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Weighted Lacunary Statistical Convergence in Intuitionistic Fuzzy Normed Linear Spaces

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Abstract

In this paper, we define new summability method which is called as $\left(\overline{N}, p_r, \theta\right)$*-summability derived by weighted mean and lacunary sequence in intuitionistic fuzzy normed linear spaces. We use* $\left(\overline{N}, p_r, \theta\right)$*- summability to define new statistical convergence type which is named as weighted lacunary statistical convergence derived by weighted mean and lacunary sequence in intuitionistic fuzzy normed linear spaces, respectively. Moreover, we study the relation between weighted lacunary statistical convergence and lacunary statistical convergence, where the base space is IFNLS.*

Keywords: *Weighted mean, Statistical convergence, Lacunary sequence, Weighted lacunary statistical convergence, Weighted lacunary density, Intuitionistic fuzzy normed linear space.*

1 Introduction

Zadeh [1] introduced fuzzy set theory in 1965. Several authors applied fuzzy set theory on different branches of pure and applied mathematics. Saadati and Park [2] introduced the concept of intuitionistic fuzzy normed linear spaces based both on the concept of intuitionistic fuzzy sets given by Atanassov [3] and the concept of fuzzy normed linear spaces given by Felbin [4].

Steinhaus [5] and Fast [6] defined the notion of statistical convergence to generalize the concept of convergence of sequences as follows:

Let N be set of natural numbers, $K \subseteq N$ and $K_n = \{k \leq n : k \in K\}$. The natural density of K is defined by $\delta(K) = \lim\limits_{n\to\infty} n^{-1}|K_n|$ if the limit exists, where $|K_n|$ denotes the cardinality of K_n. A sequence $x = (x_k)$ is statistical convergent to L if for each $\varepsilon > 0$, the set $\{k \in N : |x_k - L| \geq \varepsilon\}$ has natural density zero, i.e. for each $\varepsilon > 0$, $\lim\limits_{n\to\infty} \frac{1}{n}|\{j \leq n : |x_j - L| \geq \varepsilon\}| = 0$. In this case, we write $S - \lim x = L$.

Many years later, some researchers investigated relations between statistical convergence with some summability methods. Invetigations in this direction can be found in [7, 9]. In 1993, Fridy and Orhan [10] introduced the idea of lacunary statistical convergence. Karakuş et al. [11] and Mursaleen and Mohiuddine [12] extended the idea of statistical convergence and the idea of lacunary statistical convergence with respect to the intuitionistic fuzzy normed linear spaces, respectively. Using these definitions, a lot of researchers gave some results [13-20].

Moricz and Orhan [21] introduced statistical summability $\left(\overline{N}, p_n\right)$ as follows:

Let $p = (p_k)_{k=0}^{\infty}$ be a sequence of nonnegatif numbers such that $p_0 > 0$ and $P_n = \sum\limits_{k=0}^{n} p_k \to \infty$ as $n \to \infty$. Set $t_n = \frac{1}{P_n}\sum\limits_{k=0}^{n} p_k x_k,\ n = 0, 1, 2, ...$

$x = (x_k)$ is statistically summable to L by the weighted mean method determined by the sequence (p_k) or briefly statistically summable $\left(\overline{N}, p_n\right)$ if $S - \lim\limits_{n\to\infty} t_n = L$. In this case, we write $\overline{N}(S) - \lim x = L$. We denote by $\overline{N}(S)$ the set of all sequences which are statistically summable $\left(\overline{N}, p_n\right)$. In this case, we write $\overline{N}(S) - \lim x = L$. We denote by $\overline{N}(S)$ the set of all sequences which are statistically summable $\left(\overline{N}, p_n\right)$. In addition, if $\lim\limits_{n\to\infty} \frac{1}{P_n}\sum\limits_{k=1}^{n} p_k|x_k - L| = 0$, then the sequence $x = (x_k)$ is said to be strongly $\left(\overline{N}, p_n\right)$- summable to L. In this case, we write $|\overline{N}, p_n| - \lim x = L$.

Karakaya and Chisti [22] used $\left(\overline{N}, p_n\right)$- summability to define the concept of weighted statistical convergence which is extended definition of statistical convergence. Mursaleen et al. [23] altered the definition of weighted statistical convergence and studied its relation with the concept of $\left(\overline{N}, p_n\right)$- summability. Mursaleen et al. [23] proved that definition must be as follows:

Let define the weighted density of $K \subseteq N$ by $\delta_{\overline{N}}(K) = \lim_{n\to\infty} \frac{1}{P_n} |K_{P_n}|$ if the limit exists. A sequence $x = (x_k)$ is weighted statistically convergent (or $S_{\overline{N}}$ -convergent) to L if, for every $\varepsilon > 0$,

$$\delta_{\overline{N}}(\{k \in N : p_k|x_k - L| \geq \varepsilon\}) = 0,$$

or equivalently

$$\lim_{n\to\infty} \frac{1}{P_n} |\{k \leq P_n : p_k |x_k - L| \geq \varepsilon\}| = 0.$$

In this case, we write $S_{\overline{N}}- \lim x = L$.

Konca and Başarır [24] introduced the new summability method, called $\left|\overline{N}, p_r, \theta\right|$- summability and investigated some relations $\left|\overline{N}, p_r, \theta\right|$- summability with the lacunary strong convergence and $\left|\overline{N}, p_n\right|$- summability. Moreover, they defined a new statistical convergence type, called weighted lacunary statistical convergence and studied some relations between this concept with the concept of lacunary statistical convergence and weighted statistical convergence.

In this paper, we define new summability method, namely $\left(\overline{N}, p_r, \theta\right)$- summability derived by weighted mean and lacunary sequence in intuitionistic fuzzy normed linear space. We use $\left(\overline{N}, p_r, \theta\right)$- summability to define new statistical convergence type which is called as weighted lacunary statistical convergence derived by weighted mean and lacunary sequence in intuitionistic fuzzy normed linear space, respectively. Moreover, we study some relations between this concept with the concept of lacunary statistical convergence and weighted statistical convergence, where the base space is intuitionistic fuzzy normed linear space.

2 Basic Definitions

In this section, we give the basic definitions which we need to introduce the concept of weighted lacunary statistical convergence and new summability method in intuitionistic fuzzy normed linear spaces.

