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Some Regular Elements, Idempotents and Right Units of Semigroup B_X (D) Defined by X- Semilattices which is Union of a Chain and Two Rhombus

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Abstract

In this paper we take $Q=\{T_0,T_1,...,T_{m-6},T_{m-5},T_{m-4},T_{m-3},T_{m-2},T_{m-1},T_m\}$ $(m\geq 6)$ *subsemilattice of X- semilattice of unions D where the elements* T_i*'s are satisfying the following properties,*

$T_0\subset T_1\subset ...\subset T_{m-5}\subset T_{m-4}\subset T_{m-2}\subset T_m$, $T_0\subset T_1\subset ...\subset T_{m-5}\subset T_{m-3}\subset T_{m-2}\subset T_m$, $T_0\subset T_1\subset ...\subset T_{m-5}\subset \subset T_{m-3}\subset T_{m-1}\subset T_m$, $T_{m-4}\setminus T_{m-3}\neq\varnothing$, $T_{m-3}\setminus T_{m-4}\neq\varnothing$, $T_{m-4}\setminus T_{m-1}\neq\varnothing$, $T_{m-1}\setminus T_{m-4}\neq\varnothing$, $T_{m-2}\setminus T_{m-1}\neq\varnothing$, $T_{m-1}\setminus T_{m-2}\neq\varnothing$, $T_{m-4}\cup T_{m-3}=T_{m-2}$, $T_{m-4}\cup T_{m-1}=T_{m-2}\cup T_{m-1}=T_m$.

We will investigate the properties of regular elements and idempotents of the complete semigroup of binary relations B_X *(D) satisfying V (D,* α*)=Q. Also we investigate right units of the semmigroup* B_X *(Q). For the case where X is a finite set we derive formulas by means of which we can calculate the numbers of regular elements, idempotents and right units of the respective semigroup.*

Keywords: *Semigroup, Semilatticce, Binary relations, Right units, Regular elements.*

1 Introduction

Let X be an arbitrary nonempty set, D be a $X-$semilattice of unions, i.e. a nonempty set of subsets of the set X that is closed with respect to the set-theoretic operations of unification of elements from D, f be an arbitrary mapping from X into D. To each such a mapping f there corresponds a binary relation α_f on the set X that satisfies the condition $\alpha_f = \bigcup_{x\in X}(\{x\}\times f(x))$. The set of all such α_f $(f: X\to D)$ is denoted by $B_X(D)$. It is easy to prove that $B_X(D)$ is a semigroup with respect to the operation of multiplication of binary relations, which is called a complete semigroup of binary relations defined by a $X-$semilattice of unions D (see [2,3 2.1 p. 34]).

By $\varnothing$ we denote an empty binary relation or empty subset of the set X. The condition $(x,y)\in\alpha$ will be written in the form $x\alpha y$. Further let $x,y\in X$, $Y\subseteq X$, $\alpha\in B_X(D)$, $T\in D$, $\varnothing\neq D'\subseteq D$ and $t\in \breve{D}=\bigcup_{Y\in D} Y$. Then by symbols we denote the following sets:

$$y\alpha=\{x\in X \mid y\alpha x\},\ Y\alpha=\bigcup_{y\in Y} y\alpha,\ V(D,\alpha)=\{Y\alpha \mid Y\in D\},$$
$$X^*=\{T \mid \varnothing\neq T\subseteq X\},\ D'_t=\{Z'\in D' \mid t\in Z'\},\ D'_T=\{Z'\in D' \mid T\subseteq Z'\}.$$
$$\ddot{D}'_T=\{Z'\in D' \mid Z'\subseteq T\},\ l(D',T)=\cup(D'\setminus D'_T),\ Y_T^\alpha=\{x\in X \mid x\alpha=T\}.$$

Under the symbol $\wedge(D,D_t)$ we mean an exact lower bound of the set D_t in the semilattice D.

Definition 1.1: *An element* α *taken from the semigroup* $B_X(D)$ *called a regular element of the semigroup* $B_X(D)$ *if in* $B_X(D)$ *there exists an element* β *such that* $\alpha\circ\beta\circ\alpha=\alpha$.

Definition 1.2: *We say that a complete* $X-$*semilattice of unions* D *is an* $XI-$*semilattice of unions if it satisfies the following two conditions:*

(a) $\wedge(D,D_t)\in D$ for any $t\in\breve{D}$;

(b) $Z=\bigcup_{t\in Z}\wedge(D,D_t)$ for any nonempty element Z of D (see [2,3 definition 1.14.2]).

Definition 1.3: *Let* D *be an arbitrary complete* $X-$*semilattice of unions,* $\alpha\in B_X(D)$ *and* $Y_T^\alpha=\{x\in X \mid x\alpha=T\}$. *If*

$$V[\alpha]=\begin{cases}V(X^*,\alpha), \ if \ \varnothing\notin D,\\ V(X^*,\alpha), \ if \ \varnothing\in V(X^*,\alpha),\\ V(X^*,\alpha)\cup\{\varnothing\}, \ if \ \varnothing\notin V(X^*,\alpha) \text{ and } \varnothing\in D,\end{cases}$$

then it is obvious that any binary relation α of a semigroup $B_X(D)$ can always be written in the form $\alpha=\bigcup_{T\in V[\alpha]}(Y_T^\alpha\times T)$ the sequel, such a representation of a binary relation α will be called quasinormal.

Note that for a quasinormal representation of a binary relation α, not all sets Y_T^α $(T\in V[\alpha])$ can be different from an empty set. But for this representation the following conditions are always fulfilled:

(a) $Y_T^\alpha\cap Y_{T'}^\alpha=\varnothing$, for any $T,T'\in D$ and $T\neq T'$;

(b) $X=\bigcup_{T\in V[\alpha]}Y_T^\alpha$

(See [2, 3 definition 1.11.1]).

Definition 1.4: *We say that a nonempty element T is a non limiting element of the set D' if $T\setminus l(D',T)\neq\varnothing$ and a nonempty element T is a limiting element of the set D' if $T\setminus l(D',T)=\varnothing$ (see [2, 3 definition 1.13.1 and definition 1.13.2]).*

Definition 1.5: *The one-to-one mapping φ between the complete $X-$semilattices of unions $\phi(Q,Q)$ and D'' is called a complete isomorphism if the condition*

$$\varphi(\cup D_1)=\bigcup_{T'\in D_1}\varphi(T')$$

is fulfilled for each nonempty subset D_1 of the semilattice D' (see [2, 3 definition 6.3.2]).

Definition 1.6: *Let α be some binary relation of the semigroup $B_X(D)$. We say that the complete isomorphism φ between the complete semilattices of unions Q and D' is a complete $\alpha-$isomorphism if*

(a) $Q=V(D,\alpha)$;

(b) $\varphi(\varnothing)=\varnothing$ for $\varnothing\in V(D,\alpha)$ and $\varphi(T)\alpha=T$ for any $T\in V(D,\alpha)$ (see [2,3 definition 6.3.3]).

Lemma 1.1: *Let $Y=\{y_1,y_2,...,y_k\}$ and $D_j=\{T_1,...,T_j\}$ be some sets, where $k\geq 1$ and $j\geq 1$. Then the number $s(k,j)$ of all possible mappings of the set Y on any such subset of the set D'_j that $T_j\in D'_j$ can be calculated by the formula $s(k,j)=j^k-(j-1)^k$ (see [2, 3 Corollary 1.18.1]).*

Lemma 1.2: *Let* $D_j = \{T_1, T_2, ...T_j\}$, X *and* $Y-$ *be three such sets, that* $\varnothing \neq Y \subseteq X$. *If* f *is such mapping of the set* X, *in the set* D_j, *for which* $f(y) = T_j$ *for some* $y \in Y$, *then the number* s *of all those mappings* f *of the set* X *in the set* D_j *is equal to* $s = j^{|X \setminus Y|} \cdot \left(j^{|Y|} - (j-1)^{|Y|}\right)$ *(see [2,3 Theorem 1.18.2]).*

Lemma 1.3: *Let* D *be a complete* $X-$*semilattice of unions. If a binary relation* ε *of the form*

$$\varepsilon = \bigcup_{t \in \breve{D}} \left(\{t\} \times \wedge(D, D_t)\right) \cup \left(\left(X \setminus \breve{D}\right) \times \breve{D}\right)$$

is a right unit of $B_X(D)$, *then it is largest right unit.*

Theorem 1.1: *Let* $D = \{\breve{D}, Z_1, Z_2, ..., Z_{n-1}\}$ *be some finite* $X-$*semilattice of unions and* $C(D) = \{P_0, P_1, P_2, ..., P_{n-1}\}$ *be the family of sets of pairwise nonintersecting subsets of the set* X. *If* φ *is a mapping of the semilattice* D *on the family of sets* $C(D)$ *which satisfies the condition* $\varphi(\breve{D}) = P_0$ *and* $\varphi(Z_i) = P_i$ *for any* $i = 1, 2, ..., n-1$ *and* $\hat{D}_Z = D \setminus \{T \in D \mid Z \subseteq T\}$, *then the following equalities are valid:*

$$\breve{D} = P_0 \cup P_1 \cup P_2 \cup ... \cup P_{n-1}, \; Z_i = P_0 \cup \bigcup_{T \in \hat{D}_{Z_i}} \varphi(T).$$

(*) In the sequel these equalities will be called formal.

It is proved that if the elements of the semilattice D are represented in the form (*), then among the parameters P_i $(i = 0, 1, 2, ..., n-1)$ there exist such parameters that cannot be empty sets for *D*. Such sets P_i $(0 < i \leq n-1)$ are called basis sources, whereas sets P_j $(0 \leq j \leq n-1)$ which can be empty sets too are called completeness sources.

