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The Generalized (s,t)-Matrix Sequence's Binomial Transforms

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Abstract

In this study, the binomial transform has been applied to the generalized (s,t)-matrix sequence $\{\Re_n(s,t)\}_{n\in\mathbb{N}}$, *$(s,t)$-Fibonacci* $\{\mathcal{F}_n(s,t)\}_{n\in\mathbb{N}}$ *and (s,t)-Lucas* $\{\mathcal{L}_n(s,t)\}_{n\in\mathbb{N}}$ *matrix sequence. Moreover, using recurrence relations, the generating functions have been founded for these transforms. Finally, the relation between these transforms has been illustrated by deriving new formulas.*

Keywords: *Generalized (s,t)-matrix sequence, (s,t)-Fibonacci matrix sequence, (s,t)-Lucas matrix sequence, binomial transform.*

1 Introduction and Preliminaries

The sequences of numbers have been interested by the researchers for a long time. Recently, there have been so many studies in the literature that concern about subsequences of the generalized k-Horadam numbers such as Fibonacci, Lucas, Pell and Jacobsthal numbers(see [1-6]). They were widely used in many research areas as Physics, Engineering, Architecture, Nature and Art (see [17-20]). For example, the ratio of two consecutive Fibonacci numbers converges to the Golden Section, $\frac{1+\sqrt{5}}{2}$, which appears in modern research

[19,20], particularly Physics of the high energy particles or theoretical Physics.

In addition, there are many study matrix sequences of some special integer sequences, such as Fibonacci, Lucas, Pell, Jocabsthal, which are interested by authors [10-16,22]. For instance, in [12-13], authors defined new matrix generalizations for Fibonacci and Lucas numbers, and using essentially a matrix approach they showed some properties of these matrix sequences. In [14], authors defined *a new sequence* in which it generalizes (s,t)-*Fibonacci* and (s,t)-*Lucas sequences* at the same time. After that, by using it, they established *generalized* (s,t)-*matrix sequence*. Finally, they presented some important relationships among this new generalization, (s,t)-Fibonacci and (s,t)-Lucas sequences and their matrix sequences. In [15], Gulec and Taskara gave new generalizations for (s,t)-Pell and (s,t)-Pell Lucas sequences for Pell and Pell–Lucas numbers. Considering these sequences, they defined the matrix sequences which have elements of (s,t)-Pell and (s,t)-Pell Lucas sequences. Also, they investigated their properties. Moreover, some matrix based transforms can be introduced for a given sequence. Binomial transform is one of these transforms and there is also other ones such as rising and falling binomial transforms(see [7-9,21]).

Motivated by [9,12-14,21], the goal of this paper is to apply the binomial transforms to the generalized (s,t)-matrix sequence $\{\Re_n(s,t)\}_{n\in\mathbb{N}}$, (s,t)-Fibonacci $\{\mathcal{F}_n(s,t)\}_{n\in\mathbb{N}}$ and (s,t)-Lucas $\{\mathcal{L}_n(s,t)\}_{n\in\mathbb{N}}$ matrix sequence. Also, the generating functions of these transforms are found by recurrence relations. Finally, it is illustrated the relations between these transforms by deriving new formulas.

Now we give some preliminaries related our study. Given an integer sequence $X=\{x_0,x_1,x_2,\ldots\}$, define the binomial transform Y of the sequence X to be the sequence $Y(X)=\{y_n\}$, where y_n is given by

$$y_n=\sum_{i=0}^{n}\binom{n}{i}x_i.$$

Proposition 1 *[14] Assume that* $a,\ b\in\mathbb{Z}$, $s>0,\ t\neq 0,\ n\geqslant 0$ *and* $s^2+4t>0$. *Then the following properties are hold:*

i) The generalized (s,t)-sequence is $G_{n+2}(s,t)=sG_{n+1}(s,t)+tG_n(s,t)$, for $G_0(s,t)=a,\ G_1(s,t)=bs,$

ii) The *generalized* (s,t)-*matrix sequence is* $\Re_{n+2}(s,t)=s\Re_{n+1}(s,t)+t\Re_n(s,t)$, for $\Re_0(s,t)=\begin{pmatrix} bs & a \\ at & (b-a)s\end{pmatrix}$ and $\Re_1(s,t)=\begin{pmatrix} bs^2+at & bs \\ bst & at\end{pmatrix}$,

iii) $\Re_n(s,t)=\begin{pmatrix} G_{n+1} & G_n \\ tG_n & tG_{n-1}\end{pmatrix}$,

iv) $\Re_{n+1}^{m}=\Re_{1}^{m}\mathcal{F}_{mn}$.