Definition 2.1 *[25] A binary operation* $* : [0, 1] \times [0, 1] \to [0, 1]$ *is said to be a continuos t-norm if it satisfies the following conditions:*

(i) $*$ is associative and commutative,

(ii) $*$ is continuous,

(iii) $a * 1 = a$ for all $a \in [0,1]$,

(iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for each $a,\ b,\ c, d \in [0,1]$.

Definition 2.2 *[25] A binary operation $\circ : [0,1] \times [0,1] \to [0,1]$ is said to be a continuos t-conorm if it satisfies the following conditions:*

(i) $\circ$ is associative and commutative,

(ii) $\circ$ is continuous,

(iii) $a \circ 0 = a$ for all $a \in [0,1]$,

(iv) $a \circ b \leq c \circ d$ whenever $a \leq c$ and $b \leq d$ for each $a,\ b,\ c, d \in [0,1]$.

Definition 2.3 *[22] The five-tuple $(X,\ \mu, \upsilon, *, \circ)$ is said to be intuitionistic fuzzy normed linear space (or shortly IFNLS) is where X is a linear space over a field F, $*$ is a continuous t-norm, $\circ$ is a continuous t-conorm, μ, υ are fuzzy sets on $X \times (0,\infty)$, μ denotes the degree of membership and υ denotes the degree of nonmembership of $(x,t) \in X \times (0,\infty)$ satisfying the following conditions for every $x, y \in X$ and $s, t > 0$:*

(i) $\mu(x,t) + \upsilon(x,t) \leq 1$,

(ii) $\mu(x,t) > 0$,

(iii) $\mu(x,t) = 1$ if and only if $x = 0$,

(iv) $\mu(\alpha x,t) = \mu\left(x, \frac{t}{|\alpha|}\right)$ if $\alpha \neq 0$,

(v) $\mu(x,t) * \mu(y,s) \leq \mu(x+y, t+s)$,

(vi) $\mu(x,.) : (0,\infty) \to [0,1]$ is continuous,

(vii) $\lim_{t\to\infty} \mu(x,t) = 1$ and $\lim_{t\to 0} \mu(x,t) = 0$,

(viii) $\upsilon(x,t) < 1$,

(ix) $\upsilon(x,t) = 0$ if and only if $x = 0$,

(x) $\upsilon(\alpha x,t) = \upsilon\left(x, \frac{t}{|\alpha|}\right)$ if $\alpha \neq 0$,

(xi) $\upsilon(x,t) \circ \upsilon(y,s) \geq \upsilon(x+y, s+t)$,

(xii) $\upsilon(x,.) : (0,\infty) \to [0,1]$ is continuous,

(xiii) $\lim_{t\to\infty} \upsilon(x,t) = 0$ and $\lim_{t\to 0} \upsilon(x,t) = 1$.

In this case, (μ, υ) is called intuitionistic fuzzy norm.

Example 2.1 *[22] Let $(R, |.|)$ denote the space of real numbers with the usual norm, and let $a * b = ab$ and $a \circ b = \min\{a+b, 1\}$ for all $a, b \in [0,1]$. For all $x \in R$, every $t > 0$, consider*

$$\mu(x,t) = \tfrac{t}{t+|x|} \text{ and } \upsilon(x,t) = \tfrac{|x|}{t+|x|}.$$

Then $(X,\ \mu, \upsilon, *, \circ)$ is an IFNLS.

Definition 2.4 *[22] Let $(X,\ \mu, \upsilon, *, \circ)$ be an IFNLS. A sequence $x = (x_k)$ in X is convergent to $L \in X$ with respect to the intuitionistic fuzzy norm (μ, υ) if, for every $\varepsilon > 0$ and $t > 0$, there exists $k_0 \in N$ such that $\mu(x_k - L, t) > 1 - \varepsilon$ and $\upsilon(x_k - L, t) < \varepsilon$ for all $k \geq k_0$ where $k \in N$. It is denoted by $(\mu, \upsilon) - \lim x = L$.*

Theorem 2.1 *[26] Let $(X,\ \mu, \upsilon, *, \circ)$ be an IFNLS. Then, a sequence $x = (x_k)$ in X is convergent to $L \in X$ if and only if $\lim_{k\to\infty} \mu(x_k - L, t) = 1$ and $\lim_{k\to\infty} \upsilon(x_k - L, t) = 0$.*

Definition 2.5 *[12] A lacunary sequence is an increasing integer sequence $\theta = (k_r)$ such that $k_0 = 0$ and $h_r = k_r - k_{r-1} \to \infty$ as $r \to \infty$. The intervals determined by θ will be denoted by $I_r = (k_{r-1}, k_r]$ and the ratio $\frac{k_r}{k_{r-1}}$ will be abbreviated as q_r.*

Definition 2.6 *[13] Let $(X,\ \mu, \upsilon, *, \circ)$ be an IFNLS and $\theta = (k_r)$ be a lacunary sequence. A sequence $x = (x_k)$ in X is said to be lacunary convergent to $L \in X$ with respect to the intuitionistic fuzzy norm (μ, υ) if, for every $\varepsilon > 0$ and $t > 0$, there exists $r_0 \in N$ such that*

$$\frac{1}{h_r} \sum_{k \in I_r} \mu(x_k - L, t) > 1 - \varepsilon$$

and

$$\frac{1}{h_r} \sum_{k \in I_r} \upsilon(x_k - L, t) < \varepsilon$$

for all $r \geq r_0$.

Definition 2.7 *[12] Let $(X,\ \mu, \upsilon, *, \circ)$ be an IFNLS and $\theta = (k_r)$ be a lacunary sequence. A sequence $x = (x_k)$ in X is said to be lacunary statistical convergent to $L \in X$ with respect to the intuitionistic fuzzy norm (μ, υ) if, for every $\varepsilon > 0$ and $t > 0$,*

$$\lim_{r\to\infty} \frac{1}{h_r} \left|\{k \in I_r : \mu(x_k - L, t) \leq 1 - \varepsilon \text{ or } \upsilon(x_k - L, t) \geq \varepsilon\}\right| = 0.$$

In this case, we write $(\mu, \upsilon)^{\theta} - \lim x = L$.