It is proved that under the mapping φ the number of covering elements of the pre-image of a basis source is always equal to one, while under the mapping φ the number of covering elements of the pre-image of a completeness source either does not exist or is always greater than one. (see [5]) **.**

Theorem 1.2: *A binary relation* $\varepsilon \in B_X(D)$ *is a right units of this semigroup iff* ε *is idempotent and* $D = V(D, \varepsilon)$ *(see Theorem 4.1.3).*

Theorem 1.3: *Let* D *be a finite* $X-$*semilattice of unions and* $\alpha \in B_X(D)$; $D(\alpha)$ *be the set of those elements* T *of the semilattice* $Q = V(D, \alpha) \setminus \{\varnothing\}$ *which are nonlimiting elements of the set* $\ddot{Q}_T$. *Then a binary relation* α *having a quasinormal representation of the form* $\alpha = \bigcup_{T \in V(D,\alpha)} \left(Y_T^{\alpha} \times T\right)$ *is a regular element of the semigroup* $B_X(D)$ *iff* $V(D, \alpha)$ *is a* $XI-$*semilattice of unions and for some*

α – isomorphism φ from $V(D,\alpha)$ to some X – subsemilattice D' of the semilattice D the following conditions are fulfilled:

a) $\bigcup_{T'\in \ddot{D}(\alpha)_T} Y_{T'}^{\alpha} \supseteq \varphi(T)$ for any $T\in D(\alpha)$;

b) $Y_T^{\alpha}\cap\varphi(T)\neq\varnothing$ for any nonlimiting element T of the set $\ddot{D}(\alpha)_T$ (see [2,3 Theorem 6.3.3]).

Lemma 1.4: *Let $D=\{\breve{D},Z_1,Z_2,...,Z_{n-1}\}$ and $C(D)=\{P_0,P_1,P_2,...,P_{n-1}\}$ are the finite semilattice of unions and the family of sets of pairwise nonintersecting subsets of the set X, $\varphi=\begin{pmatrix}\breve{D} & Z_1 & Z_2...Z_{n-1}\\ P_0 & P_1 & P_2...P_{n-1}\end{pmatrix}$ is a mapping of the semilattice D on the family of the sets $C(D)$, If $\varphi(T)=P\in C(D)$ for some $\breve{D}\neq T\in D$, then $D_t=D\setminus \ddot{D}_T$ for all $t\in P$ (see [***] Lemma 11.6.1).*

Definition 1.8: *Let Q and D' be respectively some XI and X – subsemilattices of the complete X – semilattice of unions D. Then $R_{\varphi}(Q,D')$ is a subset of the semigroup $B_X(D)$ such that $\alpha\in R_{\varphi}(Q,D')$ only if the following conditions are fulfilled for the elements α and φ:*

a) The binary relation α be regular element of the semigroup $B_X(D)$;

b) $V(D,\alpha)=Q$;

c) φ is a complete α – isomorphism between the complete semilattices of unions Q and D' satisfying the conditions $a)$ and $b)$ of the Theorem 1.4 (see [2,3] definition 6.3.4).

Further ε_Q, $\Omega(Q)$ and $\Phi(Q,D')$ respectively are the identity mapping of the semilattice Q, the set of all XI – subsemilattices of the complete X – semilattice of unions D such that $Q'\in\Omega(Q)$ if there exists a complete isomorphism between the semilattices Q' and Q and the set of all complete isomorphisms of the XI – semilattice of unions into the semilattice D' such that $\varphi\in\Phi(Q,D')$ if φ is a α – isomorphism for some $\alpha\in B_X(D)$ and $V(D,\alpha)=Q$.

Next, let

$$R(Q,D')=\bigcup_{\varphi\in\Phi(Q,D')} R_{\varphi}(Q,D') \text{ and } R(D')=\bigcup_{Q'\in\Omega(Q)} R(Q',D').$$

$\bar{R}(Q,D')$ is an arbitrary element of the set $\{R_{\varphi}(Q,D')\,|\,\varphi\in\Phi(Q,D')\}$ and a mapping $T\to\bar{T}$ between the complete XI and X – semilattices of unions Q and D' is a complete isomorphism corresponding to the set $\bar{R}(Q,D')$ (see Theorem 6.3.5).

Theorem 1.4: *A regular element* α *of the semigroup* $B_X(D)$ *is idempotent iff the mapping* φ *satisfying the condition* $\varphi(T)=T\alpha$ *for any* $T\in V(D,\alpha)$ *is an identity mapping of the semilattice* $V(D,\alpha)$ *(see[2,3 Theorem 6.3.7).*

Theorem 1.5: *If* $E_X^{(r)}(Q)$ *is the set of all right units of* $B_X(Q)$, *then* $E_X^{(r)}(Q)=R_{\varepsilon_Q}(Q,Q)$ *(see Theorem 6.3.11).*

Theorem 1.6: *Let* X *be a finite set. if* φ *is a fixed element of the set* $\Phi(Q,D')$, $\Omega(Q)=m_0$ *and* q *be number of all automorphisms of the semilattice then*

$$\left|R(D')\right|=m_0\cdot q\cdot\left|R_\varphi(Q,D')\right|$$

(See Theorem 6.3.5).

2 Results

Theorem 2.1: *Let* $Q=\{T_0,T_1,...,T_{m-6},T_{m-5},T_{m-4},T_{m-3},T_{m-2},T_{m-1},T_m\}$ $(m\geq 5)$ *be a subsemilattice of the semilattice* D *and*

$$\begin{array}{l}
T_0\subset T_1\subset...\subset T_{m-5}\subset T_{m-4}\subset T_{m-2}\subset T_m,\\
T_0\subset T_1\subset...\subset T_{m-5}\subset T_{m-3}\subset T_{m-2}\subset T_m,\\
T_0\subset T_1\subset...\subset T_{m-5}\subset T_{m-3}\subset T_{m-1}\subset T_m,\\
T_{m-4}\setminus T_{m-3}\neq\varnothing,\ T_{m-3}\setminus T_{m-4}\neq\varnothing,\ T_{m-4}\setminus T_{m-1}\neq\varnothing,\\
T_{m-1}\setminus T_{m-4}\neq\varnothing,\ T_{m-2}\setminus T_{m-1}\neq\varnothing,\ T_{m-1}\setminus T_{m-2}\neq\varnothing,\\
T_{m-4}\cup T_{m-3}=T_{m-2},\ T_{m-4}\cup T_{m-1}=T_{m-2}\cup T_{m-1}=T_m.
\end{array}$$

Then Q is always an XI – semilattice of unions.

Proof: Let $P_0,P_1,...,P_{m-1}$ and C be the pairwise nonintersecting subsets of the set X and φ be a mapping of the semilattice Q onto the family of sets $\{P_0,P_1,...,P_{m-1},C\}$ that has the form

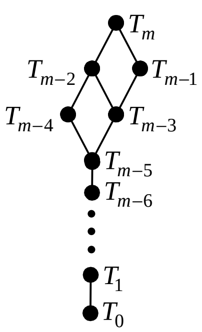

Fig. 1

$$\varphi=\begin{pmatrix}T_0 & T_1 & ... & T_{m-6} & T_{m-5} & T_{m-4} & T_{m-3} & T_{m-2} & T_{m-1} & T_m\\ P_0 & P_1 & ... & P_{m-6} & P_{m-5} & P_{m-4} & P_{m-3} & P_{m-2} & P_{m-1} & C\end{pmatrix}$$

Then the formal equalities corresponding to the semilattice Q are written as

$$\begin{aligned}
&T_m = C\cup P_0\cup P_1\cup\ldots\cup P_{m-6}\cup P_{m-5}\cup P_{m-4}\cup P_{m-3}\cup P_{m-2}\cup P_{m-1},\\
&T_{m-1} = C\cup P_0\cup P_1\cup\ldots\cup P_{m-6}\cup P_{m-5}\cup P_{m-4}\cup P_{m-3}\cup P_{m-2},\\
&T_{m-2} = C\cup P_0\cup P_1\cup\ldots\cup P_{m-6}\cup P_{m-5}\cup P_{m-4}\cup P_{m-3}\cup P_{m-1},\\
&T_{m-3} = C\cup P_0\cup P_1\cup\ldots\cup P_{m-6}\cup P_{m-5}\cup P_{m-4},\\
&T_{m-4} = C\cup P_0\cup P_1\cup\ldots\cup P_{m-6}\cup P_{m-5}\cup P_{m-3}\cup P_{m-1},\\
&T_{m-5} = C\cup P_0\cup P_1\cup\ldots\cup P_{m-6},\\
&T_{m-6} = C\cup P_0\cup P_1\cup\ldots\cup P_{m-7},\\
&\text{———————}\\
&T_1 = C\cup P_0,\\
&T_0 = C,
\end{aligned} \tag{2.1}$$

where $|C|\geq 0$, $|P_{m-5}|\geq 0$, $|P_{m-3}|\geq 0$ and $P_0, P_1, \ldots, P_{m-6}, P_{m-4}, P_{m-2}, P_{m-1} \notin \{\varnothing\}$. Further, let $t\in T_m$. Then from equalities (2.1) and from the Lemma 1.2 we have

$$\wedge(Q,Q_t) = \begin{cases} T_0, \ if \ \ t\in C,\\ T_1, \ if \ \ t\in P_0,\\ ------\\ T_{m-5}, \ if \ \ t\in P_{m-6},\\ T_{m-5}, \ if \ \ t\in P_{m-5},\\ T_{m-3}, \ if \ \ t\in P_{m-4},\\ T_{m-5}, \ if \ \ t\in P_{m-3},\\ T_{m-1}, \ if \ \ t\in P_{m-2},\\ T_{m-4}, \ if \ \ t\in P_{m-1}.\end{cases}$$

Then We have obtained that $\wedge(Q,Q_t)\in D$ for any $t\in T_m$. Furthermore, if $Q^\wedge = \{\wedge(Q,Q_t) t\in T_m\}$, then $Q^\wedge = \{T_0, T_1, \ldots, T_{m-5}, T_{m-4}, T_{m-3}, T_{m-1}\}$ and it is easy to verify that any nonempty element of the semilattice Q is the union of some elements of the set $Q^\wedge$. Now, taking into account Definition 1.2, we obtain that Q is an XI – semilattice of unions.

