Gen. Math. Notes, Vol. 20, No. 2, February 2014, pp. 136-141

ISSN 2219-7184; Copyright © ICSRS Publication, 2014
www.i-csrs.org
Available free online at http://www.geman.in

A Formula for Tetranacci-Like Sequence

Bijendra Singh[1], Pooja Bhadouria[2], Omprakash Sikhwal[3] and Kiran Sisodiya[4]

[1, 2, 4]School of Studies in Mathematics, Vikram University
Ujjain, (M.P.), India
[3]Department of Mathematics, Mandsaur Institute of Technology
Mandsaur, (M.P.), India
[1]E-mail: bijendrasingh@yahoo.com
[2]Email: pooja.kajal@yahoo.co.in
[3]Email: opbhsikhwal@rediffmail.com
[4]Email: kiran.sisodiya@yahoo.com

(Received: 16-11-13 / Accepted: 13-12-13)

Abstract

Many papers are concerning a variety of generalizations of the Fibonacci sequence. In this paper, we define a Tetranacci-Like sequence in terms of first four terms and then present the general formula for n^{th} term of the Tetranacci-Like sequence with derivation.

Keywords: *Tetranacci sequence, Tetranacci-Like sequence, Tetranacci numbers.*

1 Introduction

Many sequences have been studied for many years now. Arithmetic, Geometric, Harmonic, Fibonacci and Lucas sequences have been very well-defined in Mathematical Journals. On the other hand, Fibonacci-Like sequence, Tribonacci-Like sequence received little more attention from mathematicians.

Fibonacci sequence is a sequence obtained by adding two preceding terms with the initial conditions 0 and 1. Similarly, Tribonacci sequence is obtained by adding three preceding terms starting with 0, 0 and 1. Moreover, Fibonacci-Like sequence and Tribonacci-Like sequence defined by the same pattern but the sequences start with two and three arbitrary terms respectively.

Various properties of Fibonacci-Like sequence have been presented in the paper of B. Singh [2]. In [3], Natividad derived a formula in solving a Fibonacci-like sequence using the Binet's formula and Bueno [1] gives the formula for the k^{th} term of Natividad's Fibonacci-Like sequence. Also, Natividad [4] established a formula in solving the n^{th} term of the Tribonacci-Like sequence.

In this paper, we will derive a general formula to finding the n^{th} term of the Tetranacci-Like sequence using its first four terms and tetranacci numbers.

The Tetranacci sequence $\{M_n\}$ [5], [6] defined by the recurrence relation

$$M_n = M_{n-1} + M_{n-2} + M_{n-3} + M_{n-4} \quad for\ n \geq 4, \tag{1.1}$$

where $M_0 = M_1 = 0,\ M_2 = M_3 = 1.$

First few terms of the Tetranacci sequence are as:

Table 1: The first 15 terms of Tetranacci Numbers

n^{th} term	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Tetranacci Numbers	0	0	1	1	2	4	8	15	29	56	108	208	401	773	1490

When the first four terms of the Tetranacci sequence become arbitrary, it is known as Tetranacci-Like sequence.

2 Main Results

The Tetranacci-Like sequence is a sequence with the arbitrary initial terms or we can say that Tetranacci-Like sequence start at any desired numbers.

Let the first four terms of Tetranacci-Like sequence be $Q_1, Q_2, Q_3\ and\ Q_4$. Then we shall derive a general formula for Q_n given the first four terms.

The sequence $Q_1, Q_2, Q_3, Q_4, \ldots, Q_n$ is known as generalized Tetranacci sequence (or Tetranacci-Like sequence), if

$$Q_n = Q_{n-4} + Q_{n-3} + Q_{n-2} + Q_{n-1} \tag{1.2}$$

To find the general formula for n^th term of the Tetranacci-Like sequence, we follow a specific pattern.