Definition 2.8 *[24] Let $\theta = (k_r)$ be a lacunary sequence, $p = (p_k)$ be a sequence of positive real numbers such that $H_r := \sum_{k \in I_r} p_k$, $P_{k_r} := \sum_{k \in (0,k_r]} p_k$, $P_{k_{r-1}} := \sum_{k \in (0,k_{r-1}]} p_k$, $Q_r := \frac{P_{k_r}}{P_{k_{r-1}}}$, $P_0 = 0$ and the intervals determined by θ and (p_k) are denoted by $I_r^{'} = \left(P_{k_{r-1}}, P_{k_r}\right]$. Hence $H_r = P_{k_r} - P_{k_{r-1}}$. If we take $p_k = 1$ for all $k \in N$, then H_r, P_{k_r}, $P_{k_{r-1}}$, Q_r and $I_r^{'}$ reduce to h_r, k_r, k_{r-1}, q_r and I_r respectively.*

If $\theta = (k_r)$ is a lacunary sequence and $P_n \to \infty$ as $n \to \infty$, then $\theta^{'} = (P_{k_r})$ is a lacunary sequence, that is $P_0 = 0$, $0 < P_{k_{r-1}} < P_{k_r}$ and $H_r = P_{k_r} - P_{k_{r-1}} \to \infty$ as $r \to \infty$.

Definition 2.9 *[24] Let $\theta = (k_r)$ be a lacunary sequence. Then, a sequence $x = (x_k)$ is $\left(\overline{N}, p_r, \theta\right)$- summable to L if $\lim_{r\to\infty} \frac{1}{H_r} \sum_{k \in I_r} p_k (x_k - L) = 0$. In this case, we write $\left(\overline{N}, p_r, \theta\right) - \lim x = L$. In addition, if $\lim_{r\to\infty} \frac{1}{H_r} \sum_{k \in I_r} p_k |x_k - L| = 0$, then the sequence $x = (x_k)$ is said to be strongly $\left(\overline{N}, p_r, \theta\right)$ - summable or $\left|\overline{N}, p_r, \theta\right|$ - summable to L. In this case, we write $\left|\overline{N}, p_r, \theta\right| - \lim x = L$.*

Definition 2.10 *[24] Let define the weighted lacunary density of $K \subseteq N$ by*

$$\delta_{\left(\overline{N},\theta\right)}(K) = \lim_{r\to\infty} \frac{1}{H_r} |K_r(\varepsilon)|$$

if, the limit exists. A sequence $x = (x_k)$ is said to be weighted lacunary statistically convergent to L if, for every $\varepsilon > 0$, the set

$$K_r(\varepsilon) = \left\{ k \in I_r^{'} : p_k |x_k - L| \geq \varepsilon \right\}$$

has weighted lacunary density zero, i.e.

$$\lim_{r\to\infty} \frac{1}{H_r} \left| \left\{ k \in I_r^{'} : p_k |x_k - L| \geq \varepsilon \right\} \right| = 0.$$

In this case, we write $S_{\left(\overline{N},\theta\right)} - \lim x = L$.

3 Weighted Lacunary Statistical Convergence in IFNLS

In this section, we define new summability method in IFNLS. Later, we obtain new statistical convergence type using certain summability method and investigate the relation between these concepts, where the base space is IFNLS.

In the following definition, the concept of $\left(\overline{N}, p_n\right)$- summability in IFNLS was given.

Definition 3.1 *[27] Let $(X, \mu, \upsilon, *, \circ)$ be an IFNLS. A sequence $x = (x_k)$ in X is said to be $\left(\overline{N}, p_n\right)$- summable to $L \in X$ with respect to the intuitionistic fuzzy norm (μ, υ) (or $\left(\overline{N}, p_n\right)^{(\mu,\upsilon)}$- summable to $L \in X$) if, for every $\varepsilon > 0$ and $t > 0$, there exists $n_0 \in N$ such that*

$$\frac{1}{P_n}\sum_{k=1}^{n} \mu\left(p_k\left(x_k - L\right), t\right) > 1 - \varepsilon$$

and

$$\frac{1}{P_n}\sum_{k=1}^{n} \upsilon\left(p_k\left(x_k - L\right), t\right) < \varepsilon$$

for all $n \geq n_0$. In this case, we write $\left(\overline{N}, p_n\right)^{(\mu,\upsilon)} - \lim x = L$. If we take $p_k = 1$ for all $k \in N$, then $\left(\overline{N}, p_n\right)$- summability with respect to the intuitionistic fuzzy norm (μ, υ) is reduced to $(C, 1)$- summability with respect to the intuitionistic fuzzy norm (μ, υ) and in this case, we write $(C, 1)^{(\mu,\upsilon)} - \lim x = L$.

Now, we give the definition of $\left(\overline{N}, p_r, \theta\right)$- summability in IFNLS.

Definition 3.2 *Let $(X, \mu, \upsilon, *, \circ)$ be an IFNLS and $\theta = (k_r)$ be a lacunary sequence. A sequence $x = (x_k)$ in X is said to be $\left(\overline{N}, p_r, \theta\right)$- summable to $L \in X$ with respect to the intuitionistic fuzzy norm (μ, υ) (or $\left(\overline{N}, p_r, \theta\right)^{(\mu,\upsilon)}$- summable to $L \in X$) if, for every $\varepsilon > 0$ and $t > 0$, there exists $r_0 \in N$ such that*

$$\frac{1}{H_r}\sum_{k \in I_r} \mu\left(p_k\left(x_k - L\right), t\right) > 1 - \varepsilon$$

and

$$\frac{1}{H_r}\sum_{k \in I_r} \upsilon\left(p_k\left(x_k - L\right), t\right) < \varepsilon$$

for all $r \geq r_0$. In this case, we write $\left(\overline{N}, p_r, \theta\right)^{(\mu,\upsilon)} - \lim x = L$.

The following results are obtained for some special cases in IFNLS:

i) Let $p_k = 1$ for all $k \in N$, then $\left(\overline{N}, p_r, \theta\right)$- summability with respect to the intuitionistic fuzzy norm (μ, υ) is reduced to lacunary convergence with respect to the intuitionistic fuzzy norm (μ, υ) given in [13].

ii) Let $\theta = (k_r) = (2^r)$ for $r > 0$. Then $\left(\overline{N}, p_r, \theta\right)^{(\mu,\upsilon)}$- summability is reduced to $\left(\overline{N}, p_n\right)^{(\mu,\upsilon)}$ - summability.

iii) Let $p_k = 1$ for all $k \in N$ and $\theta = (k_r) = (2^r)$ for $r > 0$. Then $\left(\overline{N}, p_r, \theta\right)$- summability is reduced to $(C, 1)$- summability, where the base space is IFNLS.