We should note that choosing suitable values on a and b in Proposition 1, it is actually obtained (s,t)-Fibonacci, (s,t)-Lucas, and their matrix sequences in [12,13] as follows:

- For $a=b=1$, $\begin{cases} G_n = F_{n+1}, \\ \Re_n = \mathcal{F}_{n+1}, \end{cases}$
- For $a=2$, $b=1$, $\begin{cases} G_n = L_n, \\ \Re_n = \mathcal{L}_n. \end{cases}$

Throughout this paper, we will use the notations $F_n,\ L_n,\ G_n,\ \mathcal{F}_n,\ \mathcal{L}_n$ and $\Re_n$ instead of $F_n(s,t),\ L_n(s,t),\ G_n(s,t),\ \mathcal{F}_n(s,t),\ \mathcal{L}_n(s,t)$ and $\Re_n(s,t)$ respectively.

2 Binomial Transform

In this section, the binomial transforms of the generalized (s,t)-matrix sequence, (s,t)-Fibonacci matrix sequence and (s,t)-Lucas matrix sequence will be introduced.

Definition 2 *Let $\Re_n$, $\mathcal{F}_n$ and $\mathcal{L}_n$ be the generalized (s,t), (s,t)-Fibonacci and (s,t)-Lucas-matrix sequences, respectively. The binomial transforms of these matrix sequences can be expressed as follows:*

i) *The binomial transform of the generalized (s,t)-matrix sequence is $b_n=\sum_{i=0}^{n}\binom{n}{i}\Re_i$,*

ii) *the binomial transform of (s,t)-Fibonacci matrix sequence is $c_n=\sum_{i=0}^{n}\binom{n}{i}\mathcal{F}_i$,*

iii) *the binomial transform of (s,t)-Lucas matrix sequence is $d_n=\sum_{i=0}^{n}\binom{n}{i}\mathcal{L}_i$.*

The following lemma will be key of the proof of the next theorems.

Lemma 3 *For $n\geq 0$, the following equalities are hold:*

i) $b_{n+1}=\sum_{i=0}^{n}\binom{n}{i}\left(\Re_i+\Re_{i+1}\right)$,

ii) $c_{n+1}=\sum_{i=0}^{n}\binom{n}{i}\left(\mathcal{F}_i+\mathcal{F}_{i+1}\right)$,

iii) $d_{n+1}=\sum_{i=0}^{n}\binom{n}{i}\left(\mathcal{L}_i+\mathcal{L}_{i+1}\right)$.

Proof. Firstly, in here we will just prove (i), since (ii) and (iii) can be thought in the same manner with them.

i) From Definition 2 and using the well known binomial equality $\binom{n+1}{i} = \binom{n}{i} + \binom{n}{i-1}$, we obtain

$$\begin{aligned} b_{n+1} &= \sum_{i=0}^{n+1} \binom{n+1}{i} \Re_i \\ &= \Re_0 + \sum_{i=1}^{n+1} \left[\binom{n}{i} + \binom{n}{i-1} \right] \Re_i \\ &= \Re_0 + \sum_{i=1}^{n} \binom{n}{i} \Re_i + \sum_{i=0}^{n} \binom{n}{i} \Re_{i+1} \\ &= \sum_{i=0}^{n} \binom{n}{i} \Re_i + \sum_{i=0}^{n} \binom{n}{i} \Re_{i+1} \\ &= \sum_{i=0}^{n} \binom{n}{i} \left(\Re_i + \Re_{i+1} \right), \end{aligned}$$

which is desired result.

■

Note that b_{n+1} is also can be written as $b_{n+1} = b_n + \sum_{i=0}^{n} \binom{n}{i} \Re_{i+1}$.

