ i\neq k \\ u_iu_k\in E_2}}^{n-1} (d_G(u_i)+d_G(u_k))d_G(u_i,u_k)\Big) + \sum_{\substack{i,k=0 \\ i\neq k \\ u_iu_k\in E_1}}^{n-1} (d_G(u_i)+d_G(u_k)) \\
& +2\sum_{\substack{i,k=0 \\ i\neq k \\ u_iu_k\in E_2}}^{n-1} (d_G(u_i)+d_G(u_k))\Big\} \\
= \; & (r-1)\Big\{2DD(G) + \sum_{\substack{i,k=0 \\ i\neq k \\ u_iu_k\in E(G)}}^{n-1} (d_G(u_i)+d_G(u_k)) + \sum_{\substack{i,k=0 \\ i\neq k \\ u_iu_k\in E_2}}^{n-1} (d_G(u_i)+d_G(u_k))\Big\} \\
= \; & (r-1)\Big\{2DD(G) + 2M_1(G) + 2\sum_{u_iu_k\in E_2} (d_G(u_i)+d_G(u_k))\Big\}. \qquad (2)
\end{aligned}
$$

Now summing (2) over $j = 0, 1, \ldots, r-1$, we get,

$$
S_2 = \sum_{j=0}^{r-1} S_2' = r(r-1)\Big[2DD(G) + 2M_2(G) + 2\sum_{u_iu_k\in E_2} (d_G(u_i)+d_G(u_k))\Big].
$$

By Lemma 2.1, we have

$$\begin{aligned} S_3 &= \sum_{\substack{i,k=0\\ i\neq k}}^{n-1} \sum_{\substack{j,p=0,\\ j\neq p}}^{r-1} (d_{G\times K_r}(x_{ij}) + d_{G\times K_r}(x_{kp})))d_{G\times K_r}(x_{ij}, x_{kp} \\ &= \sum_{\substack{i,k=0\\ i\neq k}}^{n-1} \sum_{\substack{j,p=0,\\ j\neq p}}^{r-1} ((r-1)d_G(u_i) + (r-1)d_G(u_k))d_G(u_i,u_k) \\ &= r(r-1)^2 \sum_{\substack{i,k=0\\ i\neq k}}^{n-1} (d_G(u_i) + d_G(u_k))d_G(u_i,u_k) \\ &= 2r(r-1)^2 DD(G). \end{aligned}$$

Using (1) and the sums in S_1 to S_3, respectively, we have,

$$\begin{aligned} {}^rD'(G\times K_r) &= 2(nr-1)mr(r-1)d(G) - r(r-1)\Big[rDD(G) + 4m(r-1) \\ &\quad +M_1(G) + \sum_{u_iu_k\in E_2} d_G(u_i)d_G(u_k)\Big]. \end{aligned}$$

Using Theorem 2.2, we have the following corollaries.

Corollary 2.3. *Let G be a connected graph on $n \geq 2$ vertices with m edges. If each edge of G is on a C_3, then* ${}^rD'(G\times K_r) = 2(nr-1)mr(r-1)d(G) - r(r-1)\Big[rDD(G) + 4(r-1)m + M_1(G)\Big]$, *where $r \geq 3$.*

Corollary 2.4. *If G is a connected triangle free graph on $n \geq 2$ vertices and m edges, then* ${}^rD'(G\times K_r) = 2(nr-1)mr(r-1)d(G) - r(r-1)\Big[rDD(G) + 4(r-1)m + 2M_1(G)\Big]$,*where $r \geq 3$.*

By direct calculations we obtain expressions for the values of the degree distance of K_n,P_n and C_n. $DD(K_n) = \frac{n(n-1)}{2}$ $DD(P_n) = \frac{1}{3}n(n-1)(2n-1)$ and $DD(C_n) = \frac{n^3}{2}$ when n is even, and $\frac{n(n^2-1)}{2}$ otherwise. One can observe that $M_1(C_n) = 4n$, $n \geq 3$, $M_1(P_1) = 0$, $M_1(P_n) = 4n-6$, $n > 1$ and $M_1(K_n) = n(n-1)^2$.