Now, we give following theorem giving relation between $\left(\overline{N}, p_n\right)$- summability and $\left(\overline{N}, p_r, \theta\right)$- summability in IFNLS.

Theorem 3.1 *Let $(X, \mu, \upsilon, *, \circ)$ be an IFNLS, $\theta = (k_r)$ be a lacunary sequence and $\frac{P_n}{n} \geq 1$ for all $n \in N$. If $\left(\overline{N}, p_r, \theta\right)^{(\mu,\upsilon)} - \lim x = L$, then $\left(\overline{N}, p_n\right)^{(\mu,\upsilon)} - \lim x = L$.*

Proof: Let $\left(\overline{N}, p_r, \theta\right)^{(\mu,\upsilon)} - \lim x = L$. Then for every $\varepsilon > 0$ and $t > 0$, there exists $j_0 \in N$ such that

$$\frac{1}{H_j} \sum_{k \in I_j} \mu\left(p_k\left(x_k - L\right), t\right) > 1 - \varepsilon \tag{1}$$

and

$$\frac{1}{H_j} \sum_{k \in I_j} \upsilon\left(p_k\left(x_k - L\right), t\right) < \varepsilon \tag{2}$$

for all $j > j_0$. By inequality (1), we can find positive constant $M \in (0, 1)$ such that

$$\frac{1}{H_j} \sum_{k \in I_j} \mu\left(p_k\left(x_k - L\right), t\right) \geq 1 - M \quad . \tag{3}$$

Therefore, using inequalities (1) and (3) and $P_{k_r} \to \infty$ as $r \to \infty$, we have

$$\begin{aligned}
&\tfrac{1}{P_{k_r}} \sum_{k=1}^{k_r} \mu\left(p_k\left(x_k - L\right), t\right) \\
&= \tfrac{1}{P_{k_r}} \Bigg(\sum_{k \in I_1} \mu\left(p_k\left(x_k - L\right), t\right) + ... + \sum_{k \in I_{j_0}} \mu\left(p_k\left(x_k - L\right), t\right) \\
&+ \sum_{k \in I_{j_0+1}} \mu\left(p_k\left(x_k - L\right), t\right) + ... + \sum_{k \in I_r} \mu\left(p_k\left(x_k - L\right), t\right) \Bigg) \\
&\geq \tfrac{(1-M)}{P_{k_r}} \left(H_1 + H_2 + ... + H_{j_0}\right) + \tfrac{(1-\varepsilon)}{P_{k_r}} \left(H_{j_0+1} + H_{j_0+2} + ... + H_r\right)
\end{aligned}$$

$= \frac{(1-M)}{P_{k_r}} \left(P_{k_1} - P_{k_0} + P_{k_2} - P_{k_1} + ... + P_{k_{j_0}} - P_{k_{j_0-1}} \right)$

$+ \frac{(1-\varepsilon)}{P_{k_r}} \left(P_{k_{j_0+1}} - P_{k_{j_0}} + P_{k_{j_0+2}} - P_{k_{j_0+1}} + ... + P_{k_r} - P_{k_{r-1}} \right)$

$= (1-M) \frac{P_{k_{j_0}}}{P_{k_r}} + (1-\varepsilon) \frac{P_{k_r} - P_{k_{j_0}}}{P_{k_r}}$

$> 1 - \varepsilon (1-M)$

$> 1 - \varepsilon^{'}$.

Following similar steps and using inequation (2), it can be shown that, for every $\varepsilon > 0$ and $t > 0$, there exists $r_0 \in N$ such that

$$\frac{1}{P_{k_r}} \sum_{k=1}^{k_r} \upsilon \left(p_k \left(x_k - L \right), t \right) < \varepsilon$$

for all $r \geq r_0$. Hence, we obtain $\left(\overline{N}, p_n \right)^{(\mu,\upsilon)} - \lim x = L$.

Theorem 3.2 *Let $(X, \mu, \upsilon, *, \circ)$ be an IFNLS, $\theta = (k_r)$ be a lacunary sequence.*

(i) If $p_k \leq 1$ for all $k \in N$, $\liminf_{r\to\infty} \frac{H_r}{h_r} > 0$ and $(\mu, \upsilon)^{\theta} - \lim x = L$, then $\left(\overline{N}, p_r, \theta \right)^{(\mu,\upsilon)} - \lim x = L$.

(ii) If $p_k \geq 1$ for all $k \in N$, $\limsup_{r\to\infty} \frac{H_r}{h_r} < \infty$ and $\left(\overline{N}, p_r, \theta \right)^{(\mu,\upsilon)} - \lim x = L$, then $(\mu, \upsilon)^{\theta} - \lim x = L$.

Proof:
(i) Suppose that $p_k \leq 1$ for all $k \in N$ and $\liminf_{r\to\infty} \frac{H_r}{h_r} > 0$. By the assumption, there exists $\delta > 0$ such that $0 < \delta \leq \frac{H_r}{h_r} \leq 1$ for all $r \in N$. Let $(\mu, \upsilon)^{\theta} - \lim x = L$, then for every $\varepsilon > 0$ and $t > 0$, there exists $r_0 \in N$ such that

$$\frac{1}{h_r} \sum_{k \in I_r} \mu \left(x_k - L, t \right) > 1 - \varepsilon \tag{4}$$

and

$$\frac{1}{h_r} \sum_{k \in I_r} \upsilon \left(x_k - L, t \right) < \varepsilon \tag{5}$$

for all $r \geq r_0$. Hence, using inequation (4), for every $\varepsilon > 0$ and $t > 0$,

$$\frac{1}{H_r} \sum_{k \in I_r} \mu \left(p_k \left(x_k - L \right), t \right) \geq \frac{1}{h_r} \sum_{k \in I_r} \mu \left(x_k - L, t \right) > 1 - \varepsilon \,. \tag{6}$$

Similarly for every $\varepsilon > 0$ and $t > 0$,

$$\begin{aligned} \frac{1}{H_r}\sum_{k\in I_r}\upsilon\left(p_k\left(x_k-L\right),t\right) &\leq \frac{1}{\delta}.\frac{1}{h_r}\sum_{k\in I_r}\upsilon\left(x_k-L,t\right) \\ &< \frac{1}{\delta}.\varepsilon \qquad (7) \\ &= \varepsilon^{'} \end{aligned}$$

As a result of inequalities (6) and (7), we have $\left(\overline{N}, p_r, \theta\right)^{(\mu,\upsilon)} - \lim x = L$.