Theorem 4 *For $n \geq 0$, recurrence relation of sequences $\{b_n\}$, $\{c_n\}$ and $\{d_n\}$ are*

i) $b_{n+2} = (s+2)\, b_{n+1} - (s+1-t)\, b_n$;

with initial conditions $b_0 = \begin{pmatrix} bs & a \\ at & bs - as \end{pmatrix}$ and $b_1 = \begin{pmatrix} bs^2 + bs + at & bs + a \\ bst + at & bs - as + at \end{pmatrix}$,

ii) $c_{n+2} = (s+2)\, c_{n+1} - (s+1-t)\, c_n$;

with initial conditions $c_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $c_1 = \begin{pmatrix} s+1 & 1 \\ t & 1 \end{pmatrix}$,

iii) $d_{n+2} = (s+2)\, d_{n+1} - (s+1-t)\, d_n$;

with initial conditions $d_0 = \begin{pmatrix} s & 2 \\ 2t & -s \end{pmatrix}$ and $d_1 = \begin{pmatrix} s^2 + s + 2t & s + 2 \\ st + 2t & -s + 2t \end{pmatrix}$.

Proof. Similarly the proof of the previous theorem, only the first case (i) will be proved. We will omit the other cases since the proofs will not be different.

i) From Lemma 3, we obtain

$$\begin{aligned}
b_{n+1} &= \sum_{i=0}^{n}\binom{n}{i}\left(\Re_i+\Re_{i+1}\right)\\
&= \Re_0+\Re_1+\sum_{i=1}^{n}\binom{n}{i}\left(\Re_i+\Re_{i+1}\right)\\
&= \Re_0+\Re_1+\sum_{i=1}^{n}\binom{n}{i}\left(\Re_i+s\Re_i+t\Re_{i-1}\right)\\
&= (s+1)\sum_{i=1}^{n}\binom{n}{i}\Re_i+t\sum_{i=1}^{n}\binom{n}{i}\Re_{i-1}+\Re_0+\Re_1.
\end{aligned}$$

from Definition 2, we have

$$b_{n+1}=(s+1)\,b_n+t\sum_{i=1}^{n}\binom{n}{i}\Re_{i-1}-s\Re_0+\Re_1. \tag{1}$$

On the other hand, using the fact of $\binom{n-1}{n}=0$ and putting $n-1$ instead of n in (1), we get

$$\begin{aligned}
b_n &= (s+1)\,b_{n-1}+t\sum_{i=1}^{n-1}\binom{n-1}{i}\Re_{i-1}-s\Re_0+\Re_1\\
&= sb_{n-1}+\sum_{i=0}^{n-1}\binom{n-1}{i}\Re_i+t\sum_{i=1}^{n-1}\binom{n-1}{i}\Re_{i-1}-s\Re_0+\Re_1\\
&= sb_{n-1}+\sum_{i=1}^{n}\left[\binom{n-1}{i-1}+t\binom{n-1}{i}\right]\Re_{i-1}-s\Re_0+\Re_1\\
&= sb_{n-1}+\sum_{i=1}^{n}\left[\binom{n-1}{i-1}+t\binom{n-1}{i}+t\binom{n-1}{i-1}-t\binom{n-1}{i-1}\right]\Re_{i-1}-s\Re_0+\Re_1\\
&= sb_{n-1}+\sum_{i=1}^{n}\left[(1-t)\binom{n-1}{i-1}+t\binom{n}{i}\right]\Re_{i-1}-s\Re_0+\Re_1\\
&= sb_{n-1}+t\sum_{i=1}^{n}\binom{n}{i}\Re_{i-1}+(1-t)\sum_{i=0}^{n-1}\binom{n-1}{i}\Re_{i-1}-s\Re_0+\Re_1\\
&= (s-t+1)\,b_{n-1}+t\sum_{i=1}^{n}\binom{n}{i}\Re_{i-1}-s\Re_0+\Re_1.
\end{aligned}$$

$$t\sum_{i=1}^{n}\binom{n}{i}\Re_{i-1}-s\Re_0+\Re_1=b_n-(s-t+1)\,b_{n-1}$$

Therefore, by subsituting this expression in (1), we obtain

$$b_{n+1} = (s+2)\, b_n - (s+1-t)\, b_{n-1}, \tag{2}$$

which is completed the proof of this case.