From (1.2), we derive some of the equations as

$$Q_5 = Q_1 + Q_2 + Q_3 + Q_4$$
$$Q_6 = Q_1 + 2Q_2 + 2Q_3 + 2Q_4$$
$$Q_7 = 2Q_1 + 3Q_2 + 4Q_3 + 4Q_4$$
$$Q_8 = 4Q_1 + 6Q_2 + 7Q_3 + 8Q_4$$
$$Q_9 = 8Q_1 + 12Q_2 + 14Q_3 + 15Q_4$$
$$Q_{10} = 15Q_1 + 23Q_2 + 27Q_3 + 29Q_4$$
$$Q_{11} = 29Q_1 + 44Q_2 + 52Q_3 + 56Q_4$$

Now we write all the numerical coefficients of $Q_1, Q_2, Q_3\ and\ Q_4$ in tabular form that were shown in Table 2.

Table 2: Coefficients of Q_1, Q_2, Q_3 and Q_4 of n^th term of Tetranacci-Like sequence

Number of terms	***n^th* term of** Tetranacci-Like **sequence**	**Coefficients**			
		Q_1	Q_2	Q_3	Q_4
1	Q_5	1	1	1	1
2	Q_6	1	2	2	2
3	Q_7	2	3	4	4
4	Q_8	4	6	7	8
5	Q_9	8	12	14	15
6	Q_{10}	15	23	27	29
7	Q_{11}	29	44	52	56
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
n	Q_n	$(n-2)$	$(n-2)+(n-3)$	$(n-2)+(n-3)+(n-4)$	$(n-1)$

After a keen observation of Table 1 and Table 2, we state the following theorem.

Theorem 1: *For any real numbers* Q_1, Q_2, Q_3 *and* Q_4*, the formula for finding the n*th *term of the Tetranacci-Like sequence is*

$$Q_n = M_{n-2}Q_1 + (M_{n-2} + M_{n-3})Q_2 + (M_{n-2} + M_{n-3} + M_{n-4})Q_3 + M_{n-1}Q_4, \tag{1.3}$$

where

$Q_n =$ n$^{\text{th}}$ term of Tetranacci-Like sequence
$Q_1 =$ first term
$Q_2 =$ second term
$Q_3 =$ third term
$Q_4 =$ fourth term
$M_{n-1}, M_{n-2}, M_{n-3}, M_{n-4} =$ corresponding tetranacci numbers.

Proof: We shall prove above theorem by the Principle of Mathematical Induction method for $n \geq 5$.

First we take $n = 5$, then we get

$$Q_5 = M_3Q_1 + (M_3 + M_2)Q_2 + (M_3 + M_2 + M_1)Q_3 + M_4Q_4$$
$$Q_5 = (1)Q_1 + (1+0)Q_2 + (1+0+0)Q_3 + (1)Q_4$$
$$Q_5 = Q_1 + Q_2 + Q_3 + Q_4,$$

which is true. (by definition of Tetranacci-Like sequence)

Now, we assume that the theorem is true for some integer k (>5), i.e.

$$P(k): Q_k = M_{k-2}Q_1 + (M_{k-2} + M_{k-3})Q_2 + (M_{k-2} + M_{k-3} + M_{k-4})Q_3 + M_{k-1}Q_4 \tag{1.4}$$

We shall now prove that P(k+1) is true whenever P(k) is true, i.e.

$$P(k+1): Q_{k+1} = M_{k-1}Q_1 + (M_{k-1} + M_{k-2})Q_2 + (M_{k-1} + M_{k-2} + M_{k-3})Q_3 + M_kQ_4 \tag{1.5}$$

To verify above equation, we shall add Q_{k-1}, Q_{k-2} and Q_{k-3} on both side of P(k), then eq.(1.4) becomes

$$Q_k + Q_{k-1} + Q_{k-2} + Q_{k-3} = M_{k-2}Q_1 + (M_{k-2} + M_{k-3})Q_2 + (M_{k-2} + M_{k-3} + M_{k-4})Q_3 + M_{k-1}Q_4 + Q_{k-1} + Q_{k-2} + Q_{k-3} \tag{1.6}$$