(ii) Suppose that $p_k \geq 1$ for all $k \in N$ and $\limsup_{r\to\infty} \frac{H_r}{h_r} < \infty$. By the assumption, there exists positive constant K such that $1 \leq \frac{H_r}{h_r} \leq K < \infty$ for all $r \in N$. Let $\left(\overline{N}, p_r, \theta\right)^{(\mu,\upsilon)} - \lim x = L$, then for every $\varepsilon > 0$ and $t > 0$, there exists $r_0 \in N$ such that

$$\frac{1}{H_r}\sum_{k\in I_r}\mu\left(p_k\left(x_k-L\right),t\right) > 1-\varepsilon \qquad (8)$$

and

$$\frac{1}{H_r}\sum_{k\in I_r}\upsilon\left(p_k\left(x_k-L\right),t\right) < \varepsilon \qquad (9)$$

for all $r \geq r_0$. Hence, for every $\varepsilon > 0$ and $t > 0$,

$$\frac{1}{h_r}\sum_{k\in I_r}\mu\left(x_k-L,t\right) > \frac{1}{H_r}\sum_{k\in I_r}\mu\left(p_k\left(x_k-L\right),t\right) > 1-\varepsilon. \qquad (10)$$

Similarly, we have using inequality (9)

$$\frac{1}{h_r}\sum_{k\in I_r}\upsilon\left(x_k-L,t\right) \leq K.\frac{1}{H_r}\sum_{k\in I_r}\upsilon\left(p_k\left(x_k-L\right),t\right) < K.\frac{1}{H_r}.\varepsilon = \varepsilon^{'}. \qquad (11)$$

As a result of inequalities (10) and (11), we have $(\mu,\upsilon)^{\theta} - \lim x = L$.

In the following definition, the concept of weighted statistical convergence with respect to the intuitionistic fuzzy norm (μ,υ) was given.

Definition 3.3 *[27] Let $(X, \mu, \upsilon, *, \circ)$ be an IFNLS. A sequence $x = (x_k)$ in X is said to be weighted statistically convergent to $L \in X$ with respect to the intuitionistic fuzzy norm (μ,υ) (or $S_{\overline{N}}^{(\mu,\upsilon)}$- convergent to $L \in X$) if, for every $\varepsilon > 0$ and $t > 0$,*

$$\delta_{\overline{N}}\left(\{k\in N : \mu\left(p_k(x_k-L),t\right) \leq 1-\varepsilon \text{ or } \upsilon\left(p_k(x_k-L),t\right) \geq \varepsilon\}\right) = 0, \qquad (12)$$

or equivalently

$$\lim_{n\to\infty}\frac{1}{P_n}\left|\left\{k\le P_n:\mu\left(p_k\left(x_k-L\right),t\right)\le 1-\varepsilon \text{ or } \upsilon\left(p_k\left(x_k-L\right),t\right)\ge\varepsilon\right\}\right|=0.$$

In this case, we write $S_{\overline{N}}^{(\mu,\upsilon)}-\lim x=L$. If we take $p_k=1$ for all $k\in N$, then weighted statistically convergence is reduced to statistically convergence with respect to the intuitionistic fuzzy norm (μ,υ) defined by Karakuş et al. [11] and in this case, we write $S^{(\mu,\upsilon)}-\lim x=L$.

Now, we give new concept of statistical convergence which is defined as weighted lacunary statistical convergence in intuitionistic fuzzy normed linear spaces. We prove some relation between weighted lacunary statistical convergence with the concepts of lacunary statistical convergence and weighted statistical convergence in intuitionistic fuzzy normed linear spaces.

Definition 3.4 *Let $(X,\mu,\upsilon,*,\circ)$ be an IFNLS and $\theta=(k_r)$ be a lacunary sequence. A sequence $x=(x_k)$ in X is said to be weighted lacunary statistical convergent to $L\in X$ with respect to the intuitionistic fuzzy norm (μ,υ) (or $S_{(\overline{N},\theta)}^{(\mu,\upsilon)}$- convergent to $L\in X$) if, for every $\varepsilon>0$ and $t>0$,*

$$\delta_{(\overline{N},\theta)}\left(\left\{k\in N:\mu\left(p_k(x_k-L),t\right)\le 1-\varepsilon \text{ or } \upsilon\left(p_k(x_k-L),t\right)\ge\varepsilon\right\}\right)=0, \quad (13)$$

or equivalently

$$\lim_{r\to\infty}\frac{1}{H_r}\left|\left\{k\in I_r{}':\mu\left(p_k\left(x_k-L\right),t\right)\le 1-\varepsilon \text{ or } \upsilon\left(p_k\left(x_k-L\right),t\right)\ge\varepsilon\right\}\right|=0.$$

In this case, we write $S_{(\overline{N},\theta)}^{(\mu,\upsilon)}-\lim x=L$. If we take $p_k=1$ for all $k\in N$, then weighted lacunary statistical convergence is reduced to lacunary statistical convergence with respect to the intuitionistic fuzzy norm (μ,υ) defined by Mursaleen and Mohiuddine [12] and in this case, we write $S_{\theta}^{(\mu,\upsilon)}-\lim x=L$.