Using Corollaries 2.3 and 2.4, we obtain the reverse degree distance of the graphs $K_n \times K_r$ and $C_n \times K_r$.

Example 2.5. *(i)* ${}^rD'(K_n \times K_r) = n(n-1)r(r-1)(2-n-r)$.

(ii) $${}^rD'(C_n\times K_r) = \begin{cases} \frac{nr(r-1)}{2}\Big[2n(nr-1) - n^2r - 8r - 8\Big], & \textit{if } n \textit{ is even} \\ \frac{nr(r-1)}{2}\Big[2(n-1)(nr-1) - n^2r - 7r - 8\Big], & \textit{if } n > 3. \end{cases}$$

(*iii*) ${}^{r}D'(P_n \times K_r) = \frac{r(r-1)}{3}\Big[6(nr-1)(n-1)^2 - 2rn^3 + 3rn^2 - 13nr - 12n + 12r + 24\Big]$.

2.2 Strong Product

The strong product of graphs G and H, denoted by $G \boxtimes H$, is the graph with vertex set $V(G) \times V(H) = \{(u,v) : u \in V(G), v \in V(H)\}$ and $(u,x)(v,y)$ is an edge whenever (*i*) $u = v$ and $xy \in E(H)$, or (*ii*) $uv \in E(G)$ and $x = y$, or (*iii*) $uv \in E(G)$ and $xy \in E(H)$. Now we obtain the reverse degree distance of $G \boxtimes K_r$.

Theorem 2.6. *Let G be a connected graph with n vertices and m edges. Then* ${}^{r}D'(G \boxtimes K_r) = (nr-1)r((r-1)(m+n)+m)d(G) - r^3DD(G) - 2r^2(r-1)W(G) - 2r^2m(r-1) - nr(r-1)^2$.

Proof: Set $V(G) = \{u_1, u_2, \ldots, u_n\}$ and $V(K_r) = \{v_1, v_2, \ldots, v_r\}$. Let x_{ij} denote the vertex (u_i, v_j) of $G \boxtimes K_r$. The degree of the vertex x_{ij} in $G \boxtimes K_r$ is $d_G(u_i) + d_{K_r}(v_j) + d_G(u_i)d_{K_r}(v_j)$, that is $d_{G\boxtimes K_r}(x_{ij}) = rd_G(u_i) + (r-1)$. One can see that for any pair of vertices x_{ij}, $x_{kp} \in V(G \boxtimes K_r)$, $d_{G\boxtimes K_r}(x_{ij}, x_{ip}) = 1$ and $d_{G\boxtimes K_r}(x_{ij}, x_{kp}) = d_G(u_i, u_k)$. By the definition of reverse degree distance.

$$
\begin{aligned}
& {}^{r}D'(G \boxtimes K_r) = 2(|V(G \boxtimes K_r)| - 1)\,|E(G \boxtimes K_r)|\, d(G \boxtimes K_r) \\
& -\frac{1}{2} \sum_{x_{ij}, x_{kp} \in V(G\boxtimes K_r)} \Big(d_{G\boxtimes K_r}(x_{ij}) + d_{G\boxtimes K_r}(x_{kp})\Big) d_{G\boxtimes K_r}(x_{ij}, x_{kp}) \\
= \;& (nr-1)r((r-1)(m+n)+m)d(G) \\
& -\frac{1}{2} \sum_{i=0}^{n-1} \sum_{\substack{j,p=0 \\ j \neq p}}^{r-1} (d_{G\boxtimes K_r}(x_{ij}) + d_{G\boxtimes K_r}(x_{ip}))d_{G\boxtimes K_r}(x_{ij}, x_{ip}) \\
& -\frac{1}{2} \sum_{\substack{i,k=0 \\ i \neq k}}^{n-1} \sum_{j=0}^{r-1} (d_{G\boxtimes K_r}(x_{ij}) + d_{G\boxtimes K_r}(x_{kj}))d_{G\boxtimes K_r}(x_{ij}, x_{kj}) \\
& -\frac{1}{2} \sum_{\substack{i,k=0 \\ i \neq k}}^{n-1} \sum_{\substack{j,p=0 \\ j \neq p}}^{r-1} (d_{G\boxtimes K_r}(x_{ij}) + d_{G\boxtimes K_r}(x_{kp}))d_{G\boxtimes K_r}(x_{ij}, x_{kp}). \qquad (3)
\end{aligned}
$$