For the largest right unit ε of the semigroup $B_X(D)$ we have:

$$\varepsilon = (P_{m-2}\times T_{m-1})\cup(P_{m-4}\times T_{m-3})\cup(P_{m-1}\times T_{m-4})\cup((P_{m-6}\cup P_{m-5}\cup P_{m-3})\times T_{m-5})\cup\ldots$$
$$\cup(P_0\times T_1)\cup(C\times T_0)\cup((X\setminus T_m)\times T_m).$$

(See Lemma 1.3).

Of the formal equalities (2.1) follows that:

$$P_{m-2} = T_{m-1}\setminus T_{m-2},\ P_{m-1} = T_{m-4}\setminus T_{m-1},\ P_{m-4} = T_{m-3}\setminus T_{m-4},$$
$$P_{m-6}\cup P_{m-5}\cup P_{m-3} = (T_{m-1}\cap T_{m-4})\setminus T_{m-6},\ P_0 = T_1\setminus T_0,\ C = T_0$$

Then we have

$$\varepsilon = ((T_{m-1}\setminus T_{m-2})\times T_{m-1})\cup((T_{m-3}\setminus T_{m-4})\times T_{m-3})\cup((T_{m-4}\setminus T_{m-1})\times T_{m-4})\cup(((T_{m-1}\cap T_{m-4})\setminus T_{m-6})\times T_{m-5})\cup$$
$$\cup((T_{m-6}\setminus T_{m-7})\times T_{m-6})\cup\ldots((T_1\setminus T_0)\times T_1)\cup(T_0\times T_0)\cup((X\setminus T_m)\times T_m).$$

Theorem 2.2: *Let* $Q=\{T_0, T_1, \ldots, T_{m-6}, T_{m-5}, T_{m-4}, T_{m-3}, T_{m-2}, T_{m-1}, T_m\}$ $(m\geq 5)$ *be a subsemilattice of the semilattice* D *such that*

$$T_0 \subset T_1 \subset ... \subset T_{m-5} \subset T_{m-4} \subset T_{m-2} \subset T_m,$$
$$T_0 \subset T_1 \subset ... \subset T_{m-5} \subset T_{m-3} \subset T_{m-2} \subset T_m,$$
$$T_0 \subset T_1 \subset ... \subset T_{m-5} \subset T_{m-3} \subset T_{m-1} \subset T_m,$$
$$T_{m-4} \setminus T_{m-3} \neq \varnothing,\ T_{m-3} \setminus T_{m-4} \neq \varnothing,\ T_{m-4} \setminus T_{m-1} \neq \varnothing,$$
$$T_{m-1} \setminus T_{m-4} \neq \varnothing,\ T_{m-2} \setminus T_{m-1} \neq \varnothing,\ T_{m-1} \setminus T_{m-2} \neq \varnothing,$$
$$T_{m-4} \cup T_{m-3} = T_{m-2},\ T_{m-4} \cup T_{m-1} = T_{m-2} \cup T_{m-1} = T_m.$$

(see Fig. 1). A binary relation α of the semigroup $B_X(D)$ that has a quasinormal representation of the form $\alpha = \bigcup_{i=0}^{m} \left(Y_i^{\alpha} \times T_i\right)$, where $Q = V(D,\alpha)$, is a regular element of the semigroup $B_X(D)$ iff for some α – isomorohism φ of the semilattice Q on some X – subsemilattice $D' = \{\varphi(T_1), \varphi(T_2), ..., \varphi(T_m)\}$ of the semilattice D satisfies the conditions.

Proof: To begin with, we recall that Q is an XI – semilattice of unions (see Theorem 2.1). Now we are to find the nonlimiting element of the sets $\ddot{Q}_q^*$ of the semilattice $Q^* = Q \setminus \{\varnothing\}$ (see definition 1.4). Indeed, let $T_q \in Q^*$, where $q = 0,1,2,...,m$. Then for $q = 0,1,2,...,m$ we obtain respectively

$$l\left(\ddot{Q}_{T_m}^*, T_m\right) = \cup\left(\{T_0, T_1, ..., T_m\} \setminus \{T_m\}\right) = \cup\{T_0, T_1, ..., T_{m-1}\} = T_m,$$
$$l\left(\ddot{Q}_{T_{m-1}}^*, T_{m-1}\right) = \cup\left(\{T_0, T_1, ..., T_{m-5}, T_{m-3}, T_{m-1}\} \setminus \{T_{m-1}\}\right) = \cup\{T_0, T_1, ..., T_{m-5}, T_{m-3}\} = T_{m-3},$$
$$l\left(\ddot{Q}_{T_{m-2}}^*, T_{m-2}\right) = \cup\left(\{T_0, T_1, ..., T_{m-5}, T_{m-4}, T_{m-3}, T_{m-2}\} \setminus \{T_{m-2}\}\right) = \cup\{T_0, T_1, ..., T_{m-5}, T_{m-4}, T_{m-3},\} = T_{m-2},$$
$$l\left(\ddot{Q}_{T_{m-3}}^*, T_{m-3}\right) = \cup\left(\{T_0, T_1, ..., T_{m-5}, T_{m-3}\} \setminus \{T_{m-3}\}\right) = \cup\{T_0, T_1, ..., T_{m-5}\} = T_{m-5},$$
$$l\left(\ddot{Q}_{T_{m-4}}^*, T_{m-4}\right) = \cup\left(\{T_0, T_1, ..., T_{m-5}, T_{m-4}\} \setminus \{T_{m-4}\}\right) = \cup\{T_0, T_1, ..., T_{m-5}\} = T_{m-5},$$
$$l\left(\ddot{Q}_{T_{m-5}}^*, T_{m-5}\right) = \cup\left(\{T_0, T_1, ..., T_{m-5}\} \setminus \{T_{m-5}\}\right) = \cup\{T_0, T_1, ..., T_{m-6}\} = T_{m-6},$$
$$l\left(\ddot{Q}_{T_{m-6}}^*, T_{m-6}\right) = \cup\left(\{T_0, T_1, ..., T_{m-6}\} \setminus \{T_{m-6}\}\right) = \cup\{T_0, T_1, ..., T_{m-7}\} = T_{m-7},$$

$$l\left(\ddot{Q}_{T_1}^*, T_1\right) = \cup\left(\{T_0, T_1\} \setminus \{T_1\}\right) = \cup\{T_0\} = T_0,$$
$$l\left(\ddot{Q}_{T_0}^*, T_0\right) = \cup\left(\{T_0\} \setminus \{T_0\}\right) = \cup\{\varnothing\} = \varnothing,$$

Therefore

$$T_m \setminus l\left(\ddot{Q}_{T_m}^*, T_m\right) = T_m \setminus T_m = \varnothing,\ T_{m-1} \setminus l\left(\ddot{Q}_{T_{m-1}}^*, T_{m-1}\right) = T_{m-1} \setminus T_{m-3} \neq \varnothing,$$
$$T_{m-2} \setminus l\left(\ddot{Q}_{T_{m-2}}^*, T_{m-2}\right) = T_{m-2} \setminus T_{m-2} = \varnothing,\ T_{m-3} \setminus l\left(\ddot{Q}_{T_{m-3}}^*, T_{m-3}\right) = T_{m-3} \setminus T_{m-5} \neq \varnothing,$$
$$T_{m-4} \setminus l\left(\ddot{Q}_{T_{m-4}}^*, T_{m-4}\right) = T_{m-4} \setminus T_{m-5} \neq \varnothing,\ T_{m-5} \setminus l\left(\ddot{Q}_{T_{m-5}}^*, T_{m-5}\right) = T_{m-5} \setminus T_{m-6} \neq \varnothing,$$
$$T_{m-6} \setminus l\left(\ddot{Q}_{T_{m-6}}^*, T_{m-6}\right) = T_{m-6} \setminus T_{m-7} \neq \varnothing,$$

$$T_1 \setminus l\left(\ddot{Q}_{T_1}^*, T_1\right) = T_1 \setminus T_0 \neq \varnothing,\ T_0 \setminus l\left(\ddot{Q}_{T_0}^*, T_0\right) = T_0 \setminus \varnothing \neq \varnothing,\ if\ T_0 \neq \varnothing,$$

i.e. $T_q \setminus l\left(\ddot{Q}_{T_q}, T_q\right) \neq \varnothing$, where $q = 1, 2, ..., m-6,\ m-5,\ m-4,\ m-3,\ m-1$. Thus we have obtained that $T_m,\ T_{m-2}$ are the limiting elements of the sets $\ddot{Q}^*_{T_m}$, $\ddot{Q}^*_{T_{m-2}}$ and the T_q are the nonlimiting elements of the set $\ddot{Q}^*_{T_q}$, where $q = 1, 2, ..., m-6,\ m-5,\ m-4,\ m-3,\ m-1$. Now, in view of Theorem (1.3) a binary relation α of the semigroup $B_X(D)$ is a regular element of this semigroup iff there exists an $\alpha-$isomorphism φ of the semilattice Q on some $X-$subsemilattice $D' = \{\varphi(T_0), ..., \varphi(T_m)\}$ of the semilattice Q such that

$$
\begin{aligned}
&Y_0^\alpha \cup Y_1^\alpha \cup ... \cup Y_p^\alpha \supseteq \varphi(T_p),\ Y_0^\alpha \cup Y_1^\alpha \cup ... \cup Y_{m-5}^\alpha \cup Y_{m-3}^\alpha \supseteq \varphi(T_{m-3}),\\
&Y_0^\alpha \cup Y_1^\alpha \cup ... \cup Y_{m-5}^\alpha \cup Y_{m-3}^\alpha \cup Y_{m-1}^\alpha \supseteq \varphi(T_{m-1}),\ Y_q^\alpha \cap \varphi(T_q) \neq \varnothing
\end{aligned}
$$

for any $p = 0, 1, ..., m-4,\ m$ and $q = 1, 2, ..., m-4,\ m-3,\ m-1$.