Theorem 3.3 *Let $(X,\mu,\upsilon,*,\circ)$ be an IFNLS, $\theta=(k_r)$ be a lacunary sequence and $\liminf_{r\to\infty}Q_r>1$. If $S_{\overline{N}}^{(\mu,\upsilon)}-\lim x=L$, then $S_{(\overline{N},\theta)}^{(\mu,\upsilon)}-\lim x=L$.*

Proof: Suppose that $\liminf_{r\to\infty}Q_r>1$, then there exists a $\delta>0$ such that $Q_r\ge 1+\delta$ for sufficiently large values of r, which implies that $\frac{H_r}{P_{k_r}}\ge\frac{\delta}{1+\delta}$. If $S_{\overline{N}}^{(\mu,\upsilon)}-\lim x=L$, then for sufficiently large values of r, we have

$$\frac{1}{P_{k_r}}\left|\left\{k\le P_{k_r}:\mu\left(p_k\left(x_k-L\right),t\right)\le 1-\varepsilon \text{ or } \upsilon\left(p_k\left(x_k-L\right),t\right)\ge\varepsilon\right\}\right|$$

$$\geq \frac{1}{P_{k_r}} \left|\left\{P_{k_{r-1}} < k \leq P_{k_r} \colon \mu\left(p_k\left(x_k - L\right), t\right) \leq 1 - \varepsilon \text{ or } \upsilon\left(p_k\left(x_k - L\right), t\right) \geq \varepsilon\right\}\right|$$

$$= \frac{H_r}{P_{k_r}} \Big(\frac{1}{H_r} \left|\left\{P_{k_{r-1}} < k \leq P_{k_r} \colon \mu\left(p_k\left(x_k - L\right), t\right) \leq 1 - \varepsilon \right.\right.$$

$$\left.\left. \text{or } \upsilon\left(p_k\left(x_k - L\right), t\right) \geq \varepsilon\right\}\right| \Big)$$

$$\geq \frac{\delta}{1+\delta} \left(\frac{1}{H_r} \left| \left\{ k \in I_r{}' \colon \mu\left(p_k\left(x_k - L\right), t\right) \leq 1 - \varepsilon \text{ or } \upsilon\left(p_k\left(x_k - L\right), t\right) \geq \varepsilon \right\} \right| \right).$$

Hence, we obtain that $S^{(\mu,\upsilon)}_{\left(\overline{N},\theta\right)} - \lim x = L$ by taking the limit as $r \to \infty$.

Theorem 3.4. *Let $(X, \mu, \upsilon, *, \circ)$ be an IFNLS, $\theta = (k_r)$ be a lacunary sequence and $\limsup_{r\to\infty} Q_r < \infty$. If $S^{(\mu,\upsilon)}_{\left(\overline{N},\theta\right)} - \lim x = L$ then $S^{(\mu,\upsilon)}_{\overline{N}} - \lim x = L$.*

Proof: Suppose that $\limsup_{r\to\infty} Q_r < \infty$. Hence, there exists $K > 0$ such that $Q_r \leq K$ for all $r \in N$. Let $S^{(\mu,\upsilon)}_{\left(\overline{N},\theta\right)} - \lim x = L$ and let

$$N_r \colon = \left| \left\{ k \in I_r{}' : \mu\left(p_k\left(x_k - L\right), t\right) \leq 1 - \varepsilon \text{ or } \upsilon\left(p_k\left(x_k - L\right), t\right) \geq \varepsilon \right\} \right|. \quad (14)$$

By (13) and (14), for every $\varepsilon > 0$ and $t > 0$, there exists $r_0 \in N$ such that $\frac{N_r}{H_r} < \varepsilon$ for all $r > r_0$. Now, let define

$$M \colon = \max\left\{N_r : 1 \leq r \leq r_0\right\} \quad (15)$$

and $n \in N$ such that $k_{r-1} < n \leq k_r$. Hence

$$\frac{1}{P_n} \left|\left\{k \leq P_n : \mu\left(p_k\left(x_k - L\right), t\right) \leq 1 - \varepsilon \text{ or } \upsilon\left(p_k\left(x_k - L\right), t\right) \geq \varepsilon\right\}\right|$$

$$\leq \frac{1}{P_{k_{r-1}}} \left|\left\{k \leq P_{k_r} : \mu\left(p_k\left(x_k - L\right), t\right) \leq 1 - \varepsilon \text{ or } \upsilon\left(p_k\left(x_k - L\right), t\right) \geq \varepsilon\right\}\right|$$

$$= \frac{1}{P_{k_{r-1}}} \left(N_1 + N_2 + ... + N_{r_0} + N_{r_0+1} + N_{r_0+2} + ... + N_r\right)$$

$$\leq \frac{M.r_0}{P_{k_{r-1}}} + \varepsilon \frac{\left(P_{k_r} - P_{k_{r_0}}\right)}{P_{k_{r-1}}}$$

$$\leq \frac{M.r_0}{P_{k_{r-1}}} + \varepsilon.Q_r \leq \frac{M.r_0}{P_{k_{r-1}}} + \varepsilon K$$

which completes the proof taking the limit as $r \to \infty$.

Corollary 3.1 *Let* $(X, \mu, \upsilon, *, \circ)$ *be an IFNLS,* $\theta = (k_r)$ *be a lacunary sequence and* $1 < \liminf_{r\to\infty} Q_r \leq \limsup_{r\to\infty} Q_r < \infty$. *Then* $S_{\overline{N}}^{(\mu,\upsilon)}$*- convergence is equivalent to* $S_{\left(\overline{N},\theta\right)}^{(\mu,\upsilon)}$*- convergence.*

Proof: It follows from Theorem 3.3 and 3.4.

Theorem 3.5 *Let* $(X,\ \mu, \upsilon, *, \circ)$ *be an IFNLS,* $\theta = (k_r)$ *be a lacunary sequence and* $\frac{H_r}{h_r} \geq 1$ *for all* $r \in N$. *If a sequence* $x = (x_k)$ *in* X *is* $\left(\overline{N}, p_r, \theta\right)^{(\mu,\upsilon)}$*-summable to* $L \in X$, *then* $x = (x_k)$ *is* $S_{\left(\overline{N},\theta\right)}^{(\mu,\upsilon)}$*- convergent to* $L \in X$.

Proof: Suppose that $x = (x_k)$ is $\left(\overline{N}, p_r, \theta\right)^{(\mu,\upsilon)}$- summable to $L \in X$. For every $\varepsilon > 0$ and $t > 0$, let

$$K_r(\varepsilon) = \left\{k \in {I_r}' : \mu\left(p_k(x_k - L), t\right) \leq 1 - \varepsilon \text{ or } \upsilon\left(p_k(x_k - L), t\right) \geq \varepsilon\right\}$$

and

$${K_r}^c(\varepsilon) = \left\{k \in {I_r}' : \mu\left(p_k(x_k - L), t\right) > 1 - \varepsilon \ \text{ and } \upsilon\left(p_k(x_k - L), t\right) < \varepsilon\right\}.$$