■

The characteristic equation of sequence $\{b_n\}$ in (2) is $\lambda^2 - (s+2)\lambda + s - t + 1 = 0$. It is easy to see that the roots of this equation are

$$\lambda_1 = \frac{s+2+\sqrt{s^2+4t}}{2},\ \ \lambda_2 = \frac{s+2-\sqrt{s^2+4t}}{2},$$

and Binet's formula of sequence $\{b_n\}$ can be expressed as

$$b_n = \frac{X\lambda_1^n - Y\lambda_2^n}{\lambda_1 - \lambda_2}, \tag{3}$$

where

$$X = \begin{pmatrix} bs^2 + at + (1-\lambda_2)\, bs & bs + (1-\lambda_2)\, a \\ bst + (1-\lambda_2)\, at & at + (1-\lambda_2)(b-a)\, s \end{pmatrix}$$

and

$$Y = \begin{pmatrix} bs^2 + at + (1-\lambda_1)\, bs & bs + (1-\lambda_1)\, a \\ bst + (1-\lambda_1)\, at & at + (1-\lambda_1)(b-a)\, s \end{pmatrix}.$$

We should note that choosing suitable values on a and b in (3), it is actually obtained Binet's formulas of c_n and d_n as follows:

- For $a = b = 1$, $c_n = \frac{A\lambda_1^n - B\lambda_2^n}{\lambda_1 - \lambda_2}$, where

$$A = \begin{pmatrix} s^2 + t + (1-\lambda_2)\, s & s + 1 - \lambda_2 \\ st + (1-\lambda_2)\, t & t \end{pmatrix}$$

and

$$B = \begin{pmatrix} s^2 + t + (1-\lambda_1)\, s & s + 1 - \lambda_1 \\ st + (1-\lambda_1)\, t & t \end{pmatrix},$$

- For $a = 2, b = 1$, $d_n = \frac{C\lambda_1^n - D\lambda_2^n}{\lambda_1 - \lambda_2}$, where

$$C = \begin{pmatrix} s^2 + 2t + (1-\lambda_2)\, s & s + 2(1-\lambda_2) \\ st + (1-\lambda_2)\, 2t & 2t + (1-\lambda_2)\, s \end{pmatrix}$$

and

$$D = \begin{pmatrix} s^2 + 2t + (1-\lambda_1)\, s & s + 2(1-\lambda_1) \\ st + 2t(1-\lambda_1) & 2t + (1-\lambda_1)\, s \end{pmatrix}.$$

Theorem 5 *The generating functions of the binomial transforms are*

i)

$$b_n(s,t,x) = \frac{\Re_0 + x[\Re_1 - (s+1)\Re_0]}{1-(s+2)x+(s+1-t)x^2} = \begin{pmatrix} \frac{bs+x(at-bs)}{1-(s+2)x+(s+1-t)x^2} & \frac{a+x(bs-as-a)}{1-(s+2)x+(s+1-t)x^2} \\ \frac{at+x(bst-ast-at)}{1-(s+2)x+(s+1-t)x^2} & \frac{(b-a)s+x\left(at-bs^2+as^2-bs+as\right)}{1-(s+2)x+(s+1-t)x^2} \end{pmatrix},$$

ii)

$$c_n(s,t,x) = \frac{\mathcal{F}_0 + x[\mathcal{F}_1 - (s+1)\mathcal{F}_0]}{1-(s+2)x+(s+1-t)x^2} = \begin{pmatrix} \frac{1-x}{1-(s+2)x+(s+1-t)x^2} & \frac{x}{1-(s+2)x+(s+1-t)x^2} \\ \frac{xt}{1-(s+2)x+(s+1-t)x^2} & \frac{x(-s-1)}{1-(s+2)x+(s+1-t)x^2} \end{pmatrix},$$

iii)

$$d_n(s,t,x) = \frac{\mathcal{L}_0 + x[\mathcal{L}_1 - (s+1)\mathcal{L}_0]}{1-(s+2)x+(s+1-t)x^2} = \begin{pmatrix} \frac{s+x(2t-s)}{1-(s+2)x+(s+1-t)x^2} & \frac{2+x(-s-2)}{1-(s+2)x+(s+1-t)x^2} \\ \frac{2t+x(-st-2t)}{1-(s+2)x+(s+1-t)x^2} & \frac{-s+x\left(2t+s^2+s\right)}{1-(s+2)x+(s+1-t)x^2} \end{pmatrix}.$$

Proof. Again, we just prove the case (i) and the others will be omitted.