We shall obtain the above sums are separately.

$$\begin{aligned} S_1 &= \sum_{i=0}^{n-1} \sum_{\substack{j,p=0\\ j\neq p}}^{r-1} (d_{G\boxtimes K_r}(x_{ij}) + d_{G\boxtimes K_r}(x_{ip}))d_{G\boxtimes K_r}(x_{ij}, x_{ip}) \\ &= \sum_{i=0}^{n-1} \sum_{\substack{j,p=0\\ j\neq p}}^{r-1} \Big[rd_G(u_i) + (r-1) + rd_G(u_i) + (r-1)\Big] \\ &= 2nr(r-1)^2 + 4r^2m(r-1). \end{aligned}$$

$$\begin{aligned} S_2 &= \sum_{j=0}^{r-1} \sum_{\substack{i,k=0\\ i\neq k}}^{n-1} (d_{G\boxtimes K_r}(x_{ij}) + d_{G\boxtimes K_r}(x_{kj}))d_{G\boxtimes K_r}(x_{ij}, x_{kj}) \\ &= \sum_{j=0}^{r-1} \sum_{\substack{i,k=0\\ i\neq k}}^{n-1} \Big[(r-1) + rd_G(u_i) + (r-1) + rd_G(u_k)\Big] d_G(u_i, u_k) \\ &= \sum_{j=0}^{r-1} \sum_{\substack{i,k=0\\ i\neq k}}^{n-1} r(d_G(u_i) + d_G(u_k)))d_G(u_i, u_k) + \sum_{j=0}^{r-1} \sum_{\substack{i,k=0\\ i\neq k}}^{n-1} 2(r-1)d_G(u_i, u_k) \\ &= 2r^2DD(G) + 4r(r-1)W(G). \end{aligned}$$

$$\begin{aligned} S_3 &= \sum_{\substack{i,k=0\\ i\neq k}}^{n-1} \sum_{\substack{j,p=0,\\ j\neq p}}^{r-1} (d_{G\boxtimes K_r}(x_{ij}) + d_{G\boxtimes K_r}(x_{kp}))d_{G\boxtimes K_r}(x_{ij}, x_{kp}) \\ &= 4r(r-1)^2W(G) + 2r^2(r-1)DD(G). \end{aligned}$$

Using the sums S_1 to S_3 in (3), we have

$$\begin{aligned} {}^rD'(G \boxtimes K_r) &= (nr-1)r((r-1)(m+n) + m)d(G) - r^3DD(G) \\ &\quad -2r^2(r-1)W(G) - 2r^2m(r-1) - nr(r-1)^2. \end{aligned}$$

Using Theorem 2.6, we obtain the following example.

Example 2.7. $(i)^rD'(C_n \boxtimes K_r) = \begin{cases} \frac{r(nr-1)n^2(2r-1)}{2} \\ -\frac{nr(3r-1)}{4}(r(n^2+4)-4) & \textit{if } n \textit{ is even} \\ \frac{r(nr-1)n(n-1)(2r-1)}{2} \\ -\frac{nr(3r-1)}{4}(r(n^2+3)-4) & \textit{if } n \textit{ is odd.} \end{cases}$

$(ii)^rD'(P_n \boxtimes K_r) = r(nr-1)(n-1)((r-1)(2n-1)+(n-1)) - \frac{n(n-1)r^2(3nr-n-1)}{3} - r(r-1)(3nr-2r-n)$.