Then,

$$\begin{aligned} &\tfrac{1}{H_r}\sum_{k\in I_r} \mu\left(p_k\left(x_k - L\right), t\right) \\ &= \tfrac{1}{H_r}\sum_{\substack{k\in I_r \\ k\in K_r(\varepsilon)}} \mu\left(p_k\left(x_k - L\right), t\right) + \tfrac{1}{H_r}\sum_{\substack{k\in I_r \\ k\in {K_r}^c(\varepsilon)}} \mu\left(p_k\left(x_k - L\right), t\right) \\ &\geq \tfrac{1}{H_r}\sum_{\substack{k\in I_r \\ k\in {K_r}^c(\varepsilon)}} \mu\left(p_k\left(x_k - L\right), t\right) > \tfrac{1}{H_r}\left|{K_r}^c(\varepsilon)\right|(1-\varepsilon). \end{aligned} \tag{16}$$

As a consequence of inequality (16), we get $\lim_{r\to\infty} \frac{1}{H_r}\left|K_r^c(\varepsilon)\right| = 1$. Similarly, for every $\varepsilon > 0$ and $t > 0$,

$$\begin{aligned} &\tfrac{1}{H_r}\sum_{k\in I_r} \upsilon\left(p_k\left(x_k - L\right), t\right) \\ &= \tfrac{1}{H_r}\sum_{\substack{k\in I_r \\ k\in K_r(\varepsilon)}} \upsilon\left(p_k\left(x_k - L\right), t\right) + \tfrac{1}{H_r}\sum_{\substack{k\in I_r \\ k\in {K_r}^c(\varepsilon)}} \upsilon\left(p_k\left(x_k - L\right), t\right) \\ &\geq \tfrac{1}{H_r}\sum_{\substack{k\in I_r \\ k\in K_r(\varepsilon)}} \upsilon\left(p_k\left(x_k - L\right), t\right) \geq \tfrac{1}{H_r}\left|K_r(\varepsilon)\right|\varepsilon. \end{aligned} \tag{17}$$

By inequality (17), we have $\lim_{r\to\infty} \frac{1}{H_r}\left|K_r(\varepsilon)\right| = 0$. This completes the proof.

The following example shows that the converse of Theorem 3.5 is not valid in general.

Example 3.1 *Let* $(R,\mu,\upsilon,*,\circ)$ *be as in Example 2.1. Now, let define* $x=(x_k)$ *to be* $1,2,...,\sqrt{h_r}$ *at the first* $\sqrt{h_r}$ *integers in* I_r*, and* $x_k=0$ *otherwise and define* $p=(p_k)$ *to be* $1^2,2^2,...,h_r$ *for* $k\in I_r$*, and* $p_k=0$ *otherwise. Then, for every* $0<\varepsilon<1$ *and for every* $t>0$*, let*

$$K_r(\varepsilon)=\left\{k\in {I_r}^{'}:\mu(p_kx_k,t)\le 1-\varepsilon \ \text{ or } \ \upsilon(p_kx_k,t)\ge\varepsilon\right\}.$$

Since

$$\begin{aligned} K_r(\varepsilon) &= \left\{k\in {I_r}^{'}: \ \tfrac{t}{t+|p_kx_k|}\le 1-\varepsilon \ \text{ or } \ \tfrac{|p_kx_k|}{t+|p_kx_k|}\ge\varepsilon\right\} \\ &= \left\{k\in {I_r}^{'}: \ |p_kx_k|\ge \tfrac{\varepsilon t}{(1-\varepsilon)}\right\}, \end{aligned}$$

we have

$$\frac{1}{H_r}|K_r(\varepsilon)|\le\frac{h_r}{H_r}=\frac{6r^2}{r(r+1)(2r+1)}$$

which yields that $S^{(\mu,\upsilon)}_{(\overline{N},\theta)}-\lim x=0$. However, since $\frac{1}{H_r}\sum\limits_{k\in I_r}\mu(p_kx_k,t)\to\infty$ and $\frac{1}{H_r}\sum\limits_{k\in I_r}\upsilon(p_kx_k,t)\to\infty$ as $r\to\infty$, $x=(x_k)$ is not $(\overline{N},p_r,\theta)$- summable to 0 with respect to intuitionistic fuzzy norm (μ,υ).

Theorem 3.6 *Let* $(X,\mu,\upsilon,*,\circ)$ *be an IFNLS,* $x=(x_k)$ *in* X *and* $\frac{H_r}{h_r}\ge 1$ *for all* $r\in N$*. If* $S^{(\mu,\upsilon)}_{(\overline{N},\theta)}-\lim x=L$*,* $\mu(p_k(x_k-L),t)\ge 1-M$ *and* $\upsilon(p_k(x_k-L),t)\le M$ *for all* $k\in N$ *such that* $M\in(0,1)$*, then* $(\overline{N},p_r,\theta)^{(\mu,\upsilon)}-\lim x=L$*.*

Proof: Suppose that $S^{(\mu,\upsilon)}_{(\overline{N},\theta)}-\lim x=L$. Then for every $\varepsilon>0$ and $t>0$,

$$\begin{aligned} &\tfrac{1}{H_r}\sum_{k\in I_r}\mu(p_k(x_k-L),t) \\ &=\tfrac{1}{H_r}\sum_{\substack{k\in I_r\\ k\in K_r(\varepsilon)}}\mu(p_k(x_k-L),t)+\tfrac{1}{H_r}\sum_{\substack{k\in I_r\\ k\in {K_r}^c(\varepsilon)}}\mu(p_k(x_k-L),t) \qquad (18) \\ &=S_1(r)+S_2(r) \end{aligned}$$

where

$$S_1(r) = \frac{1}{H_r} \sum_{\substack{k \in I_r \\ k \in K_r(\varepsilon)}} \mu\left(p_k\left(x_k - L\right), t\right) \tag{19}$$

and

$$S_2(r) = \frac{1}{H_r} \sum_{\substack{k \in I_r \\ k \in K_r{}^c(\varepsilon)}} \mu\left(p_k\left(x_k - L\right), t\right). \tag{20}$$

If $k \in K_r(\varepsilon)$, then

$$S_1(r) = \frac{1}{H_r} \sum_{\substack{k \in I_r \\ k \in K_r(\varepsilon)}} \mu\left(p_k\left(x_k - L\right), t\right) \geq \frac{|K_r(\varepsilon)|}{H_r}(1 - M). \tag{21}$$