By equation (1.4), we have

$$Q_{k-1} = M_{k-3}Q_1 + (M_{k-3} + M_{k-4})Q_2 + (M_{k-3} + M_{k-4} + M_{k-5})Q_3 + M_{k-2}Q_4$$

$$Q_{k-2} = M_{k-4}Q_1 + (M_{k-4} + M_{k-5})Q_2 + (M_{k-4} + M_{k-5} + M_{k-6})Q_3 + M_{k-3}Q_4$$

$$Q_{k-3} = M_{k-5}Q_1 + (M_{k-5} + M_{k-6})Q_2 + (M_{k-5} + M_{k-6} + M_{k-7})Q_3 + M_{k-4}Q_4$$

Use above in eq. (1.6), we obtain

$$\begin{aligned} &Q_k + Q_{k-1} + Q_{k-2} + Q_{k-3} \\ &= M_{k-2}Q_1 + (M_{k-2} + M_{k-3})Q_2 + (M_{k-2} + M_{k-3} + M_{k-4})Q_3 + M_{k-1}Q_4 \\ &\quad M_{k-3}Q_1 + (M_{k-3} + M_{k-4})Q_2 + (M_{k-3} + M_{k-4} + M_{k-5})Q_3 + M_{k-2}Q_4 \\ &\quad M_{k-4}Q_1 + (M_{k-4} + M_{k-5})Q_2 + (M_{k-4} + M_{k-5} + M_{k-6})Q_3 + M_{k-3}Q_4 \\ &\quad M_{k-5}Q_1 + (M_{k-5} + M_{k-6})Q_2 + (M_{k-5} + M_{k-6} + M_{k-7})Q_3 + M_{k-4}Q_4 \end{aligned}$$

$$\begin{aligned} Q_{k+1} = {} & (M_{k-2} + M_{k-3} + M_{k-4} + M_{k-5})Q_1 + [(M_{k-2} + M_{k-3} + M_{k-4} + M_{k-5}) + \\ & (M_{k-3} + M_{k-4} + M_{k-5} + M_{k-6})]Q_2 + [(M_{k-2} + M_{k-3} + M_{k-4} + M_{k-5}) + \\ & (M_{k-3} + M_{k-4} + M_{k-5} + M_{k-6}) + (M_{k-4} + M_{k-5} + M_{k-6} + M_{k-7})]Q_3 + \\ & (M_{k-1} + M_{k-2} + M_{k-3} + M_{k-4})Q_4 \end{aligned} \tag{1.7}$$

Now by the definition of Tetranacci sequence eq. (1.7) becomes

$$Q_{k+1} = M_{k-1}Q_1 + [M_{k-1} + M_{k-2}]Q_2 + [M_{k-1} + M_{k-2} + M_{k-3}]Q_3 + M_k Q_4$$

Thus by the Mathematical Induction P(k+1) is true, whenever P(k) is true. Hence the theorem is verified.

3 Conclusion

In this paper, we have introduced Tetranacci-Like sequence using its first four terms and Tetranacci numbers and derived the general formula of n^{th} term of the Tetranacci-Like sequence. The method of Mathematical Induction has been used.

Acknowledgement

The authors would like to thanks the anonymous referee for carefully reading the paper and for their comments.

References

[1] A.C.F. Bueno, Solving the k^{th} term of Natividad's Fibonacci-like sequence, *International Journal of Mathematics and Scientific Computing*, 3(1) (2013), 8.

[2] B. Singh, O.P. Sikhwal and S. Bhatanagar, Fibonacci-Like sequence and its properties, *Int. J. Contemp. Math. Sciences*, 5(18) (2010), 859-868.

[3] L.R. Natividad, Deriving a formula in solving Fibonacci-like sequence, *International Journal of Mathematics and Scientific Computing*, 1(1) (2011), 19-21.

[4] L.R. Natividad and P.B. Policarpio, A novel formula in solving Tribonacci-like sequence, *Gen. Math. Notes*, 17(1) (2013), 82-87.

[5] M.E. Waddill, The Tetranacci sequence and generalizations, *Fibonacci Quarterly*, 30(1) (1992), 9-19.

[6] M.E. Waddill, Some properties of the Tetranacci sequence, *Modulo M*, August 30(3) (1992), 232-238.