It is clearly understood that the inclusion $Y_0^\alpha \cup Y_1^\alpha \cup ... \cup Y_m^\alpha = X \supseteq \varphi(T_m)$ is always valid. Therefore

$$
\begin{aligned}
&Y_0^\alpha \cup Y_1^\alpha \cup ... \cup Y_p^\alpha \supseteq \varphi(T_p),\ Y_0^\alpha \cup Y_1^\alpha \cup ... \cup Y_{m-5}^\alpha \cup Y_{m-3}^\alpha \supseteq \varphi(T_{m-3}),\\
&Y_0^\alpha \cup Y_1^\alpha \cup ... \cup Y_{m-5}^\alpha \cup Y_{m-3}^\alpha \cup Y_{m-1}^\alpha \supseteq \varphi(T_{m-1}),\ Y_q^\alpha \cap \varphi(T_q) \neq \varnothing
\end{aligned}
$$

for any $p = 0, 1, ..., m-4,\ m$ and $q = 1, 2, ..., m-4,\ m-3,\ m-1$.

Theorem is proved.

Corollary 2.1: *Let* $Q = \{T_0, T_1, ..., T_{m-6}, T_{m-5}, T_{m-4}, T_{m-3}, T_{m-2}, T_{m-1}, T_m\}$ $(m \geq 5)$ *be a subsemilattice of the semilattice* D *such that*

$$
\begin{aligned}
&T_0 \subset T_1 \subset ... \subset T_{m-5} \subset T_{m-4} \subset T_{m-2} \subset T_m,\\
&T_0 \subset T_1 \subset ... \subset T_{m-5} \subset T_{m-3} \subset T_{m-2} \subset T_m,\\
&T_0 \subset T_1 \subset ... \subset T_{m-5} \subset T_{m-3} \subset T_{m-1} \subset T_m,\\
&T_{m-4} \setminus T_{m-3} \neq \varnothing,\ T_{m-3} \setminus T_{m-4} \neq \varnothing,\ T_{m-4} \setminus T_{m-1} \neq \varnothing,\\
&T_{m-1} \setminus T_{m-4} \neq \varnothing,\ T_{m-2} \setminus T_{m-1} \neq \varnothing,\ T_{m-1} \setminus T_{m-2} \neq \varnothing,\\
&T_{m-4} \cup T_{m-3} = T_{m-2},\ T_{m-4} \cup T_{m-1} = T_{m-2} \cup T_{m-1} = T_m.
\end{aligned}
$$

A binary relation α of the semigroup $B_X(D)$, which has a quasinormal representation of the form $\alpha = \bigcup_{i=0}^{m}\left(Y_i^\alpha \times T_i\right)$, such that где $Q = V(D, \alpha)$ is an idempotent element of the semigroup $B_X(D)$ iff

$$
\begin{aligned}
&Y_0^\alpha \cup Y_1^\alpha \cup ... \cup Y_p^\alpha \supseteq T_p,\ Y_0^\alpha \cup Y_1^\alpha \cup ... \cup Y_{m-5}^\alpha \cup Y_{m-3}^\alpha \supseteq T_{m-3},\\
&Y_0^\alpha \cup Y_1^\alpha \cup ... \cup Y_{m-5}^\alpha \cup Y_{m-3}^\alpha \cup Y_{m-1}^\alpha \supseteq T_{m-1},\ Y_q^\alpha \cap T_q \neq \varnothing
\end{aligned}
$$

for any $p = 0,1,\ldots,m-4$ and $q = 1,2,\ldots,m-4,\ m-3,\ m-1$.

Proof: As has been shown in the proof of Theorem 2.2, all elements of the set *Q*, different from the elements T_3, T_5 are the nonlimiting elements of the semilattice *Q*. Now the corollary immediately follows from Theorem 1.4. Corollary is proved.

Corollary 2.2: *Let* $Q=\{T_0,T_1,\ldots,T_{m-6},T_{m-5},T_{m-4},T_{m-3},T_{m-2},T_{m-1},T_m\}$ $(m\ge 5)$ *be a semilattice such that*

$$\begin{array}{l}T_0\subset T_1\subset\ldots\subset T_{m-5}\subset T_{m-4}\subset T_{m-2}\subset T_m,\\ T_0\subset T_1\subset\ldots\subset T_{m-5}\subset T_{m-3}\subset T_{m-2}\subset T_m,\\ T_0\subset T_1\subset\ldots\subset T_{m-5}\subset T_{m-3}\subset T_{m-1}\subset T_m,\\ T_{m-4}\setminus T_{m-3}\ne\varnothing,\ T_{m-3}\setminus T_{m-4}\ne\varnothing,\ T_{m-4}\setminus T_{m-1}\ne\varnothing,\\ T_{m-1}\setminus T_{m-4}\ne\varnothing,\ T_{m-2}\setminus T_{m-1}\ne\varnothing,\ T_{m-1}\setminus T_{m-2}\ne\varnothing,\\ T_{m-4}\cup T_{m-3}=T_{m-2},\ T_{m-4}\cup T_{m-1}=T_{m-2}\cup T_{m-1}=T_m.\end{array}$$

A binary relation α of the semigroup $B_X(Q)$ that has a quasinormal representation of the form $\alpha=\bigcup_{i=0}^{m}\left(Y_i^\alpha\times T_i\right)$, such that $Q=V(Q,\alpha)$, is a right unit of the semigroup $B_X(Q)$ iff

$$\begin{array}{l}Y_0^\alpha\cup Y_1^\alpha\cup\ldots\cup Y_p^\alpha\supseteq T_p,\ Y_0^\alpha\cup Y_1^\alpha\cup\ldots\cup Y_{m-5}^\alpha\cup Y_{m-3}^\alpha\supseteq T_{m-3},\\ Y_0^\alpha\cup Y_1^\alpha\cup\ldots\cup Y_{m-5}^\alpha\cup Y_{m-3}^\alpha\cup Y_{m-1}^\alpha\supseteq T_{m-1},\ Y_q^\alpha\cap T_q\ne\varnothing\end{array}$$

for any $p = 0,1,\ldots,m-4$ and $q = 1,2,\ldots,m-4,\ m-3,\ m-1$.

Proof: By assumption, $Q=V(Q,\alpha)$. Now the validity of the corollary immediately follows from Corollary 2.2 and from Theorem 1.2.

Corollary is proved.

Theorem 2.3: *Let* $Q=\{T_0,T_1,\ldots,T_{m-6},T_{m-5},T_{m-4},T_{m-3},T_{m-2},T_{m-1},T_m\}$ $(m\ge 5)$ *be a subsemilattice of the semilattice* D *such that*

$$\begin{array}{l}T_0\subset T_1\subset\ldots\subset T_{m-5}\subset T_{m-4}\subset T_{m-2}\subset T_m,\\ T_0\subset T_1\subset\ldots\subset T_{m-5}\subset T_{m-3}\subset T_{m-2}\subset T_m,\\ T_0\subset T_1\subset\ldots\subset T_{m-5}\subset T_{m-3}\subset T_{m-1}\subset T_m,\\ T_{m-4}\setminus T_{m-3}\ne\varnothing,\ T_{m-3}\setminus T_{m-4}\ne\varnothing,\ T_{m-4}\setminus T_{m-1}\ne\varnothing,\\ T_{m-1}\setminus T_{m-4}\ne\varnothing,\ T_{m-2}\setminus T_{m-1}\ne\varnothing,\ T_{m-1}\setminus T_{m-2}\ne\varnothing,\\ T_{m-4}\cup T_{m-3}=T_{m-2},\ T_{m-4}\cup T_{m-1}=T_{m-2}\cup T_{m-1}=T_m.\end{array}$$

If the $XI-$semilattices Q and $D'=\{\overline{T}_0,\overline{T}_1,\ldots,\overline{T}_{m-6},\overline{T}_{m-5},\overline{T}_{m-4},\overline{T}_{m-3},\overline{T}_{m-2},\overline{T}_{m-1},\overline{T}_m\}$ are $\alpha-$isomorphic and $|\Omega(Q)|=m_0$, then the following equality is valid:

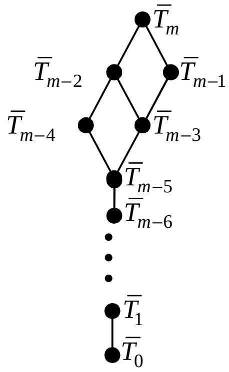

Fig. 2

$$\left|R(D')\right| = m_0 \cdot \left(2^{\left|\overline{T}_1 \setminus \overline{T}_0\right|} - 1\right) \cdot \left(3^{\left|\overline{T}_2 \setminus \overline{T}_1\right|} - 2^{\left|\overline{T}_2 \setminus \overline{T}_1\right|}\right) \cdots \left((m-5)^{\left|\overline{T}_{m-6} \setminus \overline{T}_{m-7}\right|} - (m-6)^{\left|\overline{T}_{m-6} \setminus \overline{T}_{m-7}\right|}\right) \cdot$$
$$\cdot (m-4)^{\left|\left(\left(\overline{T}_{m-1} \cap \overline{T}_{m-4}\right) \setminus \overline{T}_{m-5}\right)\right|} \cdot \left((m-4)^{\left|\overline{T}_{m-5} \setminus \overline{T}_{m-6}\right|} - (m-5)^{\left|\overline{T}_{m-5} \setminus \overline{T}_{m-6}\right|}\right) \cdot$$
$$\cdot \left((m-3)^{\left|\overline{T}_{m-4} \setminus \overline{T}_{m-1}\right|} - (m-4)^{\left|\overline{T}_{m-4} \setminus \overline{T}_{m-1}\right|}\right) \cdot \left((m-3)^{\left|\overline{T}_{m-3} \setminus \overline{T}_{m-4}\right|} - (m-4)^{\left|\overline{T}_{m-3} \setminus \overline{T}_{m-4}\right|}\right) \cdot$$
$$\cdot \left((m-2)^{\left|\overline{T}_{m-1} \setminus \overline{T}_{m-2}\right|} - (m-3)^{\left|\overline{T}_{m-1} \setminus \overline{T}_{m-2}\right|}\right) \cdot (m+1)^{\left|X \setminus \overline{T}_m\right|}.$$

Proof: In the first place, we note that the semilattice Q has only one automorphisms (i.e. $\left|\Phi(Q,Q)\right| = 1$). Let $\alpha \in \overline{R}(Q, D')$ and a quasinormal representation of a regular binary relation α have the form

$$\alpha = \bigcup_{i=1}^{m} \left(Y_i^{\alpha} \times T_i\right). \tag{2.2}$$

Then according to Theorem 1.2 the condition $\alpha \in \overline{R}(Q, D')$ is fulfilled if

$$Y_0^{\alpha} \cup Y_1^{\alpha} \cup \ldots \cup Y_p^{\alpha} \supseteq \overline{T}_p,\ p = 0,1,\ldots,m-4;\ Y_0^{\alpha} \cup Y_1^{\alpha} \cup \ldots \cup Y_{m-5}^{\alpha} \cup Y_{m-3}^{\alpha} \supseteq \overline{T}_{m-3}, \quad (2.3)$$
$$Y_0^{\alpha} \cup Y_1^{\alpha} \cup \ldots \cup Y_{m-5}^{\alpha} \cup Y_{m-3}^{\alpha} \cup Y_{m-1}^{\alpha} \supseteq \overline{T}_{m-1};\ Y_q^{\alpha} \cap \overline{T}_q \neq \varnothing, \quad (2.4)$$
$$q = 1, 2, \ldots, m-4,\ m-3,\ m-1. \quad (2.5)$$

Now, assume that f_{α} is a mapping of the set X in D such that $f_{\alpha}(t) = t\alpha$ for any $t \in X$. $f_{0\alpha}$, f_{m-5}, f_{m-4}, f_{m-3}, f_{m-1}, $f_{k\alpha}$ $(k = 1, 2, \ldots, m-6)$ and f_{m+1} are respectively the restrictions of the mapping f_{α} on the sets

$$\overline{T}_0,\ \left(\overline{T}_{m-1} \cap \overline{T}_{m-4}\right) \setminus \overline{T}_{m-6},\ \overline{T}_{m-4} \setminus \overline{T}_{m-1},\ \overline{T}_{m-3} \setminus \overline{T}_{m-4},\ \overline{T}_{m-1} \setminus \overline{T}_{m-2},$$
$$\overline{T}_1 \setminus \overline{T}_0, \ldots, \overline{T}_{m-6} \setminus \overline{T}_{m-7},\ X \setminus \overline{T}_m$$

We have, by assumption, that these sets do not intersect pair wise and the set-theoretic union of these sets is equal to X.

Let us establish the properties of the mappings $t \in X$. $f_{0\alpha}$, f_{m-5}, f_{m-4}, f_{m-3}, f_{m-1}, $f_{k\alpha}$ $(k = 1, 2, \ldots, m-6)$ and f_{m+1}.

1) $t \in \overline{T}_0$. Hence by virtue of the inclusions (2.3) we have $t \in Y_0^\alpha$, i.e., $t\alpha = T_0$ by the definition of the set Y_0^α. Thus $f_{0\alpha}(t) = T_0$ for any $t \in \overline{T}_0$.

2) $t \in \left(\overline{T}_{m-1} \cap \overline{T}_{m-4}\right) \setminus \overline{T}_{m-6},$. In that case, by virtue of inclusion $(2.4),(2.3)$ we have

$$t \in \left(\overline{T}_{m-1} \cap \overline{T}_{m-4}\right) \setminus \overline{T}_{m-1} \cap \overline{T}_{m-4} \subseteq$$
$$\subseteq \left(Y_0^\alpha \cup Y_1^\alpha \cup \ldots \cup Y_{m-5}^\alpha \cup Y_{m-3}^\alpha \cup Y_{m-1}^\alpha\right) \cap \left(Y_0^\alpha \cup Y_1^\alpha \cup \ldots \cup Y_{m-5}^\alpha \cup Y_{m-4}^\alpha\right) = Y_0^\alpha \cup Y_1^\alpha \cup \ldots \cup Y_{m-5}^\alpha.$$

Therefore $t\alpha \in \{T_0, T_1, \ldots, T_{m-5}\}$ by the definition of the sets $Y_0^\alpha, Y_1^\alpha, \ldots, Y_{m-5}^\alpha$. Thus $f_{m-5\alpha}(t) \in \{T_0, T_1, \ldots, T_{m-5}\}$ for any $t \in \left(\overline{T}_{m-1} \cap \overline{T}_{m-4}\right) \setminus \overline{T}_{m-6}$.

On the other hand, the inequality $Y_{m-5}^\alpha \cap \overline{T}_{m-5} \neq \varnothing$ is true. Therefore $t_{m-5} \in Y_{m-5}^\alpha$ for some element $t_{m-5} \in \overline{T}_{m-5}$. Hence it follows that $t_{m-5}\alpha = T_{m-5}$. Furthermore, if $t_{m-5} \in \overline{T}_{m-6}$, then $t_{m-5}\alpha \in \{T_0, T_1, \ldots, T_{m-6}\}$. However the latter condition contradicts the equality $t_{m-5}\alpha = T_{m-5}$. The contradiction obtained shows that $t_{m-5} \in \overline{T}_{m-5} \setminus \overline{T}_{m-6}$. Thus $f_{m-5\alpha}(t_{m-5}) = T_{m-5}$ for some $t_{m-5} \in \overline{T}_{m-5} \setminus \overline{T}_{m-6}$.

3) $t \in T_{m-4} \setminus T_{m-1}$. In that case, by virtue of inclusion (2.3) we have $t \in T_{m-4} \setminus T_{m-1} \subseteq \overline{T}_{m-4} \subseteq Y_0^\alpha \cup Y_1^\alpha \cup \ldots \cup Y_{m-5}^\alpha \cup Y_{m-4}^\alpha$. Therefore $t\alpha \in \{T_0, T_1, \ldots, T_{m-5}, T_{m-4}\}$ by the definition of the sets $Y_0^\alpha, Y_1^\alpha, \ldots Y_{m-5}^\alpha, Y_{m-4}^\alpha$. Thus $f_{m-4\alpha}(t) \in \{T_0, T_1, \ldots, T_{m-5}, T_{m-4}\}$ for any $t \in \overline{T}_{m-4} \setminus \overline{T}_{m-1}$.

On the other hand, the inequality $Y_{m-4}^\alpha \cap \overline{T}_{m-4} \neq \varnothing$ is true. Therefore $t_{m-4} \in Y_{m-4}^\alpha$ for some element $t_{m-3} \in \overline{T}_{m-4}$. Hence it follows that $t_{m-4}\alpha = T_{m-4}$. Furthermore, if $t_{m-4} \in \overline{T}_{m-1}$, then $t_{m-4}\alpha \in \{T_0, T_1, \ldots, T_{m-5}, T_{m-3}, T_{m-1}\}$. However the latter condition contradicts the equality $t_{m-4}\alpha = T_{m-4}$. The contradiction obtained shows that $t_{m-4} \in \overline{T}_{m-4} \setminus \overline{T}_{m-1},$. Thus $f_{m-4\alpha}(t_{m-4}) = T_{m-4}$ for some $t_{m-4} \in \overline{T}_{m-4} \setminus \overline{T}_{m-1}$.

4) $t \in \overline{T}_{m-3} \setminus \overline{T}_{m-4},$. In that case, by virtue of inclusion (2.3) we have $t \in \overline{T}_{m-3} \setminus \overline{T}_{m-4}, \subseteq \overline{T}_{m-3} \subseteq Y_0^\alpha \cup Y_1^\alpha \cup \ldots \cup Y_{m-5}^\alpha \cup Y_{m-3}^\alpha$. Therefore $t\alpha \in \{T_0, T_1, \ldots, T_{m-5}, T_{m-3}\}$ by the definition of the sets $Y_0^\alpha, Y_1^\alpha, \ldots Y_{m-5}^\alpha, Y_{m-3}^\alpha$. Thus $f_{m-3\alpha}(t) \in \{T_0, T_1, \ldots, T_{m-5}, T_{m-3}\}$ for any $t \in \overline{T}_{m-3} \setminus \overline{T}_{m-4}$.