Since $S_{(\overline{N},\theta)}^{(\mu,\upsilon)} - \lim x = L$,

$$\lim_{r\to\infty} S_1(r) \geq 0. \tag{22}$$

If $k \in K_r{}^c(\varepsilon)$, then we have

$$S_2(r) = \frac{1}{H_r} \sum_{\substack{k \in I_r \\ k \in K_r{}^c(\varepsilon)}} \mu\left(p_k\left(x_k - L\right), t\right) > \frac{\left|K_r^c(\varepsilon)\right|}{H_r}(1 - \varepsilon) \tag{23}$$

which yields that

$$\lim_{r\to\infty} S_2(r) > (1 - \varepsilon). \tag{24}$$

Using equalities (19)-(20) and inequalities (21)-(23), we get

$$\lim_{r\to\infty} \frac{1}{H_r} \sum_{k \in I_r} \mu\left(p_k\left(x_k - L\right), t\right) = 1. \tag{25}$$

Similarly, we obtain

$$\lim_{r\to\infty} \frac{1}{H_r} \sum_{k \in I_r} \upsilon\left(p_k\left(x_k - L\right), t\right) = 0. \tag{26}$$

As a result of equalities (25) and (26), we have

$$\left(\overline{N}, p_r, \theta\right)^{(\mu,\upsilon)} - \lim x = L. \tag{27}$$

Theorem 3.7 $(X, \mu, \upsilon, *, \circ)$ *be an IFNLS,* $\theta = (k_r)$ *be a lacunary sequence. Then the following statements are true:*

(i) If $p_k \leq 1$ for all $k \in N$, $\liminf\limits_{r\to\infty} \frac{H_r}{h_r} > 0$ and $S_\theta^{(\mu,\upsilon)} - \lim x = L$, then $S_{(\overline{N},\theta)}^{(\mu,\upsilon)} - \lim x = L$.

(ii) If $p_k \geq 1$ for all $k \in N$, $\limsup_{r\to\infty} \frac{H_r}{h_r} < \infty$ and $S^{(\mu,\upsilon)}_{(\overline{N},\theta)} - \lim x = L$, then $S^{(\mu,\upsilon)}_{\theta} - \lim x = L$.

Proof:

(i) Suppose that $p_k \leq 1$ for all $k \in N$ and $\liminf_{r\to\infty} \frac{H_r}{h_r} > 0$. By the assumption, there exists $\delta > 0$ such that $0 < \delta \leq \frac{H_r}{h_r} \leq 1$ for all $r \in N$. Let $S^{(\mu,\upsilon)}_{\theta} - \lim x = L$, then for every $\varepsilon > 0$ and $t > 0$, we have

$$\frac{1}{H_r}\left|\left\{k \in I_r{}' \colon \mu\left(p_k\left(x_k - L\right), t\right) \leq 1 - \varepsilon \text{ or } \upsilon\left(p_k\left(x_k - L\right), t\right) \geq \varepsilon\right\}\right|$$

$$= \frac{1}{H_r}\left|\left\{P_{k_{r-1}} < k \leq P_{k_r} \colon \mu\left(p_k\left(x_k - L\right), t\right) \leq 1 - \varepsilon \text{ or } \upsilon\left(p_k\left(x_k - L\right), t\right) \geq \varepsilon\right\}\right|$$

$$\leq \frac{1}{\delta}.\frac{1}{h_r}\left|\left\{P_{k_{r-1}} \leq k_{r-1} < k \leq P_{k_r} \leq k_r \colon \ \mu\left(x_k - L, t\right) \leq 1 - \varepsilon\right.\right.$$

$$\left.\left.\text{or } \ \upsilon\left(x_k - L, t\right) \geq \varepsilon\right\}\right|$$

$$\leq \frac{1}{\delta}.\frac{1}{h_r}\left|\left\{k_{r-1} < k \leq k_r \colon \ \mu\left(x_k - L, t\right) \leq 1 - \varepsilon \ \text{ or } \upsilon\left(x_k - L, t\right) \geq \varepsilon\right\}\right|$$

Hence, we obtain the result by taking the limit as $r \to \infty$.

(ii) Suppose that $p_k \geq 1$ for all $k \in N$ and $\limsup_{r\to\infty} \frac{H_r}{h_r} < \infty$. By the assumption, there exists positive constant K such that $1 \leq \frac{H_r}{h_r} \leq K < \infty$. Let $S^{(\mu,\upsilon)}_{(\overline{N},\theta)} - \lim x = L$, then for every $\varepsilon > 0$ and $t > 0$, we have

$$\frac{1}{h_r}\left|\left\{k \in I_r \colon \ \mu\left(x_k - L, t\right) \leq 1 - \varepsilon \ \text{ or } \upsilon\left(x_k - L, t\right) \geq \varepsilon\right\}\right|$$

$$= \frac{1}{h_r}\left|\left\{k_{r-1} < k \leq k_r \colon \ \mu\left(x_k - L, t\right) \leq 1 - \varepsilon \ \text{ or } \upsilon\left(x_k - L, t\right) \geq \varepsilon\right\}\right|$$

$$\leq K.\frac{1}{H_r}\left|\left\{k_{r-1} \leq P_{k_{r-1}} < k \leq k_r \leq P_{k_r} \colon \ \mu\left(p_k\left(x_k - L\right), t\right) \leq 1 - \varepsilon\right.\right.$$

$$\left.\left.\text{or } \upsilon\left(p_k\left(x_k - L\right), t\right) \geq \varepsilon\right\}\right|$$

$$\leq K.\frac{1}{H_r}\left|\left\{k \in I_r{}' \colon \ \mu\left(p_k\left(x_k - L\right), t\right) \leq 1 - \varepsilon \ \text{ or } \upsilon\left(p_k\left(x_k - L\right), t\right) \geq \varepsilon\right\}\right|.$$

Hence, we obtain the result by taking the limit as $r \to \infty$.

4 Conclusion

In this paper, we introduced the definitions of $(\overline{N}, p_r, \theta)$-summability and weighted lacunary statistical convergence with respect to the intuitionistic fuzzy norm (μ, υ) and investigated their relationship.Our aim is to unify these two methods and use weighted mean and lacunary sequence to introduce the concept of weighted lacunary statistical convergence with respect to the intuitionistic fuzzy norm (μ, υ). Our investigation of $(\overline{N}, p_r, \theta)$- summability and weighted lacunary statistical convergence in intuitionistic fuzzy normed linear spaces also provides a tool to deal with certain summability methods and generalized statistical convergence types.

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