i) Let $b_n(s,t,x)$ be generating function for the binomial transform of generalized (s,t)-matrix sequence. Then, we can write

$$b_n(s,t,x) = b_0 + xb_1 + ... + x^n b_n + ... \tag{4}$$

By multiplying equation (4) with $-(s+2)x$ and $(s+1-t)x^2$, respectively, then we have

$$-(s+2)xb_n(s,t,x) = -(s+2)xb_0 - (s+2)x^2b_1 - ... - (s+2)x^{n+1}b_n - ... \tag{5}$$

$$(s+1-t)x^2b_n(s,t,x) = (s+1-t)x^2b_0 + (s+1-t)x^3b_1 + ... + (s+1-t)x^{n+2}b_n + ... \tag{6}$$

Considering (4), (5) and (6), we obtain the following equation as

$$b_n(s,t,x)\left(1-(s+2)x+(s+1-t)x^2\right) = b_0 + x(b_1 - (s+2)b_0).$$

Finally, from Theorem 4, we get

$$b_n(s,t,x) = \begin{pmatrix} \frac{bs+x(at-bs)}{1-(s+2)x+(s+1-t)x^2} & \frac{a+x(bs-as-a)}{1-(s+2)x+(s+1-t)x^2} \\ \frac{at+x(bst-ast-at)}{1-(s+2)x+(s+1-t)x^2} & \frac{(b-a)s+x\left(at-bs^2+as^2-bs+as\right)}{1-(s+2)x+(s+1-t)x^2} \end{pmatrix}.$$

■

Note that we can get the following relations between the generating functions of the generalized (s,t), (s,t)-Fibonacci and (s,t)-Lucas-matrix sequences and the generating functions of the binomial transforms of these sequences, respectively.

i) Let $r(x) = \dfrac{\Re_0 + x[\Re_1 - s\Re_0]}{1 - sx - tx^2}$ be the ordinary generating function of the sequence $\{\Re_n\}$. By using the transformation of $\frac{1}{1-x} r\left(\frac{x}{1-x}\right)$, we obtain the generating function of the binomial transform sequence $\{b_n\}$ in Theorem 5-(i).

ii) Let $f(x) = \dfrac{\mathcal{F}_0 + x[\mathcal{F}_1 - s\mathcal{F}_0]}{1 - sx - tx^2}$ be the ordinary generating function of the sequence $\{\mathcal{F}_n\}$. By using the transformation of $\frac{1}{1-x} f\left(\frac{x}{1-x}\right)$, we obtain the generating function of the binomial transform sequence $\{c_n\}$ in Theorem 5-(ii).

iii) Let $g(x) = \dfrac{\mathcal{L}_0 + x[\mathcal{L}_1 - s\mathcal{L}_0]}{1 - sx - tx^2}$ be the ordinary generating function of the sequence $\{\mathcal{L}_n\}$. By using the transformation of $\frac{1}{1-x} g\left(\frac{x}{1-x}\right)$, we obtain the generating function of the binomial transform sequence $\{d_n\}$ in Theorem 5-(iii).

Theorem 6 *For $n, m \in \mathbb{N}_0$, we have*

$$c_{n+m} = c_n c_m.$$

Proof. We use the second principle of finite induction on n to prove this theorem. Let $n = 0$. Then the Theorem yields $c_0 c_m = \mathcal{F}_0 c_m = c_m$ since $\mathcal{F}_0 = I$. Now assume that

$$c_{n+m} = c_n c_m, \text{ for } n \leq N.$$

Then, by considering Theorem 5, we obtain

$$\begin{aligned} c_{N+1+m} &= (s+2)\, c_{N+m} - (s+1-t)\, c_{N+m-1} \\ &= (s+2)\, c_N c_m - (s+1-t)\, c_{N-1} c_m \\ &= [(s+2)\, c_N - (s+1-t)\, c_{N-1}]\, c_m \\ &= c_{N+1} c_m. \end{aligned}$$

■

Theorem 7 *The relations between the transforms $\{b_n\}$, $\{c_n\}$ and $\{d_n\}$ would be illustrated by following way.*

i) $b_{n+1} - b_n = \Re_1 c_n$,

ii) $c_{n+1} - c_n = \mathcal{F}_1 c_n$,

iii) $d_{n+1} - d_n = \mathcal{L}_1 c_n$.

Proof. By considering Definition 2, Lemma 3 and choosing suitable values on a, b and m in Proposition 1-(iv), the proof of Theorem is clear. ■

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