On the other hand, the inequality $Y_{m-3}^\alpha \cap \overline{T}_{m-3} \neq \varnothing$ is true. Therefore $t_{m-3} \in Y_{m-3}^\alpha$ for some element $t_{m-3} \in \overline{T}_{m-3}$. Hence it follows that $t_{m-3}\alpha = T_{m-3}$. Furthermore, if $t_{m-3} \in \overline{T}_{m-4}$, then $t_{m-3}\alpha \in \{T_0, T_1, \ldots, T_{m-5}, T_{m-4}\}$. However the latter condition contradicts the equality $t_{m-3}\alpha = T_{m-3}$. The contradiction obtained shows that $t_{m-3} \in \overline{T}_{m-3} \setminus \overline{T}_{m-4},$. Thus $f_{m-3\alpha}(t_{m-3}) = T_{m-3}$ for some $t_{m-3} \in \overline{T}_{m-3} \setminus \overline{T}_{m-4}$.

5) $t\in\overline{T}_{m-1}\setminus\overline{T}_{m-2}$. In that case, by virtue of inclusion (2.4) we have $t\in\overline{T}_{m-1}\setminus\overline{T}_{m-2}\subseteq\overline{T}_{m-1}\subseteq Y_0^{\alpha}\cup Y_1^{\alpha}\cup\ldots\cup Y_{m-5}^{\alpha}\cup Y_{m-3}^{\alpha}\cup Y_{m-1}^{\alpha}$.

Therefore $t\alpha\in\{T_0,T_1,\ldots,T_{m-5},T_{m-3},T_{m-1}\}$ by the definition of the sets $Y_0^{\alpha},Y_1^{\alpha},\ldots Y_{m-5}^{\alpha},Y_{m-3}^{\alpha},Y_{m-1}^{\alpha}$. Thus $f_{m-1\alpha}(t)\in\{T_0,T_1,\ldots,T_{m-5},T_{m-3},T_{m-1}\}$ for any $t\in\overline{T}_{m-1}\setminus\overline{T}_{m-2}$.

On the other hand, the inequality $Y_{m-1}^{\alpha}\cap\overline{T}_{m-1}\neq\varnothing$ is true. Therefore $t_{m-2}\in Y_{m-1}^{\alpha}$ for some element $t_{m-2}\in\overline{T}_{m-1}$. Hence it follows that $t_{m-2}\alpha=T_{m-1}$. Furthermore, if $t_{m-2}\in\overline{T}_{m-2}$, then $t_{m-2}\alpha\in\{T_0,T_1,\ldots,T_{m-5},T_{m-4},T_{m-3},T_{m-2}\}$. However the latter condition contradicts the equality $t_{m-2}\alpha=T_{m-1}$. The contradiction obtained shows that $t_{m-2}\in\overline{T}_{m-1}\setminus\overline{T}_{m-2}$, . Thus $f_{m-1\alpha}(t_{m-2})=T_{m-1}$ for some $t_{m-2}\in\overline{T}_{m-1}\setminus\overline{T}_{m-2}$.

6) $t\in\overline{T}_k\setminus\overline{T}_{k-1}$ $(k=1,2,\ldots,m-6)$. In that case, by virtue of inclusion (2.3) we have

$$t\in\overline{T}_k\setminus\overline{T}_{k-1}\subseteq\overline{T}_k\subseteq Y_0^{\alpha}\cup Y_1^{\alpha}\cup Y_2^{\alpha}\cup\ldots\cup Y_k^{\alpha}.$$

Therefore $t\alpha\in\{T_0,T_1,\ldots,T_k\}$ by the definition of the sets $Y_0^{\alpha},Y_1^{\alpha},\ldots,Y_k^{\alpha}$. Thus $f_{k\alpha}(t)\in\{T_0,T_1,\ldots,T_k\}$ for any $t\in\overline{T}_k\setminus\overline{T}_{k-1}$.

On the other hand, the inequality $Y_k^{\alpha}\cap\overline{T}_k\neq\varnothing$ is true. Therefore $t_k\in Y_k^{\alpha}$ for some element $t_k\in\overline{T}_k$. Hence it follows that $t_k\alpha=T_k$. Furthermore, if $t_k\in\overline{T}_{k-1}$, then $t_k\alpha\in\{T_0,T_1,\ldots,T_{k-1}\}$. However the latter condition contradicts the equality $t_k\alpha=T_k$. The contradiction obtained shows that $t_k\in\overline{T}_k\setminus\overline{T}_{k-1}$. Thus $f_{k\alpha}(t_k)=T_k$ for some $t_k\in\overline{T}_k\setminus\overline{T}_{k-1}$.

7) $t\in X\setminus\overline{T}_m$. Then by virtue of the condition $X=\bigcup_{i=0}^{m}Y_i^{\alpha}$ we have $t\in\bigcup_{i=0}^{m}Y_i^{\alpha}$. Hence we obtain $t\alpha\in\{T_0,T_1,T_2,\ldots,T_m\}$. Thus $f_{m\alpha}(t)\in\{T_0,T_1,T_2,\ldots,T_m\}$ for any $t\in X\setminus\overline{T}_m$.

Therefore for a binary relation $\alpha\in\overline{R}(Q,D')$ that has a representation of form 2.2 there always exists a uniquely defined system

$$\left(f_{0\alpha},f_{m-5\alpha},f_{m-4\alpha},f_{m-3\alpha},f_{m-1\alpha},f_{k\alpha},f_{m+1\alpha}\right),\tag{2.6}$$

where $(k=1,2,\ldots,m-6)$. It is obvious that to different elements of the set $\overline{R}(Q,D')$ there correspond different ordered sets of form (2.6).
Now let

$$f_1:\overline{T}_0\to\{T_0\},\ f_{m-5}:(\overline{T}_{m-1}\cap\overline{T}_{m-4})\setminus\overline{T}_{m-6}\to\{T_0,T_1,\ldots,T_{m-5}\},\ f_{m-4}:\overline{T}_{m-4}\setminus\overline{T}_{m-1}\to\{T_0,T_1,\ldots,T_{m-5},T_{m-4}\},$$
$$f_{m-3}:\overline{T}_{m-3}\setminus\overline{T}_{m-4}\to\{T_0,T_1,\ldots,T_{m-5},T_{m-3}\},\ f_{m-1}:\overline{T}_{m-1}\setminus\overline{T}_{m-2}\to\{T_0,T_1,\ldots,T_{m-5},T_{m-3},T_{m-1}\},$$
$$f_k:\overline{T}_k\setminus\overline{T}_{k-1}\to\{T_0,T_1,\ldots,T_k\}\ k=1,2,\ldots,m-6,\ f_{m+1}:X\setminus\overline{T}_m\to\{T_0,T_1,\ldots,T_m\}$$

be the mappings satisfying the following conditions:

8) $f_1(t)=T_0$ for any $t\in\overline{T}_0$;

9) $f_{m-5}(t)\in\{T_0,T_1,...,T_{m-5}\}$ for any $t\in(\overline{T}_{m-1}\cap\overline{T}_{m-4})\setminus\overline{T}_{m-6}$ and $f_{m-5\alpha}(t_{m-5})=T_{m-5}$ for some $t_{m-5}\in\overline{T}_{m-5}\setminus\overline{T}_{m-6}$;

10) $f_{m-4}(t)\in\{T_0,T_1,...,T_{m-5},T_{m-4}\}$ for any $t\in\overline{T}_{m-4}\setminus\overline{T}_{m-1}$ and $f_{m-4\alpha}(t_{m-4})=T_{m-4}$ for some $t_{m-3}\in\overline{T}_{m-4}\setminus\overline{T}_{m-1},$;

11) $f_{m-3}(t)\in\{T_0,T_1,...,T_{m-5},T_{m-3}\}$ for any $t\in\overline{T}_{m-3}\setminus\overline{T}_{m-4}$ and $f_{m-3\alpha}(t_{m-3})=T_{m-3}$ for some $t_{m-3}\in\overline{T}_{m-3}\setminus\overline{T}_{m-4},$;

12) $f_{m-1}(t)\in\{T_0,T_1,...,T_{m-5},T_{m-3},T_{m-1}\}$ for any $t\in\overline{T}_{m-1}\setminus\overline{T}_{m-2}$, and $f_{m-1\alpha}(t_{m-2})=T_{m-1}$ for some $t_{m-2}\in\overline{T}_{m-1}\setminus\overline{T}_{m-2}$;

13) $f_k(t)\in\{T_0,T_1,...,T_k\}$ for any $t\in\overline{T}_k\setminus\overline{T}_{k-1}$ and $f_k(t_k)=T_k$ for some $t_k\in\overline{T}_k\setminus\overline{T}_{k-1}$;

14) $f_{m+1}(t)\in\{T_0,T_1,T_2,...,T_m\}$ for any $t\in X\setminus\overline{T}_m$.

Now we write the mapping $f:X\to D$ as follows:

$$f(t)=\begin{cases} f_0(t), \ if \ \ t\in\overline{T}_0, \\ f_{m-5}(t), \ if \ \ t\in(\overline{T}_{m-1}\cap\overline{T}_{m-4})\setminus\overline{T}_{m-6}, \\ f_{m-4}(t), \ if \ \ t\in \ \overline{T}_{m-4}\setminus\overline{T}_{m-1}, \\ f_{m-3}(t), \ if \ \ t\in \ \overline{T}_{m-3}\setminus\overline{T}_{m-4}, \\ f_{m-1}(t), \ if \ \ t\in\overline{T}_{m-1}\setminus\overline{T}_{m-2}, \\ f_k(t), \ if \ \ t\in\overline{T}_k\setminus\overline{T}_{k-1}, k=1,2,...,m-6, \\ f_{m+1}(t), \ if \ t\in X\setminus\overline{T}_m. \end{cases}$$

To the mapping f we put into correspondence the relation $\beta=\bigcup_{t\in X}(\{t\}\times f(t))$.