2.3 Join

The join $G+H$ of two graphs G and H with disjoint vertex sets $V(G)$ and $V(H)$ is the graph on the vertex set $V(G)\cup V(H)$ and the edge set $E(G)\cup E(H)\cup\{uv \mid u\in V(G),\ v\in V(H)\}$. Now we compute the reverse degree distance of join of two connected graphs.

Theorem 2.8. *Let G and H be graphs with n_1, n_2 vertices and m_1, m_2 edges, respectively. Then ${}^rD'(G+H) = 2(n_1+n_2-1)(m_1+m_2+n_1n_2) - M_1(G) - M_1(H) - \overline{M}_1(G) - \overline{M}_1(H) - 6n_1m_2 - 6n_2m_1 - n_1n_2(n_1+n_2-1)$.*

Proof: Set $V(G)=\{u_1,u_2,\ldots,u_n\}$ and $V(H)=\{v_1,v_2,\ldots,v_m\}$. By definition of the join of two graphs, one can see that,

$$d_{G+H}(x) = \begin{cases} d_G(x) + |V(H)|\,, & \text{if } x\in V(G)\\ d_H(x)+|V(G)|\,, & \text{if } x\in V(H)\end{cases}$$

and $$d_{G+H}(u,v) = \begin{cases} 0, & \text{if } u=v\\ 1, & \text{if } uv\in E(G) \text{ or } uv\in E(H) \text{ or } (u\in V(G) \text{ and } v\in V(H))\\ 2, & \text{otherwise.}\end{cases}$$

Therefore,

$$\begin{aligned} &{}^rD'(G+H) = 2(|V(G+H)|-1)\,|E(G+H)|\,d(G+H)\\ &-\frac{1}{2}\sum_{u,v\in V(G+H)}(d_{G+H}(u)+d_{G+H}(v))d_{G+H}(u,v)\\ =\;& 4(n_1n_2-1)(m_1+m_2-n_1n_2) - \sum_{uv\in E(G)}\big(d_G(u)+n_2+d_G(v)+n_2\big)\\ &-2\sum_{uv\notin E(G)}\big(d_G(u)+n_2+d_G(v)+n_2\big)\\ &-\sum_{uv\in E(H)}\big(d_H(u)+n_1+d_H(v)+n_1\big) - 2\sum_{uv\in E(H)}\big(d_H(u)+n_1+d_H(v)+n_1\big)\\ &-\sum_{u\in V(G),\ v\in V(H)}\big(d_G(u)+n_2+d_H(v)+n_1\big)\Big)\\ =\;& 2(n_1+n_2-1)(m_1+m_2+n_1n_2) - M_1(G) - M_1(H) - \overline{M}_1(G) - \overline{M}_1(H)\\ &-6n_1m_2 - 6n_2m_1 - n_1n_2(n_1+n_2-1). \end{aligned}$$

Using Theorem 2.8, we have the following corollary.

Corollary 2.9. *Let G be graph on n vertices and m edges. Then ${}^rD'(G+K_t) = (n+t-1)(2m+t(t-1)) - M_1(G) - \overline{M}_1(G) - t(t-1)(3n+t-1) - 6mt$.*

One can observe that $\overline{M_1}(K_n)=0$, $\overline{M_1}(P_n)=2(n-2)^2$ and $\overline{M_1}(C_n)=2n(n-3)$.

Using $M_1(C_n)$,$M_1(P_n)$,$\overline{M_1}(P_n)$ and $\overline{M_1}(C_n)$ and Corollary 2.9, we compute the formulae for reciprocal degree distance of fan and wheel graphs, $P_n + K_1$ and $C_n + K_1$.

Example 2.10. *(i)* ${}^{r}D'(P_n + K_1) = n^2 - 10n + 4$.
(ii) ${}^{r}D'(C_n + K_1) = n^2 - 22n$.

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