Now let $Y_i^\beta=\{t\in X\,|\,t\beta=T_i\}$, where $i=0,1,2,...,m$. With this notation, the binary relation β is represented as $\beta=\bigcup_{i=0}^{m}(Y_i^\beta\times T_i)$. Moreover, from the defnition of the binary relation β we immediately obtain

$$\begin{array}{l} Y_0^\beta\cup Y_1^\beta\cup...\cup Y_p^\beta\supseteq\overline{T}_p, \ p=0,1,...,m-6, \\ Y_0^\beta\cup Y_1^\beta\cup...\cup Y_{m-5}^\beta\cup Y_{m-3}^\beta\supseteq\overline{T}_{m-3}, \\ Y_0^\beta\cup Y_1^\beta\cup...\cup Y_{m-5}^\beta\cup Y_{m-3}^\beta\cup Y_{m-1}^\beta\supseteq\overline{T}_{m-1}, \\ Y_q^\beta\cap\overline{T}_q\neq\varnothing, \ q=1,2,...,m-4, \ m-3, \ m-1. \end{array}$$

since $f_{m-5\alpha}(t_{m-5}) = T_{m-5}$ for some $t_{m-5} \in \overline{T}_{m-5} \setminus \overline{T}_{m-6}$; $f_{m-4\alpha}(t_{m-4}) = T_{m-4}$ for some $t_{m-3} \in \overline{T}_{m-4} \setminus \overline{T}_{m-1}$; $f_{m-3\alpha}(t_{m-3}) = T_{m-3}$ for some $t_{m-3} \in \overline{T}_{m-3} \setminus \overline{T}_{m-4}$; $f_{m-1\alpha}(t_{m-2}) = T_{m-1}$ for some $t_{m-2} \in \overline{T}_{m-1} \setminus \overline{T}_{m-2}$; $f_k(t_k) = T_k$ for some $t_k \in \overline{T}_k \setminus \overline{T}_{k-1}$, $k = 1, 2, ..., m-6$.

Hence by virtue of Theorem 2.2 we conclude that the binary relation β is a regular element of the semigroup $B_X(D)$ that belongs to the set $\overline{R}(Q, D')$.

Therefore there exists a one-to-one correspondence between the binary relations α of the set $\overline{R}(Q, D')$ that have representations of form (2.2) and ordered systems of the form (2.6).

The numbers of all mappings of the form $f_{0\alpha}$, f_{m-5}, f_{m-4}, f_{m-3}, f_{m-1}, $f_{k\alpha}$

$(k = 1, 2, ..., m-6)$ and f_{m+1} $\left(\alpha \in \overline{R}(Q, D')\right)$ are equal respectively to 1,

$$(m-4)^{\left|((\overline{T}_{m-1} \cap \overline{T}_{m-4}) \setminus \overline{T}_{m-6}) \setminus (\overline{T}_{m-5} \setminus \overline{T}_{m-6})\right|} \left((m-4)^{\left|\overline{T}_{m-5} \setminus \overline{T}_{m-6}\right|} - (m-5)^{\left|\overline{T}_{m-5} \setminus \overline{T}_{m-6}\right|}\right),$$

$$(m-3)^{\left|\overline{T}_{m-4} \setminus \overline{T}_{m-1}\right|} - (m-4)^{\left|\overline{T}_{m-4} \setminus \overline{T}_{m-1}\right|}, \ (m-3)^{\left|\overline{T}_{m-3} \setminus \overline{T}_{m-4}\right|} - (m-4)^{\left|\overline{T}_{m-3} \setminus \overline{T}_{m-4}\right|},$$

$$(m-2)^{\left|\overline{T}_{m-1} \setminus \overline{T}_{m-2}\right|} - (m-3)^{\left|\overline{T}_{m-1} \setminus \overline{T}_{m-2}\right|}, \ (k+1)^{\left|\overline{T}_k \setminus \overline{T}_{k-1}\right|} - k^{\left|\overline{T}_k \setminus \overline{T}_{k-1}\right|}, \ (k = 1, 2, ..., m-6)$$

$$(m+1)^{\left|X \setminus \overline{T}_m\right|}.$$

Therefore the equality

$$\begin{aligned} \left|\overline{R}(Q, D')\right| = {} & \left(2^{\left|\overline{T}_1 \setminus \overline{T}_0\right|} - 1\right) \cdot \left(3^{\left|\overline{T}_2 \setminus \overline{T}_1\right|} - 2^{\left|\overline{T}_2 \setminus \overline{T}_1\right|}\right) \cdots \left((m-5)^{\left|\overline{T}_{m-6} \setminus \overline{T}_{m-7}\right|} - (m-6)^{\left|\overline{T}_{m-6} \setminus \overline{T}_{m-7}\right|}\right) \cdot \\ & \cdot (m-4)^{\left|((\overline{T}_{m-1} \cap \overline{T}_{m-4}) \setminus \overline{T}_{m-5})\right|} \cdot \left((m-4)^{\left|\overline{T}_{m-5} \setminus \overline{T}_{m-6}\right|} - (m-5)^{\left|\overline{T}_{m-5} \setminus \overline{T}_{m-6}\right|}\right) \cdot \left((m-3)^{\left|\overline{T}_{m-4} \setminus \overline{T}_{m-1}\right|} - (m-4)^{\left|\overline{T}_{m-4} \setminus \overline{T}_{m-1}\right|}\right) \cdot \\ & \cdot \left((m-3)^{\left|\overline{T}_{m-3} \setminus \overline{T}_{m-4}\right|} - (m-4)^{\left|\overline{T}_{m-3} \setminus \overline{T}_{m-4}\right|}\right) \cdot \left((m-2)^{\left|\overline{T}_{m-1} \setminus \overline{T}_{m-2}\right|} - (m-3)^{\left|\overline{T}_{m-1} \setminus \overline{T}_{m-2}\right|}\right) \cdot (m+1)^{\left|X \setminus \overline{T}_m\right|}. \end{aligned}$$

is valid.

Now, using the equalities $\left|\Omega(Q)\right| = m_0$, $\left|\Phi(Q, D')\right| = 1$ and Theorem 1.6, we Obtain

$$\begin{aligned} \left|R(D')\right| = {} & m_0 \cdot \left(2^{\left|\overline{T}_1 \setminus \overline{T}_0\right|} - 1\right) \cdot \left(3^{\left|\overline{T}_2 \setminus \overline{T}_1\right|} - 2^{\left|\overline{T}_2 \setminus \overline{T}_1\right|}\right) \cdots \left((m-5)^{\left|\overline{T}_{m-6} \setminus \overline{T}_{m-7}\right|} - (m-6)^{\left|\overline{T}_{m-6} \setminus \overline{T}_{m-7}\right|}\right) \cdot \\ & \cdot (m-4)^{\left|((\overline{T}_{m-1} \cap \overline{T}_{m-4}) \setminus \overline{T}_{m-5})\right|} \cdot \left((m-4)^{\left|\overline{T}_{m-5} \setminus \overline{T}_{m-6}\right|} - (m-5)^{\left|\overline{T}_{m-5} \setminus \overline{T}_{m-6}\right|}\right) \cdot \left((m-3)^{\left|\overline{T}_{m-4} \setminus \overline{T}_{m-1}\right|} - (m-4)^{\left|\overline{T}_{m-4} \setminus \overline{T}_{m-1}\right|}\right) \cdot \\ & \cdot \left((m-3)^{\left|\overline{T}_{m-3} \setminus \overline{T}_{m-4}\right|} - (m-4)^{\left|\overline{T}_{m-3} \setminus \overline{T}_{m-4}\right|}\right) \cdot \left((m-2)^{\left|\overline{T}_{m-1} \setminus \overline{T}_{m-2}\right|} - (m-3)^{\left|\overline{T}_{m-1} \setminus \overline{T}_{m-2}\right|}\right) \cdot (m+1)^{\left|X \setminus \overline{T}_m\right|}. \end{aligned}$$

Theorem is proved.

Corollary 1.4. *Let* $Q=\{T_0,T_1,...,T_{m-6},T_{m-5},T_{m-4},T_{m-3},T_{m-2},T_{m-1},T_m\}$ $(m\geq 5)$ *be a subsemilattice of the semilattice* D *such that*

$$\begin{aligned}
&T_0\subset T_1\subset ...\subset T_{m-5}\subset T_{m-4}\subset T_{m-2}\subset T_m,\\
&T_0\subset T_1\subset ...\subset T_{m-5}\subset T_{m-3}\subset T_{m-2}\subset T_m,\\
&T_0\subset T_1\subset ...\subset T_{m-5}\subset T_{m-3}\subset T_{m-1}\subset T_m,\\
&T_{m-4}\setminus T_{m-3}\neq\varnothing,\ T_{m-3}\setminus T_{m-4}\neq\varnothing,\ T_{m-4}\setminus T_{m-1}\neq\varnothing,\\
&T_{m-1}\setminus T_{m-4}\neq\varnothing,\ T_{m-2}\setminus T_{m-1}\neq\varnothing,\ T_{m-1}\setminus T_{m-2}\neq\varnothing,\\
&T_{m-4}\cup T_{m-3}=T_{m-2},\ T_{m-4}\cup T_{m-1}=T_{m-2}\cup T_{m-1}=T_m.
\end{aligned}$$

If $E_X^{(r)}(Q)$ is the set of all right units of the semigroup $B_X(Q)$, then

$$\begin{aligned}
&\left|E_X^{(r)}(Q)\right|=\left(2^{|T_1\setminus T_0|}-1\right)\cdot\left(3^{|T_2\setminus T_1|}-2^{|T_2\setminus T_1|}\right)\cdots\left((m-5)^{|\bar{T}_{m-6}\setminus\bar{T}_{m-7}|}-(m-6)^{|\bar{T}_{m-6}\setminus\bar{T}_{m-7}|}\right)\cdot\\
&\cdot(m-4)^{|((T_{m-1}\cap T_{m-4})\setminus T_{m-5})|}\cdot\left((m-4)^{|T_{m-5}\setminus T_{m-6}|}-(m-5)^{|T_{m-5}\setminus T_{m-6}|}\right)\cdot\left((m-3)^{|T_{m-4}\setminus T_{m-1}|}-(m-4)^{|T_{m-4}\setminus T_{m-1}|}\right)\cdot\\
&\cdot\left((m-3)^{|T_{m-3}\setminus T_{m-4}|}-(m-4)^{|T_{m-3}\setminus T_{m-4}|}\right)\cdot\left((m-2)^{|T_{m-1}\setminus T_{m-2}|}-(m-3)^{|T_{m-1}\setminus T_{m-2}|}\right)\cdot(m+1)^{|X\setminus T_m|}.
\end{aligned}$$

Proof: By virtue of Theorem 6.3.1 we have $E_X^{(r)}(Q)=R_{\varepsilon_Q}(Q,Q)$, where ε_Q is the identity mapping of the semilattice Q. Hence, taking into account the equalities $Q=D'$ $\left|R_{\varepsilon_Q}(Q,Q)\right|=\left|\bar{R}(Q,D')\right|$ and

$$\begin{aligned}
&\left|R(D')\right|=m_0\cdot\left(2^{|\bar{T}_1\setminus\bar{T}_0|}-1\right)\cdot\left(3^{|\bar{T}_2\setminus\bar{T}_1|}-2^{|\bar{T}_2\setminus\bar{T}_1|}\right)\cdots\left((m-5)^{|\bar{T}_{m-6}\setminus\bar{T}_{m-7}|}-(m-6)^{|\bar{T}_{m-6}\setminus\bar{T}_{m-7}|}\right)\cdot\\
&\cdot(m-4)^{|((\bar{T}_{m-1}\cap\bar{T}_{m-4})\setminus\bar{T}_{m-5})|}\cdot\left((m-4)^{|\bar{T}_{m-5}\setminus\bar{T}_{m-6}|}-(m-5)^{|\bar{T}_{m-5}\setminus\bar{T}_{m-6}|}\right)\cdot\left((m-3)^{|\bar{T}_{m-4}\setminus\bar{T}_{m-1}|}-(m-4)^{|\bar{T}_{m-4}\setminus\bar{T}_{m-1}|}\right)\cdot\\
&\cdot\left((m-3)^{|\bar{T}_{m-3}\setminus\bar{T}_{m-4}|}-(m-4)^{|\bar{T}_{m-3}\setminus\bar{T}_{m-4}|}\right)\cdot\left((m-2)^{|\bar{T}_{m-1}\setminus\bar{T}_{m-2}|}-(m-3)^{|\bar{T}_{m-1}\setminus\bar{T}_{m-2}|}\right)\cdot(m+1)^{|X\setminus\bar{T}_m|}.
\end{aligned}$$

We obtain

$$\begin{aligned}
&\left|E_X^{(r)}(Q)\right|=\left(2^{|T_1\setminus T_0|}-1\right)\cdot\left(3^{|T_2\setminus T_1|}-2^{|T_2\setminus T_1|}\right)\cdots\left((m-5)^{|\bar{T}_{m-6}\setminus\bar{T}_{m-7}|}-(m-6)^{|\bar{T}_{m-6}\setminus\bar{T}_{m-7}|}\right)\cdot\\
&\cdot(m-4)^{|((T_{m-1}\cap T_{m-4})\setminus T_{m-5})|}\cdot\left((m-4)^{|T_{m-5}\setminus T_{m-6}|}-(m-5)^{|T_{m-5}\setminus T_{m-6}|}\right)\cdot\left((m-3)^{|T_{m-4}\setminus T_{m-1}|}-(m-4)^{|T_{m-4}\setminus T_{m-1}|}\right)\cdot\\
&\cdot\left((m-3)^{|T_{m-3}\setminus T_{m-4}|}-(m-4)^{|T_{m-3}\setminus T_{m-4}|}\right)\cdot\left((m-2)^{|T_{m-1}\setminus T_{m-2}|}-(m-3)^{|T_{m-1}\setminus T_{m-2}|}\right)\cdot(m+1)^{|X\setminus T_m|}.
\end{aligned}$$

Corollary 1.5: *Let* $Q=\{T_0,T_1,T_2,...,T_6\}$ *be a subsemilattice of the semilattice D and*

$$\begin{aligned}
&T_0\subset T_1\subset T_2\subset T_4\subset T_6,\ T_0\subset T_1\subset T_3\subset T_4\subset T_6\\
&T_0\subset T_1\subset T_3\subset T_5\subset T_6,\ T_2\setminus T_3\neq\varnothing,\ T_3\setminus T_2\neq\varnothing,\\
&T_2\setminus T_5\neq\varnothing,\ T_5\setminus T_2\neq\varnothing,\ T_4\setminus T_5\neq\varnothing,\ T_5\setminus T_4\neq\varnothing,\\
&T_2\cup T_3=T_4,\ T_2\cup T_5=T_4\cup T_5=T_6.
\end{aligned}$$

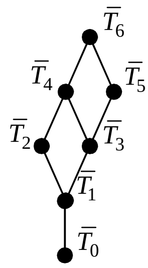

Fig. 4

(see Fig. 13.6.4). If the $XI-$semilattices Q and $D'=\{\overline{T}_1,\overline{T}_2,...,\overline{T}_6\}$ are $\alpha-$isomorphic and $|\Omega(Q)|=m_0$, the equality is valid.

$$|R(D')| = m_0 \cdot 2^{|((\overline{T}_5\cap\overline{T}_2)\setminus\overline{T}_1)|} \cdot \left(2^{|\overline{T}_1\setminus\overline{T}_0|}-1\right)\cdot \cdot\left(3^{|\overline{T}_2\setminus\overline{T}_5|}-2^{|\overline{T}_2\setminus\overline{T}_5|}\right)\cdot\left(3^{|\overline{T}_3\setminus\overline{T}_2|}-2^{|\overline{T}_3\setminus\overline{T}_2|}\right)\cdot\left(4^{|\overline{T}_5\setminus\overline{T}_4|}-3^{|\overline{T}_5\setminus\overline{T}_4|}\right)\cdot 7^{|X\setminus\overline{T}_6|}.$$

Proof: The corollary immediately follows from Theorem 1.3.

Corollary 1.6: *Let* $Q=\{T_0,T_1,T_2,T_3,T_4,T_5,T_6,T_7\}$ *is a subsemilattice of the semilattice* D *such that*

$$\begin{aligned}&T_0\subset T_1\subset T_2\subset T_3\subset T_5\subset T_7,\ T_0\subset T_1\subset T_2\subset T_4\subset T_5\subset T_7,\\&T_0\subset T_1\subset T_2\subset T_4\subset T_6\subset T_7,\ T_4\setminus T_3\neq\varnothing,\ \ T_3\setminus T_4\neq\varnothing,\\&T_6\setminus T_3\neq\varnothing,\ T_3\cup T_6\neq\varnothing,\ T_6\setminus T_5\neq\varnothing,\ T_5\cup T_6\neq\varnothing,\\&T_4\cup T_3=T_5,\ T_6\cup T_3=T_6\cup T_5=T_7\end{aligned}$$

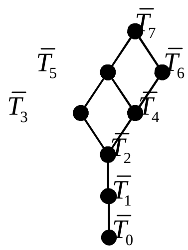

Fig. 5

(see Fig. 13.3.7). If the $XI-$semilattices $Q=\{T_0,T_1,T_2,T_3,T_4,T_5,T_6,T_7\}$ and $D'=\{\overline{T}_0,\overline{T}_1,\overline{T}_2,\overline{T}_3,\overline{T}_4,\overline{T}_5,\overline{T}_6,\overline{T}_7\}$ are $\alpha-$isomorphic and $|\Omega(Q)|=m_0$, then the following statement is true:

$$|R(D')| = m_0\cdot\left(2^{|\overline{T}_1\setminus\overline{T}_0|}-1\right)\cdot 3^{|((\overline{T}_6\cap\overline{T}_3)\setminus\overline{T}_2)|}\cdot\left(3^{|\overline{T}_2\setminus\overline{T}_1|}-2^{|\overline{T}_2\setminus\overline{T}_1|}\right)\cdot \cdot\left(4^{|\overline{T}_3\setminus\overline{T}_6|}-3^{|\overline{T}_3\setminus\overline{T}_6|}\right)\cdot\left(4^{|\overline{T}_4\setminus\overline{T}_3|}-3^{|\overline{T}_4\setminus\overline{T}_3|}\right)\cdot\left(5^{|\overline{T}_6\setminus\overline{T}_5|}-4^{|\overline{T}_6\setminus\overline{T}_5|}\right)\cdot 8^{|X\setminus\overline{T}_7|}.$$

Proof: The corollary immediately follows from Theorem 1.3. □

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